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ORIGINAL ARTICLE

Five dimensional FRW cosmological models in a scalar-tensor theory of gravitation

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Abstract A five dimensional FRW cosmological space-time is considered in the scalar-tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A 113:467, 2003) in the presence of a perfect fluid source. Cosmological models corresponding to stiff fluid, disordered radiation, dust and false vacuum are obtained. Some physical and kinematical properties of each of the models are also studied.

Keywords FRW models · Scalar-tensor theory · Prefect fluid source

1 Introduction

Einstein's general theory of relativity has been successful in finding different cosmological models for the universe. However in recent years there have been several modifications of Einstein's theory of gravitation to incorporate certain desirable features lacking in the original theory. For example, Mach's principle is not fully incorporated into Einstein's theory through the field equations. Also, the recent scenario of early inflation and late time accelerated expansions of the universe (Rises et al. 1998; Perlmutter et al. 1999) is not explained by general theory of relativity. Hence to incorporate the above desirable features there have been several modifications of general relativity. Significant among them are scalar-tensor theories of gravitation formulated by Brans and Dicke (1961), and Saez and Ballester (1986) and modified theories of gravity like $f(R)$ theory of gravity formulated by Nojiri and Odinstov (2003) and $f(R,T)$ theory of gravity proposed by Harko et al. (2011). In recent years there has been an immense interest in constructing cosmological models of the universe to study the origin, physics and ultimate fate of the universe. In particular, cosmological models of Brans-Dicke and Saez-Ballester scalar-tensor theories of gravitation are attraction more and more attention of scientists.

Brans-Dicke (1961) theory is a well known competitor of Einstein's theory of gravitation. It is the simplest example of a scalar-tensor theory in which the gravitational interaction is mediated by a scalar field ϕ as well as the tensor field g_{ij} of the Einstein's theory. In this theory, the scalar field ϕ has the dimension of the universal gravitational constant. Subsequently, Saez and Ballester (1986) developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field ϕ in a simple manner. In spite of the dimensionless character of the scalar field an anti gravity regime appears in this theory. Also, this theory gives satisfactory description of the weak fields and suggests a possible way to solve 'missing matter' problem in non-flat FRW cosmologies. The field equations given by Saez and Ballester for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij} \quad (1)$$

and the scalar field ϕ satisfies the equation.

$$2\phi^n\phi_{;i}^{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \quad (2)$$

Also, we have

$$T^{ij}_{;j} = 0 \quad (3)$$

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which is a consequence of the field Eqs. (1) and (2).This equation, physically represents the conservation of the matter source. It also gives us equations of motion in this theory. This equation strengthens this theory of gravitation. Here ω and n are constants and comma and semicolon denote partial and covariant differentiation respectively. The other symbols have their usual meaning as in general relativity.

Singh and Agrawal (1991), Reddy and Rao (2001), Reddy et al. (2006), Mohanty and Sahoo (2004a, 2004b), Adhav et al. (2007), and Tripathy et al. (2008) are some of the authors who have studied several aspects of the Saiz-Ballester theory in four and five dimensional space. Friedman-Robertson-Walker (FRW) model describes spatially homogeneous and isotropic universe in four dimensions. The study of higher dimensional space time is important at early stages of evaluation of the universe because it has physical relevance to the early universe before the universe has undergone compactification transitions (Witten 1984; Appelquist et al. 1987; Chodos and Detweller 1980; Marchiano 1984), Reddy et al. (2013) and Naidu et al. (2013) have discussed five dimensional anisotropic Kaluza-Klein cosmological models in Brans-Dicke and Saez-Ballester scalar-tensor theories of gravitation respectively.

Motivated by the above investigations and discussions we focus our attention, in this paper, to explore the five dimensional FRW Cosmological models in Saez-Ballester theory of gravitation. The paper is organized as follows. In Sect. 2, we derive the field equations of Saez-Ballester theory for a five dimensional FRW space-time in the presence of perfect fluid source. In Sect. 3, some cosmological models representing stiff fluid, radiation, dust and inflation are obtained. A consolidated physical behavior of the models is studied in Sect. 4. The last section contains some conclusions.

2 Metric and field equations

We consider the five dimensional FRW metric in the form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \left(1-kr^2\right)d\varphi^2\right] \tag{4}$$

The non-vanishing components of the Einstein tensor for the metric (4) are

$$\begin{aligned} G_o^o &= 6\frac{(\dot{a})^2}{a^2} + \frac{6k}{a^2} \\ G_1^1 = G_2^2 = G_3^3 = G_4^4 &= 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + \frac{3k}{a^2} \end{aligned} \tag{5}$$

where an overhead dot indicates ordinary differentiation with respect to t and $k = +1, -1, 0$ for closed, open and flat models respectively.

The energy momentum tensor for a perfect fluid distribution is given by

$$T_j^i = (\rho + p)u^i u_j - p g_{ij}, \quad i, j = 0, 1, 2, 3, 4 \tag{6}$$

Here

$$u^i u_i = 1, \qquad u^i u_j = 0 \tag{7}$$

So that we have

$$\begin{aligned} &T_0^0 = \rho, \qquad T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p \\ &T = T_0^0 + T_1^1 + T_2^2 + T_3^3 + T_4^4 = \rho - 4p \end{aligned} \tag{8}$$

Using co moving coordinates, the field equations (1)–(3) with the help of Eqs. (5)–(8), for the metric (4), can be written as

$$6\frac{(\dot{a})^2}{a^2} + \frac{6k}{a^2} - \frac{\omega}{2}\frac{(\dot{\phi})^2}{\phi^2} = -8\pi\rho \tag{9}$$

$$3\frac{\ddot{a}}{a} + 3\frac{(\dot{a})^2}{a^2} + \frac{3k}{a^2} + \frac{\omega}{2}\frac{(\dot{\phi})^2}{\phi^2} = 8\pi p \tag{10}$$

$$\frac{\ddot{\phi}}{\phi} + 4\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{n}{2}\frac{(\dot{\phi})^2}{\phi^2} = 0 \tag{11}$$

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + p) = 0 \tag{12}$$

3 Solutions and the models

The field equations (9)–(12) constitute a system of three independent equations [Eq. (12) being the consequence of Eqs. (9)–(11)] in four unknowns a, ρ, p and ϕ. Hence we need an extra condition to get a determinate solution. Here we find five dimensional FRW flat models; so that we take $k = 0$. We find the solutions of the field equations for the following physically important cases.

3.1 Case (i): Zeldovich or Stiff fluid model

In this case $\rho = p$ so that the field equations yield the solution for the metric potential as

$$a^2 = 2\sqrt{a_0 t - t_0} \tag{13}$$

along with the scalar field given by

$$\phi^{\frac{n+2}{2}} = \left(\frac{n+2}{8}\right)\phi_0 \log(a_0 t - t_0) + \varphi_0 \tag{14}$$

where a_0, t_0, ϕ_0 and φ_0 are constants of integration.

Now by choosing suitably the coordinates and integration constants (i.e., $a_0 = 1$, $\phi_0 = 1$ and $\varphi_0 = 0$) we can write the five dimensional FRW flat models, in this particular case as

$$ds^2 = dt^2 - 2\sqrt{t - t_0}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + d\varphi^2\right] \tag{15}$$

with the scalar field ϕ given by

$$\phi^{\frac{n+2}{2}} = \left(\frac{n+2}{8}\right)\log(t - t_0) \tag{16}$$

Equation (15) represents five dimensional flat FRW cosmological model corresponding to stiff fluid model in Saez-Ballester scalar-tensor theory of gravitation with the following physical and kinematical parameters which is important in the physical discussion of cosmology.

The spatial volume of the model is given by

$$V = a^4 = 4(t - t_0) \tag{17}$$

Hubble's parameter H is given by

$$H = \frac{1}{4}\left(\frac{1}{t - t_0}\right) \tag{18}$$

The deceleration parameter q is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 3 \tag{19}$$

The pressure and density in this model are given by

$$8\pi\rho = 8\pi p = \frac{2\omega}{(n+2)^2(t-t_0)^2[\log(t-t_0)]^2} - \frac{3}{8(t-t_0)^2} \tag{20}$$

From the above results we observe that the volume scale factor of the universe increases with the growth of cosmic time which shows the spatial expansion of the universe. At the initial epoch, i.e., at $t = 0$, the Hubble parameter, the energy density and pressure assume infinitely large values where as with the growth of cosmic time they decrease to null values as $t \to \infty$. Also, it can be observed that the deceleration parameter of the model $q > o$. If $q < o$ the model accelerates and when $q > o$, the model decelerates in the standard way. Here the model in five dimensions decelerates in the standard way which is not in accordance with the present day scenario of accelerated expansion of the universe. However, the universe will accelerate in finite time after compactification transition and cosmic re-collapse where the universe in turn inflates, decelerates and then accelerates (Nojiri and Odinstov 2003).

3.2 Case (ii): $\rho = 4p$ (radiating model in five dimensions)

In this particular case the field equations (9)–(11) reduce to (for $k = 0$) two independent equations

$$6\frac{(\dot{a})^2}{a^2} - \omega\frac{(\dot{\phi})^2}{\phi^2} = -32\pi p \tag{21}$$

$$3\frac{\ddot{a}}{a} + 3\frac{(\dot{a})^2}{a^2} + \frac{\omega}{2}\frac{(\dot{\phi})^2}{\phi^2} = 8\pi p \tag{22}$$

To obtain a determinate solution of the above field equations we use the special law of variation for the Hubble's parameter proposed by Berman (1983) which is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \text{constant} \tag{23}$$

where q is the constant deceleration parameter of the model of the universe.

The solutions of Eq. (23) yields

$$a(t) = (ct + d)^{\frac{1}{1+q}} \tag{24}$$

where $c \neq 0$ and d are constants of integration. Also Eq. (24) implies $1 + q > 0$. Now with the help of Eq. (24) the field equations (21)–(22) yield the metric coefficients as (after redefining the constants)

$$a(t) = (t - t_0)^{\frac{1}{1+q}} \tag{25}$$

along with the scalar field given by

$$\phi^{\frac{n+2}{2}} = \left(\frac{n+2}{2}\right)\left(\frac{1+q}{q-3}\right)(t - t_0)^{\frac{q-3}{1+q}} \tag{26}$$

Now the metric (4) (with $k = 0$), in this case, can be written as

$$ds^2 = dt^2 - (t - t_0)^{\frac{2}{1+q}} \times \left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + d\varphi^2\right] \tag{27}$$

Equation (27) along with Eq. (26) represent a flat FRW five dimensional cosmological model in Saez-Ballester scalar-tensor theory of gravitation which can be considered as analogous to radiating model of general relativity in four dimensions. This universe describes the early stages of evolution of the universe with the following physical and kinematical parameters of the model which are important in the physical discussion of cosmology.

The spatial volume V is given by

$$V = a^4(t) = (t - t_0)^{\frac{4}{1+q}} \tag{28}$$

Hubble's parameter is

$$H = \frac{\dot{a}}{a} = \frac{1}{(1+q)(t-t_0)} \tag{29}$$

The pressure and density, in this universe, are given by

$$\rho = 4p = \frac{1}{8\pi}\left[\frac{2\omega(q-3)^2 - 6(n+2)^2}{(n+2)^2(1+q)^2(t-t_0)^2}\right] \tag{30}$$

A study of the above results reveals that the universe in this case also behaves in the similar way as in the case (ii) of stiff fluid universe since $1+q>0$.

3.3 Case (iii): $\rho = 0$ (dust model)

In this particular case, for $k=0$, Eqs. (9)–(11) yield the independent equations

$$6\frac{(\dot{a})^2}{a^2} - \frac{\omega}{2}\frac{(\dot{a})^2}{\phi^2} = -8\pi\rho \tag{31}$$

$$3\frac{\ddot{a}}{a} + 3\frac{(\dot{a})^2}{a^2} + \frac{\omega}{2}\frac{(\dot{\phi})^2}{\phi^2} = 0 \tag{32}$$

Proceeding in the similar way as in the case (ii) we obtain the same model given by Eq. (27) with the energy density given by Eq. (30) and the scalar field given by Eq. (26). The physical behavior of the universe is the same as in case (ii) except that $p=0$ here.

3.4 Case (iv): $\rho + p = 0$ (false vacuum or inflationary model)

In this particular case field equations (9)–(11) reduce to

$$3\frac{\ddot{a}}{a} - 3\frac{(\dot{a})^2}{a^2} + \omega\frac{(\dot{\phi})^2}{\phi^2} = 0 \tag{33}$$

Proceeding in the similar way as in the case (ii) we obtain the same model given by Eq. (27) along with the scalar field given by Eq. (26) satisfying the relation [by the use of Eqs. (25) and (26) in Eq. (33)]

$$4\omega(q-3)^2 - 3(n+2)^2(q+1) = 0 \tag{34}$$

The energy density and pressure in this model are given by

$$\rho = -p = \frac{1}{8\pi}\left[\frac{2\omega(q-3)^2 - 6(n+2)^2}{(n+2)^2(1+q)^2(t-t_0)^2}\right] \tag{35}$$

It can be observed from Eq. (35) that p is negative which shows that the universe is in an accelerating mode which is in accordance with the recent scenario of the accelerated expansion of the universe with exotic pressure.

4 Physical discussion

The model given by Eq. (27) represents a five dimensional FRW flat model for radiating, dust and false vacuum in the scalar-tensor theory of gravitation proposed by Saez and Ballester. For all the above models spatial volume is given by Eq. (28) which shows spatial expansion of the models with the increase in cosmic time t. The average Hubble parameter in each case is given by Eq. (29) which diverges at $t=t_0$ and will vanish for infinitely large t. It may be observed that the pressure and density in all the models diverge at $t=t_0$ and vanish as $t\to\infty$. It is interesting to note that in the case of false vacuum the pressure becomes negative which shows that the universe, in this particular case, is in an accelerating mode. This is in accordance with the recent scenario of early inflation and late time acceleration of the universe with exotic pressure. It may also be observed that in the case of stiff fluid model the deceleration parameter is positive and hence, in this case, the model decelerates in the standard way.

5 Conclusions

In this paper, we have considered a five dimensional FRW space-time in scalar-tensor theory of gravitation proposed by Saez and Ballester (1986) in the presence of perfect fluid distribution. Solving the field equations of this theory we have presented cosmological models corresponding to stiff fluid, disordered radiation, dust and false vacuum in a five dimensional flat FRW space-time. It is observed that the model in each case is free from initial singularity. The models obtained here will help to understand Saez-Ballester cosmology in five dimensions.

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