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ORIGINAL ARTICLE

Gravitinos tunneling from traversable Lorentzian wormholes

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Abstract Recent research shows that Hawking radiation (HR) is also possible around the trapping horizon of a wormhole. In this article, we show that the HR of gravitino (spin-3/2) particles from the traversable Lorentzian wormholes (TLWH) reveals a negative Hawking temperature (HT). We first introduce the TLWH in the past outer trapping horizon geometry (POTHG). Next, we derive the Rarita-Schwinger equations (RSEs) for that geometry. Then, using both the Hamilton-Jacobi (HJ) ansätz and the WKB approximation in the quantum tunneling method, we obtain the probabilities of the emission/absorption modes. Finally, we derive the tunneling rate of the emitted gravitino particles, and succeed to read the HT of the TLWH.

Keywords Hawking radiation · Gravitino · Quantum tunneling · Lorentzian wormhole · Spin-3/2 particles

1 Introduction

An interesting phenomenon that corresponds to spontaneous emissions (as if a black body radiation) from a black hole (BH) is the HR. It is a semi-classical outcome of the quantum field theory (Hawking 1975, 1976). HR dramatically changed our way of looking to the BHs; they are not absolutely black and cold objects, rather they emit energy with a characteristic temperature: HT. Event horizon, where is an irreversible point (in classical manner) for any object including photons is the test-bed of the gedanken experiment for the HR. The studies concerning this phenomenon have been carrying on by using different methods. In particular, the quantum tunneling (Parikh and Wilczek 2000) of particles with different spins from the various BHs have gained momentum in the recent years (the reader may be referred to Vanzo et al. 2011; Jing 2003; Kerner and Mann 2006, 2008a,b; Yale and Mann 2009; Yang et al. 2014; Sharif and Javed 2013a,b; Kruglov 2014; Li and Ren 2008; Ran 2014; Chen et al. 2015a,b; Sakalli et al. 2012, 2014; Sakalli and Ovgun 2015a,b; Gecim and Sucu 2015; Jan and Gohar 2014; Singh et al. 2014; Dehghani 2015 and references cited therein). Recently, it has been shown that HR of the bosons with spin-0 (scalar particles) and spin-1 (vector particles) from the TLWH (Morris and Thorne 1988), which is a bridge or tunnel between different regions of the spacetime is possible by using the POTHG (Gonzalez-Diaz 2010; Martin-Moruno and Gonzalez-Diaz 2009; Sakalli and Ovgun 2015c). Wormhole has been extensively studied in different areas (Garattini 2015; Kuhfittig 2015; Rahaman et al. 2014a,b, 2015; Halilsoy et al. 2014). However, HT of the TLWH appears to be negative because of the phantom energy (exotic matter: the sum of the pressure and energy density is negative) that supports the broadness of the wormhole throat (Morris and Thorne 1988). In addition, it is a well-known fact that the virtual particle-antiparticle pairs are created near the horizon. In a BH spacetime the real particles with positive energy and temperature are emitted towards spatial infinity (Wald 1976). However, in the POTHG which is analog to the white hole geometry, the antiparticles come out from the horizon (Helou 2015a). In other words, our analysis predicts that the energy spectrum of the antiparticles leads to a negative temperature for the TLWH. For the subject of the white hole radiation, the reader may refer to Peltola and Makela (2006).

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As it is shown by Caldwell et al. (2003), the dark matter (DM) (Hurst et al. 2015) could have a phantom energy. In this regard, the phantom energy can keep apart every bound object until the Cosmos eventuates in the Big-Rip (Chimento and Lazkoz 2004). On the other hand, DM does not emit, reflect or absorb light, making it not just dark but entirely transparent. But if the DM particles strolling around a BH or a wormhole can produce gamma-rays would give a possibility to study the radiation of this mysterious matter (Liew 2013; Allahverdi et al. 2015). DM has many candidates, and gravitino (spin-3/2) is one of them (Kawasaki and Moroi 1995; Davidson et al. 2008). Gamma-ray decay of the gravitino DM has been very recently studied in Allahverdi et al. (2015). So, HR of the gravitinos from the BHs and/or wormholes could make an impact on the production of the DM. Behaviors of the gravitino's wave function are governed by the RSEs (Yale and Mann 2009; Corley 1999). So, our main motivation in this paper is to investigate the HR of the gravitino tunneling from the TLWH geometry. Using the RSEs and HJ method, we aim to regain the standard HT of the TLWH.

The structure of this paper is as follows. In Sect. 2, we introduce the $3+1$ dimensional TLWH (Martin-Moruno and Gonzalez-Diaz 2009) and analyzes the RSEs for the gravitino particles in the POTHG of the TLWH (Hayward 1994, 1998, 2009; Misner and Sharp 1964; Aminneborg et al. 1998). We show that the RSEs are separable when a suitable HJ änsatz is employed. Then the radial equation can be reduced to a coefficient matrix equation that makes us possible to compute the probabilities of the emission/absorption of the gravitinos. Finally, we calculate the tunneling rate of the radiated gravitinos, and retrieve the standard HT of the TLWH. We summarize and discuss our results in Sect. 3.

2 Quantum tunneling of gravitinos from 3 + 1 dimensional TLWH

For the wave equation of the gravitino (spin-3/2) particles, we start with the massless (the mass has no remarkable effect in the computation of the quantum tunneling Yale and Mann 2009) RSEs (Corley 1999; Majhi and Samanta 2010; Chen and Huang 2015; Chen et al. 2013):

$$i\gamma^{\nu}(D_{\nu})\Psi_{\mu}=0, \tag{1}$$

$$\gamma^{\mu}\Psi_{\mu}=0, \tag{2}$$

where $\Psi_{\mu}\equiv\Psi_{\mu a}$ is a vector-valued spinor and the γ^{μ} matrices satisfy $\{\gamma^{\mu},\gamma^{\nu}\}=2g^{\mu\nu}$. The first equation is the Dirac equation applied to every vector index of Ψ, while the second is a set of additional constraints to ensure that no ghost state propagates; that is, to ensure that Ψ represents only spin-3/2 fermions, with no spin-1/2 mixed states (Yale and Mann 2009; Majhi and Samanta 2010).

The covariant derivative obeys

$$D_{\mu}=\partial_{\mu}+\frac{i}{2}\Gamma_{\mu}^{\alpha\beta}J_{\alpha\beta}, \tag{3}$$

where

$$\begin{aligned}&\Gamma_{\mu}^{\alpha\beta}=g^{\beta\gamma}\Gamma_{\mu\gamma}^{\alpha},\\ &J_{\alpha\beta}=\frac{i}{4}[\gamma^{\alpha},\gamma^{\beta}],\\ &\{\gamma^{\alpha},\gamma^{\beta}\}=2g^{\alpha\beta}\times I.\end{aligned} \tag{4}$$

The metric of TLWH in the generalized retarded Eddington-Finkelstein coordinates (REFCs), which is the POTHG, is given by (Martin-Moruno and Gonzalez-Diaz 2009)

$$ds^2=-Fdu^2-2dudr+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right), \tag{5}$$

where $F=1-2M/r$. Misner-Sharp energy is represented by $M=\frac{1}{2}r(1-\partial^a r\partial_a r)$ which becomes $M=\frac{1}{2}r_h$ on the trapping horizon (r_h) (Misner and Sharp 1964). Marginal surfaces having $F(r_h)=0$ are the past marginal surfaces in the REFCs (Gonzalez-Diaz 2010).

For solving the RSEs, we use the following Dirac γ-matrices:

$$\begin{aligned}&\gamma^{u}=\frac{1}{\sqrt{F}}\begin{pmatrix}-i & -\sigma^3\\ -\sigma^3 & i\end{pmatrix}, \qquad \gamma^{r}=\sqrt{F}\begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix},\\ &\gamma^{\theta}=\frac{1}{r}\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}, \qquad \gamma^{\phi}=\frac{1}{r\sin\theta}\begin{pmatrix}0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix},\end{aligned} \tag{6}$$

where the Pauli matrices are given by

$$\begin{aligned}&\sigma^1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \qquad \sigma^2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\\ &\sigma^3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.\end{aligned} \tag{7}$$

Gravitino wave function (ψ) has two spin states [spin up (i.e. positive r-direction) and spin down (i.e. negative r-direction)]:

$$\psi_{\nu\uparrow}=(a_{\nu},0,c_{\nu},0)e^{\frac{i}{\hbar}S_{\uparrow}(u,r,\theta,\phi)}, \tag{8}$$

$$\psi_{\nu\downarrow}=(0,b_{\nu},0,d_{\nu})e^{\frac{i}{\hbar}S\downarrow(u,r,\theta,\phi)}, \tag{9}$$

where $S(u,r,\theta,\phi)$ denotes the gravitino action which is going to be expanded in powers of $\hbar$, and $a_{\nu},b_{\nu},c_{\nu},d_{\nu}$ are the arbitrary constants. Here we shall only consider the spin up case, since the spin down case is fully analogous with it. The action for the spin-up states can be chosen as follows

$$\begin{aligned}S_{\uparrow}(u,r,\theta,\phi)&=S_{\uparrow0}(u,r,\theta,\phi)+\hbar S_{\uparrow1}(u,r,\theta,\phi)\\ &\quad+\hbar^2S_{\uparrow2}(u,r,\theta,\phi)+\cdots.\end{aligned} \tag{10}$$

Therefore, the corresponding RSEs become

$$\frac{1}{\sqrt{F}}\left[(ia_0-c_0)(\partial_u S_{\uparrow0})\right]+\sqrt{F}(-c_0\partial_r S_{\uparrow0})=0, \tag{11}$$

$$\frac{1}{r\sin\theta}(-ic_0\partial_{\phi}S_{\uparrow0})+\frac{1}{r}(-c_0\partial_{\theta}S_{\uparrow0})=0, \tag{12}$$

$$\frac{1}{\sqrt{F}}\left[(-a_0 - ic_0)(\partial_u S_{\uparrow 0})\right] + \sqrt{F}(-a_0\partial_r S_{\uparrow 0}) = 0, \tag{13}$$

$$\frac{1}{r\sin\theta}(-ia_0\partial_\phi S_{\uparrow 0}) + \frac{1}{r}(-a_0\partial_\theta S_{\uparrow 0}) = 0, \tag{14}$$

with the constraints equations:

$$\frac{-a_0 + c_0}{\sqrt{F}} + \sqrt{F}c_1 + \frac{d_2}{r} - \frac{id_3}{r\sin\theta} = 0, \tag{15}$$

$$\frac{-d_0}{\sqrt{F}} - \sqrt{F}d_1 + \frac{c_2}{r} + \frac{ic_3}{r\sin\theta} = 0, \tag{16}$$

$$\frac{a_0}{\sqrt{F}} + \sqrt{F}a_1 + \frac{b_2}{r} - \frac{ib_3}{r\sin\theta} = 0, \tag{17}$$

$$\frac{-b_0 + id_0}{\sqrt{F}} - \sqrt{F}b_1 + \frac{a_2}{r} + \frac{ia_3}{r\sin\theta} = 0. \tag{18}$$

Equations (15–18) are not important here. Because these equations give an independent wave solution, so that they have no effect on the action (Yale and Mann 2009).

Afterwards, the separation of variables method is applied to the action $S_{\uparrow 0}(u, r, \theta, \phi)$:

$$S_{\uparrow 0} = Eu - W(r) - j_\theta\theta - j_\phi\phi + K, \tag{19}$$

where E and (j_θ, j_ϕ) are energy and angular constants, respectively. However, K is an arbitrary complex constant. Thus, Eqs. (11–14) reduce to

$$-\frac{ia_0}{\sqrt{F}}E + \frac{c_0}{\sqrt{F}}E - c_0\sqrt{F}W' = 0, \tag{20}$$

$$\frac{-c_0}{r}\left(j_\theta + \frac{i}{\sin\theta}j_\phi\right) = 0, \tag{21}$$

$$\frac{a_0}{\sqrt{F}}E + \frac{ic_0}{\sqrt{F}}E - a_0\sqrt{F}W' = 0, \tag{22}$$

$$\frac{-a_0}{r}\left(j_\theta + \frac{i}{\sin\theta}j_\phi\right) = 0. \tag{23}$$

Equations (21) and (23) are about the solutions of (j_θ, j_ϕ), and they do not have contribution to the tunneling rate. For this reason, we simply ignore them. Namely, the master equations for the tunneling rate are Eqs. (20) and (22). To analyze them, we first consider the case of $a_0 = ic_0$ (Hui-Ling and Shu-Zheng 2009). Using Eqs. (20) and (22), we now have a solution for $W(r)$ as

$$W_1 = \int \frac{2E}{F}dr. \tag{24}$$

The integrand has a simple pole at $r = r_h$. Choosing the contour as a half loop going around this pole from left to right and integrating, one obtains

$$W_1 = \frac{i2\pi E}{F'(r_h)} = \frac{i\pi E}{\kappa|_H}. \tag{25}$$

where $\kappa|_H = \partial_r F/2|_{r=r_H}$ is the surface gravity at the horizon. On the other hand, if one sets $a_0 = -ic_0$, this time Eqs. (20) and (22) admit the following solution for $W(r)$:

$$W_2 = 0. \tag{26}$$

Hence, we can derive the ingoing/outgoing imaginary action solutions as

$$\operatorname{Im} S_1 = \operatorname{Im} W_1 + \operatorname{Im} K, \tag{27}$$

$$\operatorname{Im} S_2 = \operatorname{Im} W_2 + \operatorname{Im} K = \operatorname{Im} K. \tag{28}$$

We can now set S_1 for the action of absorbed (ingoing) gravitinos. We can tune their probability:

$$\Gamma_{in} = \exp(-2\operatorname{Im} S_1), \tag{29}$$

to %100 by letting

$$K = -\frac{i\pi E}{\kappa|_H}. \tag{30}$$

Consequently, the probability of the emitted (outgoing) gravitinos becomes

$$\Gamma_{out} = \exp(-2\operatorname{Im} S_2) = \exp\left(\frac{2\pi E}{\kappa|_H}\right). \tag{31}$$

Recalling the definition of the tunneling rate:

$$\Gamma = \frac{\Gamma_{out}}{\Gamma_{in}} = \exp\left(\frac{2\pi E}{\kappa|_H}\right), \tag{32}$$

which is also equivalent to the Boltzmann factor: $\Gamma = \exp(-E/T)$, we read the HT of the TLWH as follows

$$T_H = -\frac{\kappa|_H}{2\pi}, \tag{33}$$

which is a negative temperature. This result implies that if the trapping horizon remains in the past outer region, the wormhole throat would have a negative temperature (Martin-Moruno and Gonzalez-Diaz 2009). The phantom energy, which is the special case of the exotic matter could be the reason of that negative temperature (Gonzalez-Diaz 2010; Martin-Moruno and Gonzalez-Diaz 2009; Sakalli and Ovgun 2015c; Gonzalez-Diaz and Siguenza 2004; Saridakis et al. 2009; Velten et al. 2013; Helou 2015b). On the other hand, when $K = 0$ in the action (19), it is possible to obtain the positive temperature: $T_H = +\frac{\kappa|_H}{2\pi}$. Although, the latter remark contradicts with the previous results (Martin-Moruno and Gonzalez-Diaz 2009; Sakalli and Ovgun 2015c) (and whence, one may easily get rid of the case of $K = 0$), however Hong and Kim (2006) showed that possibility of negative/positive temperature of the wormhole depends on the exotic matter distribution.

3 Conclusion

In this work, we have studied the HR of the gravitino particles from the TLWH in 3 + 1 dimensions. TLWH has been introduced in the POTHG. We have analyzed the RSEs in the background of the TLWH with the help of HJ method. The probabilities of the emitted/absorbed gravitino particles from the trapped horizon of the TLWH have been computed. After comparing the obtained tunneling rate with the

Boltzmann factor, we have recovered the standard HT of the TLWH, which is a negative temperature. This is the special condition in which the high-energy states are more occupied than lower-energy states (Braun et al. 2013). Another possibility of the negative temperature may originate from the exotic matter distribution of the wormhole (Hong and Kim 2006). Meanwhile, very recently it has been claimed by Helou (2015a) that HR does not occur in the POTHG. In fact, the latter debatable remark is based on the study of Firouzjaee and Ellis (2015) stating that cosmic matter flux may turn the HR off. On the other hand, Hayward show that switching off the radiation causes the wormhole to collapse to a Schwarzschild BH (Hayward 2002).

In summary, gravitinos can tunnel through wormhole [simply this can be thought as a wormhole with one entrance (BH) and one exit (white hole)]. In such a case, gravitinos tunnel from the BH with positive temperature, while they tunnel through the white hole with negative temperature. Thus our calculations are based on the exit of the wormhole, just as the white hole case. Besides, we have shown that positive temperature can be obtained by tuning the K-constant in the action (19) to zero. However, the latter result is a debatable issue, and it demands much deeper analysis. This will be our next venture in this line of study.

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