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ORIGINAL ARTICLE

Study of the nature of dark matter in halos of dwarf galaxies

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Abstract The kinematics of dwarf galaxies are strongly influenced by dark matter down to small galactocentric radii. So they are good candidates to investigate the nature of Dark Matter. In the present work we have carried out mass modeling of a number of recently observed dwarf galaxies Swaters et al. in Astron. Astrophys. 493:871, 2009. We have used a Navarro-Frenk-White (NFW) halo, Freeman disc along with a gaseous disc for modeling the observed rotation curves of those dwarf galaxies. For comparison we also used a Burkert halo, Freeman disc and gaseous disc. For both the scenario we have performed Kolmogorov–Smirnov (KS) test between the observed and predicted rotational velocity profiles. The tests are rejected for NFW halo almost in 50 per cent cases but they are accepted almost for all cases for Burkert halo, preferring a Burkert halo model generally for dwarf galaxies. The above results reveal a constant density core of dark matter (DM) in the halos of dwarf galaxies compared to a cuspy nature of NFW halo and a possible challenge to Λ-CDM scenario for the nature of dark matter in most of the dwarf galaxies.

Keywords Galaxies:dwarfs · Rotation curves:modeling

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1 Introduction

The scenario for structure formation in the Universe originates from the idea of an inflationary Universe along with small initial fluctuations in the cosmic mass distribution. The above model requires both baryonic as well as non-baryonic dark matter and some form of vacuum energy or quintessence. Unfortunately the nature of dark matter still remains unknown. The most popular candidate is cold dark matter (CDM) particles. These particles have negligible thermal velocity with respect to expansion velocity of the Universe. So it is expected that they have high phase space density. Numerical simulations show that most of the CDM particles having very low entropy preserve almost their initial extremely high phase space density and settle in the centers of virialized dark halos and thus create a cuspy density profile (Navarro et al. 1996; Fukushige and Makino 1997; Ghigna et al. 1998; Klypin et al. 1999; White and Springel 2000). Several newer DM only simulations (e.g., Stadel et al. 2009; Navarro et al. 2010) showed that the inner DM profile is better approximated by an Einasto profile than a NFW one. That profile has an inner slope flattening towards the center of the halo to values of alpha $= -0.8$, at 100 pc from the halo center (Stadel et al. 2009).

Modelling of dark matter halos with CDM particles seem to disagree with a number of observations. Primarily the number of subhalos around the Milky Way galaxy, identified as satellite galaxies is an order of magnitude smaller than predicted by CDM (Kauffmann et al. 1993; Klypin et al. 1999; Moore et al. 1999). Moreover the observed rotation curves (RC) of dwarf and low surface brightness galaxies (LSB) seem to indicate that their dark halos have constant density core rather than a cusy one (Flores and Primack 1994; Moore 1994; Burkert 1995; Burkert and Silk 1997; McGaugh and de Blok 1998; Dalcanton and Bernstein 2000; Firmani et al. 2001; Frusciante et al. 2012).

Due to these discrepancies numerous alternatives to the CDM paradigm have been proposed e.g. broken scale invariants (BSI) (Kamionkowski and Liddle 2000; White and Croft 2000), warm dark matter (Sommer-Larsen and Dolgov 2001; Hogan and Dalcanton 2000), scalar field dark matter (SFDM) (Hu and Peebles 2000; Peebles and Vilenkin 1999; Peebles 2000) and various sorts of self interacting or annihilating dark matter (SIDM) (Carlson et al. 1992; Bento et al. 2000). But most of these alternatives are unable to solve both the above mentioned problems simultaneously and often face their own problems (Moore et al. 1999; Hogan and Dalcanton 2000; Burkert 2000; Yoshida et al. 2000).

In some situations instead of alternative, the nature of dark matter has been modified. There are two models able to solve the cusp-core problem based on baryonic physics: (a) Several studies have suggested that reionization and supernova feed back can help to suppress star formation and also decrease central densities in low mass dark matter halos (e.g., Navarro et al. 1996; Mashchenko et al. 2006; Governato et al. 2010; Binney et al. 2001; Peñarrubia et al. 2012). But the above theories are hard to reconcile with the low ejection efficiencies found in more detailed hydrodynamic simulations (Mac Low and Ferrara 1999; Strickland and Stevens 2000). Hence we can at present say that the long term popular CDM theory is facing biggest challenge to date. But before abandoning CDM theory on the ground that it is inconsistent with observations it is advisable to examine observational evidences in large numbers and more closely with various types of mass modelling. Oman et al. (2015) showed that the galaxies obtained with this mechanism, but also changing the DM particles (e.g., to self-interacting DM, or WDM), do not really solve the problem. (b) The other baryonic mechanism able to solve the cusp-core problem is the scenario in which cores are formed through exchange of angular momentum between DM and baryons through dynamical friction (El-Zant et al. 2001, 2004; Romano-Diaz et al. 2008, 2009; Del Popolo and Kroupa 2009; Cole et al. 2011; Inoue and Saitoh 2011). The cusp-core problem can be solved not only changing the DM nature but also with theories of modified gravity (e.g., $f(R)$) theories.

The cusp-core problem is not only typical of galaxies but there are evidences (e.g., Newman et al. 2013a, 2013b; Martizzi et al. 2012; Del Popolo 2012a, 2014a) that the inner DM profiles of clusters are flatter than those predicted by the Λ-CDM model.

In the present work we have scrutinized the rotation curves for a large number of recently observed dwarf galaxies (Swaters et al. 2009). We have carried out mass modelling both (i) with a Freeman stellar disc, a thin gaseous disc, a NFW halo as well as (ii) with a Freeman stellar disc, gaseous disc, Burkert halo (Burkert 1995). In Sect. 2 the data used, have been discussed. Section 3 describes the mass modelling and Sect. 4 gives results and discussions.

2 Data set

Swaters et al. (2009) have derived rotation curves from HI observations for a sample of 62 late type dwarf galaxies. These galaxies have been observed as part of the Westerbork HI survey of Spiral and Irregular Galaxies (WHISP) project. For each galaxy the rotational velocity data have been listed at v_1, v_2, v_3, v_4, where v_i is the rotational velocity at ih ($i = 1, 2, 3, 4$) and h is the disc scale length in kpc. The rotational velocity has been given also at v_{last} where v_{last} is the rotational velocity at last observed point r_{last} from the center of the corresponding galaxy. Also depending upon the quality of rotation curves an index q is given whose values are 1, 2, 3, 4 respectively. ‘1’ means the galaxy has well determined velocity data (viz. Table 2 of Swaters et al. 2009). These late type dwarf galaxies have rotation curves similar to those of late type spiral galaxies i.e. the rotation curves rise up to two disc scale lengths and then start to flatten. The light distribution in the central part is closely related to the inner rise of the rotation curves. This indicates that stronger central concentration of light also has higher mass densities even for low surface brightness galaxies. We have used the values of v_i’s ($i = 1, 2, 3, 4$, ‘last’) for the galaxies for which $q = 1$ and all v_i’s are given. In this way we have considered a data set of 10 galaxies. They are listed in Table 1.

3 Mass modelling

We model each dwarf galaxy consisting of two components, namely the stellar and gaseous discs, embedded in a dark halo. The stellar component is modeled as exponential disc (Freeman 1970) with disc scale length h. Any bulge component is assumed to be negligible in terms of mass. For the dark halo we consider two different mass distributions: (i) an Navarro Frenk White (NFW) profile (Navarro et al. 1996) and (ii) the cored profile of the Halo Universal Rotation Curve (URCH) (Salucci et al. 2008). NFW profile is an outcome of numerical simulation for CDM particles for structure formation, whereas URCH potential by design fits the broad range of rotation curve shapes of spiral galaxies.

3.1 Mathematical model

We consider the motion of gas in the galactic disc with cylindrical coordinates (r, θ, z) and z-axis as the axis of rotational symmetry. The Navier–Stokes equation of motion along radial direction for gases in a viscous gaseous disc with a Freeman stellar disc embedded in a spherical dark halo is,

Table 1 List of dwarf galaxies and velocity data from Swaters et al. (2009) used in the present analysis

Serial no.	Galaxy name	v_1 ($\mathrm{km\,s^{-1}}$)	v_2 ($\mathrm{km\,s^{-1}}$)	v_3 ($\mathrm{km\,s^{-1}}$)	v_4 ($\mathrm{km\,s^{-1}}$)	v_{last} ($\mathrm{km\,s^{-1}}$)
1	UGC-731	50	63	73	74	74
2	UGC-2034	29	37	40	45	47
3	UGC-4499	38	58	66	71	74
4	UGC-6446	58	70	75	78	80
5	UGC-7399	55	79	89	92	109
6	UGC-7603	30	47	59	60	64
7	UGC-8490	48	66	74	77	78
8	UGC-9211	35	53	63	66	65
9	UGC-10310	55	79	89	92	109
10	UGC-12060	61	72	74	75	74

$$\frac{\partial u}{\partial t}+\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)u-\frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left[\frac{4}{3}\frac{\partial \chi}{\partial r}+\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}\right)\right] + F_r+F_D+F_H \tag{1}$$

where the velocity vector $\bar{v}=(u,v,w)$, μ is the coefficient of viscosity and

$$\chi=\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}+\frac{u}{r} \tag{2}$$

Due to rotational symmetry we have taken $\frac{\partial}{\partial\theta}\equiv 0$ for any variable. At the plane of the disc, $z=0$. At any (r,z) let $w=-zf(r)$, then $w=0$ at $z=0$ and $\frac{\partial w}{\partial z}\neq 0$ at $z=0$.

F_r, F_D and F_H are the gravitational forces due to gaseous disc, stellar disc and dark halo respectively.

Let us consider the density of the gaseous disc is $\rho(r)=Ar^{-n}$ where A and n are constants. Then the mass of gas within the gaseous disc of radius r is $M(r)=\int_0^r 2\pi r\rho dr=\int_0^r 2\pi r Ar^{-n}dr$

$$M(r)=\frac{2\pi Ar^{2-n}}{2-n}. \tag{3}$$

Hence $n<2$ (since, $M(r)>0$). Hence

$$F_r=-\frac{GM}{r^2}=-\frac{2\pi AG}{2-n}r^{-n} \tag{4}$$

$$F_D=-\frac{1}{2}\frac{GM_D}{R_D}(3.2)^2\left(\frac{r}{R_D^2}\right)[I_0K_0-I_1K_1]$$
(Freeman 1970) (5)

where I, K are modified Bessel functions (Watson 1931) and R_D is the disc scale length (here corresponding h for dwarf galaxies considered) and M_D is the stellar disc mass. In our problem we have taken M_D as a constant stellar disc mass of all galaxies i.e. $M_D\sim(3.5\pm1.8)\times10^8\ M_\odot$ (Frusciante et al. 2012) instead of a parameter. Here we have considered $3.2R_D$ as the periphery of the stellar disc (Salucci et al. 2008).

$$F_{H,NFW}=-\frac{v_{vir}^2 g(c)r_{vir}}{r^2 g(cx)} \quad \text{(Navarro et al. 1996)} \tag{6}$$

where $g(c)=[\ln(1+c)-\frac{c}{1+c}]^{-1}$, $x=\frac{r}{r_{vir}}$, $c=$ concentration parameter $=\frac{r_{vir}}{r_s}$, r_{vir} is the radius at which mean density is 200 times the critical density of closure and r_s is the core radius of the halo.

$$F_{H,URCH}=-\frac{6.4\rho_0 r_0^3}{r}\left[\ln\left(1+\frac{r}{r_0}\right)-\arctan\left(\frac{r}{r_0}\right)+\frac{1}{2}\ln\left(1+\frac{r^2}{r_0^2}\right)\right]$$
(Frusciante et al. 2012) (7)

The equation of continuity is,

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\left[\frac{\partial}{\partial r}(r\rho u)+\frac{\partial}{\partial z}(r\rho w)\right]=0 \tag{8}$$

The density distribution of the gas is given by $\rho\sim r^{-n}$, $0<n<1$. Then considering steady motion and using the above density law Eq. (8) becomes

$$\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}+\frac{u}{r}=\frac{nu}{r} \tag{9}$$

Then $\chi=\frac{nu}{r}$.

The equation of state of gas is,

$$p=\frac{R\rho T}{\mu'} \tag{10}$$

where R is the Riemann's constant, μ' is the mean molecular weight and T is the temperature. Substituting the values of w in Eq. (8) we get for steady state $\frac{\partial u}{\partial r}+\frac{1-n}{r}u(r)=f(r)$.

Integrating, $\frac{u(r)}{r^{n-1}}=\int\frac{f(r)}{r^{n-1}}dr+\text{const.}$

Since u decreases as r increases and finally tends to zero we assume, $f(r)=Kr^{-m}$, $m>0$. Then

$$u(r)=\frac{Kr^{1-m}}{2-m-n} \tag{11}$$

Table 2 List of model parameters for best choice

Serial no.	Galaxy name	c	r_{virial} (kpc)	r_s (kpc)	$\frac{\rho_0}{10^{-24}}$ ($\mathrm{gm\,cm^{-3}}$)	r_0 (kpc)	R_D (kpc)
1	UGC-731	8.9	95.5	10.73	4.1	3.1	1.65
2	UGC-2034	8.9	95.3	10.83	4.12	3.12	1.29
3	UGC-4499	8.4	95.4	11.36	4.13	3.13	1.49
4	UGC6446	8.7	95.1	10.93	4.14	3.14	1.87
5	UGC-7399	8.6	95.2	11.07	4.11	3.17	0.79
6	UGC-7603	8.5	94.9	11.2	4.17	3.15	0.9
7	UGC-8490	8.3	95.7	11.5	4.15	3.16	0.66
8	UGC-9211	8.2	95.6	12.0	4.2	3.2	1.32
9	UGC-10310	7.9	96.0	10.92	4.22	3.22	1.66
10	UGC-12060	8.88	96.0	10.75	4.0	3.16	1.76

where $m > 1$ and $2 - m - n \neq 0$. Hence for $w \sim zf(r)$, $u \sim rf(r)$, where $f(r) \sim r^{-m}$ (Ghosh et al. 1989). As a result, in the galactic disc (i.e. $z = 0$) Eq. (1) becomes

$$u\frac{\partial u}{\partial r} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{4}{3}\frac{\mu}{\rho}\frac{\partial \chi}{\partial r} - \frac{\mu}{\rho}\frac{\partial^2 w}{\partial z \partial r} + F_r + F_D + F_H \tag{12}$$

Substituting from (2), (4), (6), (9), (10), (11) and using $\chi = \frac{nu}{r}$ the rotation velocity for NFW halo is (from (12)),

$$\begin{aligned} v^2 = {} & \frac{K^2(1-m)}{(2-m-n)^2} r^{2-2m} - \frac{4\mu K m n}{3A(2-m-n)} r^{n-m} - \frac{nRT}{\mu'} \\ & + \frac{2\pi A G}{2-n} r^{1-n} + \left[\frac{1}{2}\frac{GM_D}{R_D}(3.2x)^2 (I_0K_0 - I_1K_1)\right] \\ & + \frac{v_{vir}^2 g(c)}{x g(cx)} \end{aligned} \tag{13}$$

where $x = \frac{r}{R_D}$. $R_D = h$ in our case. Similarly substituting from (2), (4), (7), (9), (11) and using $\chi = \frac{nu}{r}$ the rotation velocity for URCH halo is (from (12)),

$$\begin{aligned} v^2 = {} & \frac{K^2(1-m)}{(2-m-n)^2} r^{2-2m} - \frac{4\mu K m n}{3A(2-m-n)} r^{n-m} - \frac{nRT}{\mu'} \\ & + \frac{2\pi A G}{2-n} r^{1-n} + \left[\frac{1}{2}\frac{GM_D}{R_D}(3.2x)^2 (I_0K_0 - I_1K_1)\right] \\ & + \frac{6.4\rho_0 r_0^3}{r}\left[\ln\left(1 + \frac{r}{r_0}\right) - \arctan\left(\frac{r}{r_0}\right) \right. \\ & \left. + \frac{1}{2}\ln\left(1 + \frac{r^2}{r_0^2}\right)\right] \end{aligned} \tag{14}$$

3.2 Initial values of the parameters

For the present problem $m > 1$, $n < 1$, $m + n - 2 > 0$. At the periphery of the galactic disc (here at $3.2R_D$) we assume u is very very small i.e. $u \sim 1\ \mathrm{km\,s^{-1}}$ and density is the same as that of intergalactic medium (i.e. 10^{-2} H atom $\mathrm{cm^{-3}}$). Then we get $\frac{K}{(2-m-n)} \sim 3.2R_D \times 10^{26}$ (in cgs). We have assumed that the gaseous disc has the same size as that of the stellar disc and the density of the gaseous disc has the same value of inter galactic medium i.e. 10^{-2} H atom $\mathrm{cm^{-3}}$ at the periphery of the disc. This gives $A = 1.67 \times 10^{-26} \times (3.2R_D)^{1.2}$. The value of dynamical coefficient of viscosity μ for unionized hydrogen gas is, $\mu = 5.7 \times T^{\frac{1}{2}}\ \mathrm{g\,cm^{-1}\,s^{-1}}$ (Lang 1978). Hence for $T = 100$ K, $\mu = 5.7 \times 10^{-4}\ \mathrm{g\,cm^{-1}\,s^{-1}}$.

$$v_{vir} = 757 - 76.9c, \qquad R^2 = 93.7\ \% \tag{15}$$

Hence for an assumed value of c, we choose v_{vir} following Eq. (14) for reproducing the velocity curve. For URCH halo the parameters are ρ_0 and r_0 i.e. the density and size of the core. For the best choice of input parameters r_{vir}, c, r_s, ρ_0, r_0 (listed in Table 2) the Kolmogorov–Smirnov tests (KS) (Appendix) have been listed in Tables 3 and 4 respectively.

4 Results and discussion

Using Eqs. (13) and (14) we have computed the velocity curves under the corresponding mass modelling using both the NFW as well as URCH/Burkert halos. The predicted values are compared with the observed ones through Kolmogorov–Smirnov 2-sample test and the results are listed in Tables 3 and 4 for the best choice of model parameters, already mentioned in Sect. 3 (viz. Table 2). Representative predicted velocity curves with the observed values are shown in Figs. 1 and 2 for dwarf galaxy UGC 731. It is clear from Tables 3 and 4, that in case of NFW halo the tests are rejected in most of the cases (almost 50 %) but in case of Burkert halo all the tests are accepted except one. Hence Burkert halo is most likely for halos of dwarf galaxies.

Now we have mentioned that ΛCDM model is highly successful in describing the observations of the Universe and its large scale structure and evolution (Komatsu et al. 2011; Del Popolo 2007, 2013, 2014b). But it has some problems in describing the small scales. The main problems are

Table 3 KS tests for galaxies for NFW halos

Serial no. of galaxies	D	p-Value	Remark
1	0.2	1.0	Rejected
2	0.8	0.07937	Rejected
3	0.8	0.07937	Rejected
4	0.6	0.3571	Accepted
5	0.6	0.3571	Accepted
6	0.8	0.07937	Rejected
7	0.6	0.3571	Accepted
8	0.1	0.007937	Rejected
9	0.4	0.873	Accepted
10	0.4	0.873	Accepted

Table 4 KS tests for galaxies for URCH/Burkert halos

Serial no. of galaxies	D	p-Value	Remark
1	0.4	0.873	Accepted
2	0.8	0.07937	Rejected
3	0.4	0.873	Accepted
4	0.4	0.873	Accepted
5	0.6	0.3571	Accepted
6	0.4	0.873	Accepted
7	0.6	0.3571	Accepted
8	0.6	0.3571	Accepted
9	0.6	0.3571	Accepted
10	0.6	0.3291	Accepted

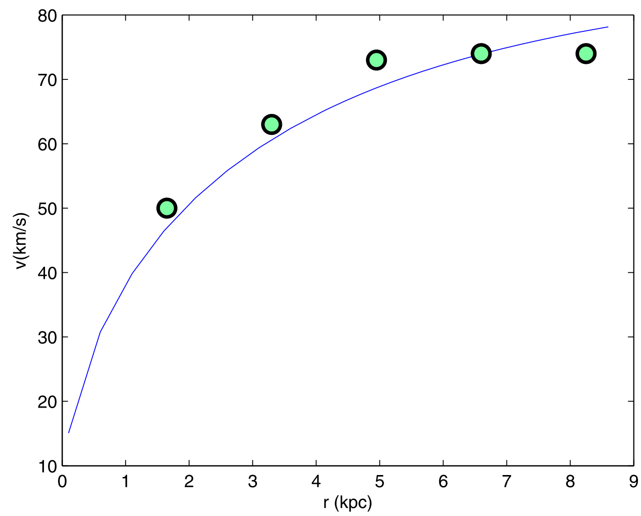

Fig. 1 The predicted (*blue curve*) and observed (*green circles*) rotation curves for UGC731 dwarf galaxy (Swaters et al. 2009) for a NFW halo

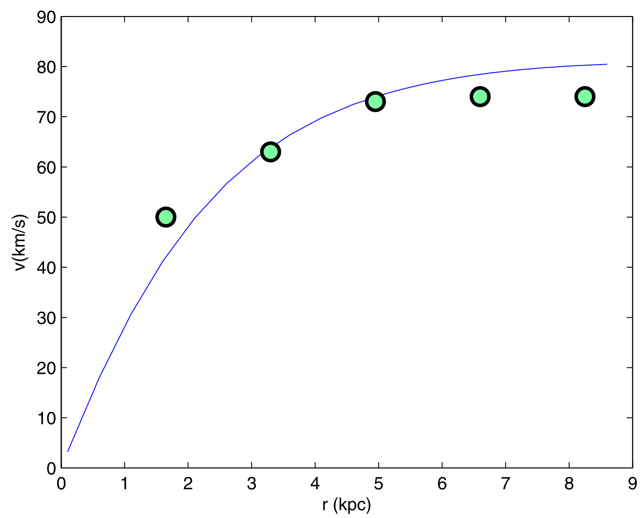

Fig. 2 The predicted (*blue curve*) and observed (*green circles*) rotation curves for UGC731 dwarf galaxy (Swaters et al. 2009) for a Burkert halo

(i) cusp/core problem i.e. ΛCDM particles show a cuspy nature of DM halo (NFW halo) though in practice a constant density core (Burkert/URCH halo) of dark matter in dwarf and LSB galaxies has the privilege over the former as is seen in our case (viz. Table 4). Secondly (ii) the 'missing satellite' problem i.e. the number of subhalos predicted in N-body simulation is much larger than the observed one (Moore et al. 1999) (iii) the angular momentum loss in Smooth Particle Hydrodynamics (SPH). Simulations give rise to discs of dwarf galaxies whose angular momentum is completely different from that simulated from ΛCDM particles. (iv) Another small scale problem of the ΛCDM model that has been revealed recently is the Too-Big-To-Fail problem (Boylan-Kolchin et al. 2011, 2012). Regarding cusp/core density profile, many theories have been developed. For example, Del Popolo and Hiotelis (2014) have suggested that presence of bulge has a great influence on the halo density profile. The presence of bulge (as in giant galaxy) instigates a stiffer density profile having slopes $\alpha \sim 0.65$–0.85 whereas in absence of bulges (as in dwarf and LSB galaxies) the slope is much flatter (viz. $\alpha \sim 0.55$). In a previous work Del Popolo (2012a, 2012b) has shown on the basis of a semi analytic model that DM density profile for dwarf galaxies is greatly influenced by tidal forces of neighboring galaxies. They have treated angular momentum and baryon fraction as parameters to understand the process. They have shown that dwarfs who suffered a smaller tidal torque are characterized by steeper density profile than dwarfs suffering higher torque. Also dwarfs having smaller baryon fraction have steeper profiles. Del Popolo (2009) has also suggested that the angular momentum and dynamical friction are able to overcome the competing effect of adiabatic contraction (AC) and this eliminates the cuspy profile to constant density core profile. In a work Burkert (1995) has shown that for a constant density core DM halo, the halo parameters are coupled through scaling relation (similar to Tully Fisher relation) which can be explained by ΛCDM model if one assumes that all the halos formed from density fluctuations with the same primordial amplitude. This result rules out a baryonic halo through violent dynamical process. Unified baryonic solutions to the quoted problems, and others mentioned above have been proposed. They are based on the action of baryons located in the inner parts of the haloes (Zolotov et al. 2012; Del Popolo 2014a, 2014b). Hence it is clear from the present work that a constant density core of the halos of dwarf galaxies are strongly preferred but the nature of this DM halo still remains to be explored in greater detail in future.

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Appendix: Kolmogorov–Smirnov one sample test

The test for goodness of fit usually involves examining a random sample from some unknown distribution in order to test the null hypothesis that the unknown distribution function is in fact a known, specified function. We usually use Kolmogorov–Smirnov test to check the normality assumption in Analysis of Variance. However it can be used for other continuous distributions also. A random sample $X_1, X_2, \dots, X_n$ is drawn from some population and is compared with $F^*(x)$ in some way to see if it is reasonable to say that $F^*(x)$ is the true distribution function of the random sample.

One logical way of comparing the random sample with $F^*(x)$ is by means of the empirical distribution function $S(x)$. The empirical distribution function $S(x)$ is a function of x, which equals the fraction of X_is that are less than or equal to x for each x. The empirical distribution function $S(x)$ is useful as an estimator of $F(x)$, the unknown distribution function of the X_is.

We can compare the empirical distribution function $S(x)$ with hypothesized distribution function $F^*(x)$ to see if there is good agreement. One of the simplest measures is the largest distance between the two functions $S(x)$ and $F^*(x)$, measured in a vertical direction. This is the statistic suggested by Kolmogorov (1956). Let the test statistic T be the greatest (denoted by "sup" for supremum) vertical distance between $S(x)$ and $F^*(x)$. In symbols we say

$$T = \sup_x \left| F^*(x) - S(x) \right|$$

For testing

$$H_0 : F(x) = F^*(x) \quad \text{for all } x$$

against

$$H_1 : F(x) \neq F^*(x) \quad \text{for at least one value of } x.$$

If T exceeds the 1-α quantile then we reject H_0 at the level of significance α. The approximate p-value can be found by interpolation.

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