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ORIGINAL ARTICLE

Bianchi type-V bulk viscous string cosmological model in a self-creation theory of gravitation

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Abstract A spatially homogenous and anisotropic Bianchi type space-time is considered in the frame work of second self-creation theory of gravitation proposed by Barber (Gen. Relativ. Gravit. 14:117, 1982) in the presence of bulk viscous fluid containing one dimensional cosmic strings. Solving the field equations of this theory an exact cosmological model is obtained using some physically plausible conditions. It is observed that strings in this model do not survive. Some physical and kinematical properties of the model are also discussed.

Keywords Second self-creation cosmology · Bulk viscous string model · Bianchi-V space-time

1 Introduction

In recent years there have been several modifications of Einstein's theory in order to incorporate certain desirable features lacking in the theory. The most significant among them are the scalar-tensor theories of gravitation formulated by Brans and Dicke (1961) and Saez and Ballester (1986). Brans-Dicke (BD) theory of gravitation introduces an additional scalar field ϕ besides the metric tensor g_{ij} and dimensionless coupling constant ω. Subsequently, Barber (1982) has proposed two self-creation theories of gravitation by modifying BD theory and general relativity in which the mass of the universe seem to be created out of self-contained gravitational, scalar and matter fields. Barber (1982) has also pointed out that the first theory is not satisfactory since the equivalence principle is violated. Later Brans (1987), also, showed that the first theory is internally inconsistent. The second theory is a modification of general relativity to a variable G theory in which the scalar field ϕ does not gravitate, but simply divides the matter tensor acting as a reciprocal of gravitational constant. Also the scalar field couples to the trace of energy momentum tensor. The field equations in Barber's (1982) second self-creation theory of gravitation are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi \phi^{-1} T_{ij} \tag{1}$$

and

$$\Box\phi = \phi^{;k}_{;k} = \frac{8\pi}{3} \mu T \tag{2}$$

where μ is a coupling constant to be determined from experiments. The measurement of the deflection of light restricts the value of coupling constant to $|\mu| \leq 10^{-1}$, T_{ij} is the energy momentum tensor, $G = \phi^{-1}$, T is the trace of the energy momentum tensor, i.e. $T = T^i_i$ and the other symbols have their usual meaning.

It is well known that Friedmann-Robertson-Walker (FRW) line element describes, satisfactorily, the present day universe. However, the cosmic microwave back ground radiation experiments indicate that the universe at its early stages has anisotropic back ground (Bennett et al. 2003; Spergel et al. 2003) Hence, Bianchi models, which are spatially homogenous and anisotropic, are suitable to describe early stages of evolution of the universe. Barber (1982) and Soleng (1987) have discussed FRW models in Barber's second self-creation theory while Shanti and Rao (1991), Reddy and Naidu (2009, 2012) and Mahanta et al. (2013)

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and Reddy et al. (2014) have investigated Bianchi models in this theory.

Investigation of bulk viscous string cosmological models are significant to discuss the early stages of evolution of the universe. Bulk viscosity contributes negative pressure term stimulating repulsive gravity which overcomes attracting gravity of matter and gives an impetus for rapid expansion of the universe. Also, Murphy (1973) and Heller and Klimek (1975) have shown that the big bang singularity can be avoided by the introduction of bulk viscosity. The importance of bulk viscosity in the cosmological models is also discussed clearly by Mohanty and Mishra (2001). Barrow (1986), Pavon et al. (1991), Martens et al. (1995), Lima et al. (1993) and Mohanty and Pradhan (1992) are some of the authors who have investigated bulk viscous cosmological models in general relativity. Also, Johri (1994), Pimental (1994), Banerjee and Beesham (1996) and Singh et al. (1996) have discussed bulk viscous models in BD theory of gravitation.

Strings are line like structures which may arise due to spontaneous symmetry breaking during phase transition in the early universe. Massive strings would then serve as seeds for the large scale structures like galaxies and cluster of galaxies in the universe. Several important aspects of string both in general relativity and in modified theories of gravitation have been investigated in Bianchi type space-times by Stachel (1980), Letelier (1983), Vilenkin et al. (1987), Banerjee et al. (1990), Reddy (2003a, 2003b), Katore and Rane (2006), Sahoo (2008) and Tripathy et al. (2009).

Bulk viscous cosmic string models in general relativity and in modified theories of gravitation play a significant role in the early stages of evolution of the universe. Naidu et al. (2013a, 2013b), Reddy et al. (2013a, 2013b, 2013c, 2013d) and Vidyasagar et al. (2014a, 2014b) have studied Bianchi type bulk viscous cosmic string models in modified theories of gravitation.

Inspired by the above discussion and investigations, we study, here, Bianchi type-V universe in the presence of bulk viscous fluid containing one dimensional cosmic strings in Barber's second self-creation theory of gravitation. The paper is organized as follows: In Sect. 2, we derive the field equations of second self-creation theory of gravitation in Bianchi type-V space time in the presence of bulk viscous cosmic string source. Section 3 deals with the solutions of the field equations and the model. Section 4 is devoted to the physical discussion of the model. The last section contains some conclusions.

2 Metric and field equations

Bianchi type-V metric is given by

$$ds^2 = dt^2 - A^2 dx^2 - e^{2\alpha x}\left(B^2 dy^2 + C^2 dz^2\right) \tag{3}$$

where α is positive constant which can be taken as unity. Here A, B and C are functions of cosmic time t only.

The energy momentum tensor for a bulk viscous cosmic string source is given by

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p} g_{ij} - \lambda x_i x_j \tag{4}$$

and

$$\bar{p} = p - 3\zeta H \tag{5}$$

where ρ is the rest energy density of the fluid, $\zeta(t)$ is the coefficient of bulk viscosity, $3\zeta H$ is usually known as bulk viscous pressure, H is the Hubble's parameter, u^i is the four velocity of the fluid, p is isotropic pressure and x^i is the directions of string and λ is the string tension density. Also, $u^i = \delta^i_4$ is four velocity vector which satisfies

$$g_{ij} u^i u_j = -x^i x_j = 1; \qquad u^i x_i = 0 \tag{6}$$

Here we, also, consider ρ, $\bar{p}$, λ and ϕ as functions of time t only.

From Eqs. (4) and (6) we have

$$T^1_1 = \lambda - \bar{p}, \qquad T^2_2 = T^3_3 = -\bar{p}, \qquad T^4_4 = \rho \tag{7}$$

So that the trace of the energy momentum tensor is

$$T = T^i_i = T^1_1 + T^2_2 + T^3_3 + T^4_4 = \rho + \lambda - 3\bar{p} \tag{8}$$

Now, using commoving coordinate system, the field equations (1) and (2) for the metric (3) can be written, with the help of Eqs. (4) and (6)–(8), as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = -8\pi\phi^{-1}(\bar{p} - \lambda) \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -8\pi\phi^{-1}\bar{p} \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -8\pi\phi^{-1}\bar{p} \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3}{A^2} = 8\pi\phi^{-1}\rho \tag{12}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{13}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi}{3}\mu(\rho + \lambda - 3\bar{p}) \tag{14}$$

where an overhead dot denotes differentiation with respect to time t.

The following are the physical geometrical parameters to be used in solving the field equations of self-creation theory field equations for the space-time given by Eq. (3).

The average scale factor $a(t)$ of the Bianchi type-V space-time is defined as

$$a(t) = (ABC)^{\frac{1}{3}} \tag{15}$$

The spatial volume of the metric is

$$V = a^3 = ABC \tag{16}$$

The directional Hubble parameters are

$$H_1 = \frac{\dot{A}}{A}, \qquad H_2 = \frac{\dot{B}}{B}, \qquad H_3 = \frac{\dot{C}}{C} \tag{17}$$

The average Hubble parameter is

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{18}$$

The dynamical scalar expansion θ and shear scalar σ^2 are

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{19}$$

$$\begin{aligned}\sigma^2 &= \frac{1}{2}\sigma^{ij}\sigma_{ij} \\ &= \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right]\end{aligned} \tag{20}$$

The average anisotropic parameter Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{21}$$

where H_i $(i = 1, 2, 3)$ represent the directional Hubble parameters.

3 Solutions and the model

The field equations yield the following independent equations

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = 8\pi\phi^{-1}\lambda \tag{22}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} = 0 \tag{23}$$

$$A^2 = KBC \tag{24}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3}{A^2} = 8\pi\phi^{-1}\rho \tag{25}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{26}$$

where K is a constant of integration which can be taken as unity without loss of generality so that we have

$$A^2 = BC \tag{27}$$

Now Eqs. (22)–(27) are a system of five independent equations in seven unknowns $A, B, C, p, \rho, \lambda, \zeta$ and ϕ.

Hence to find a determinate solution we use the following conditions.

(i) The scalar expansion θ is proportional to the shear scalar σ^2 so that we have (Collins et al. 1980)

$$B = C^m \tag{28}$$

where $m \neq 1$ is a positive constant which takes care of anisotropy of space-time.

(ii) For a barotropic fluid, the combined effects of the proper pressure and the bulk viscous pressure can be expressed as

$$\begin{aligned}\bar{p} &= p - 3\zeta H = \varepsilon\rho \\ p &= \varepsilon_0\rho\end{aligned} \tag{29}$$

where $\varepsilon = \varepsilon_0 - \beta$ $(0 < \varepsilon_0 \leq 1)$ and $\varepsilon, \varepsilon_0$ and β are constants.

Now using Eqs. (28) and (27) in Eq. (23) we obtain the expressions for metric potentials as

$$\begin{aligned}A &= \left[\frac{3(m+1)}{2}(c_1 t + c_2)\right]^{\frac{1}{3}} \\ B &= \left[\frac{3(m+1)}{2}(c_1 t + c_2)\right]^{\frac{2m}{3(m+1)}} \\ C &= \left[\frac{3(m+1)}{2}(c_1 t + c_2)\right]^{\frac{2}{3(m+1)}}\end{aligned} \tag{30}$$

where c_1 and c_2 are constants of integration. The scalar field ϕ in the model is obtained from Eqs. (26)–(28) as

$$\phi = \frac{2\phi_0}{3(m+1)c_1}\log\left[\frac{3(m+1)}{2}(c_1 t + c_2)\right] + \psi_0 \tag{31}$$

where ψ_0 is a constant of integration which can be set equal to unity.

Now, by a suitable choice of integration constants (i.e. choosing $c_1 = 1, c_2 = 0$) we can now write the metric (3) with the help of Eqs. (30) as

$$\begin{aligned}ds^2 = dt^2 &- \left[\frac{3(m+1)}{2}t\right]^{\frac{2}{3}} dx^2 \\ &- e^{2x}\left[\left\{\frac{3(m+1)}{2}t\right\}^{\frac{4m}{3(m+1)}} dy^2\right.\end{aligned}$$

$$+\left\{\frac{3(m+1)}{2}t\right\}^{\frac{4}{3(m+1)}}dz^2\Bigg] \tag{32}$$

and the scalar field in the model is

$$\phi = \frac{2\phi_0}{3(m+1)}\log\left[\frac{3(m+1)}{2}t\right] \tag{33}$$

4 Some physical and kinematical properties of the model

Equation (32) along with Eq. (33) represents Bianchi type-V universe in Barber's second self-creation theory of gravitation with the following physical and geometrical parameters which are very important in the discussion of cosmological models of the universe.

- Spatial volume

$$V = \left[\frac{3(m+1)}{2}\right]t \tag{34}$$

- The mean Hubble parameter is

$$H = \frac{1}{3t} \tag{35}$$

- The scalar expansion is

$$\theta = 3H = \frac{1}{t} \tag{36}$$

- The shear scalar is

$$\sigma^2 = \frac{(m-1)^2}{(m+1)^2t^2} \tag{37}$$

- The average anisotropy parameter is

$$\Delta = \frac{2(m-1)^2}{3(m+1)^2} \tag{38}$$

From Eqs. (32) and (35) we get the energy density given by

$$8\pi\phi^{-1}\rho = \frac{2m^2+8m+2}{9(m+1)^2t^2} - 3\left[\left(\frac{3m+1}{2}\right)t\right]^{\frac{-2}{3}} \tag{39}$$

Also, the isotropic pressure, bulk viscous pressure, tension density in the string and coefficient of bulk viscosity are given by

$$8\pi\phi^{-1}p = \varepsilon_0\left[\frac{2m^2+8m+2}{9(m+1)^2t^2} - 3\left[\left(\frac{3m+1}{2}\right)t\right]^{\frac{-2}{3}}\right] \tag{40}$$

$$8\pi\phi^{-1}\bar{p} = \varepsilon\left[\frac{2m^2+8m+2}{9(m+1)^2t^2} - 3\left[\left(\frac{3m+1}{2}\right)t\right]^{\frac{-2}{3}}\right] \tag{41}$$

$$\lambda = 0 \tag{42}$$

and

$$8\pi\phi^{-1}\zeta = (\varepsilon_0-\varepsilon)\left[\frac{2m^2+8m+2}{9(m+1)^2t} - 3\left[\frac{3m+1}{2}\right]^{\frac{-2}{3}}t^{\frac{1}{3}}\right] \tag{43}$$

where ϕ is given by Eq. (33)

The deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 2$$

We shall now discuss the behavior of the universe given by Eq. (32) using the above results. The spatial volume increases with cosmic time. The Hubble's parameter, scalar expansion, shear scalar, the pressure energy density and the coefficient of bulk viscosity diverge at $t=0$ and vanish for a large t. We observe from Eq. (42) that we have 'zero tension' cosmic strings which in turn means cosmic strings in this universe do not survive. Hence Eq. (32) along with Eq. (33) represents Bianchi type bulk viscous universe in this theory. We can observe that when $m=1$, the universe becomes isotropic and shear free. The deceleration parameter $q>0$ and hence the universe decelerates initially. But there will be a transition from decelerated phase to accelerated phase at late times in accordance with the observations (Calledwell et al. 2006). Also it can be seen that the bulk viscosity decreases with time and leads to inflationary model (Padmanabhan and Chitre 1987).

5 Conclusions

Here we have discussed spatially homogenous and anisotropic Bianchi type-V universe in the presence of bulk viscous cosmic strings in the second self-creation theory of gravitation formulated by Barber (1982). To obtain a determinate solution of the non-linear field equations we have used the barotropic equation of state and the proportionality of the scalar expansion and shear scalar of the space-time. The model obtained represents bulk viscous cosmological model in this theory since we have observed, interestingly, that in this particular universe strings do not survive. The universe is spatially expanding, shearing and non-rotating and becomes isotropic at late times so that there is a transition from decelerated phase to accelerated phase which is quite in agreement with the present day observations.

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Compliance with ethical standards The authors declare that they have no potential conflict and will abide by the ethical standards of this journal.

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