Astrophys Space Sci (2015) 357:105
DOI 10.1007/s10509-015-2332-5

ORIGINAL ARTICLE

Dirac and scalar particles tunnelling from topological massive warped-AdS_3 black hole

G. Gecim[1] · Y. Sucu[1]

Received: 28 October 2014 / Accepted: 26 March 2015 / Published online: 13 May 2015

Abstract We investigate the Dirac and scalar particles tunnelling as a radiation of Warped AdS_3 black holes in Topological Massive Gravity. Using Hamilton-Jacobi method, we discuss tunnelling probability and Hawking temperature of the spin-1/2 and spin-0 particles for the black hole. We observe the tunnelling probability and Hawking temperature to be same for the spin-1/2 and spin-0. We show that the tunnelling process may occur, for both Dirac and scalar particles.

Keywords Hawking radiation · Particle tunnelling · Warped AdS_3 black holes

1 Introduction

A self-consistent quantum gravity theory hasn't been constructed yet. Therefore, the quantum mechanical properties of a classical gravitational field is studied by the quantum mechanical behaviour of a physical system effected from it. In particular, thanks to the extension of standard Quantum theory to curved spacetime, some events, such as particle creation and thermal radiation of a black hole, can be predicted. Moreover, the black holes as the most popular concepts of the classical gravity are just understood by the quantum mechanical concepts. From this point of view, the solutions of the relativistic quantum mechanical wave equations in a gravitational background became an important tool for getting information about its nature (Parker 1968, 1969, 1971). For this reason, the relativistic quantum mechanical wave equations in a curved spacetime background have been extensively studied (Brill and Wheeler 1957; Chandrasekhar 1976; Barut and Duru 1987; Sucu and Unal 2004).

The nature of black holes have been started to be understood by thermodynamical and quantum mechanical concepts since 1970 (Greif 1969; Carter 1972; Bekenstein 1973; Hawking 1974, 1975, 1976). Among these concepts, especially, thermal radiation, known as Hawking radiation in the literature, has been investigated as a quantum tunnelling effect of the relativistic particles from a black hole (Shankaranarayanan et al. 2001; Srinivasan and Padmanabhan 1999; Vagenas 2002; Kraus and Wilczek 1995a,b; Parikh and Wilczek 2000; Vagenas 2001; Arzano et al. 2005; Kerner and Mann 2006, 2008a). Thanks to the studies, a black hole temperature, which is called hawking temperature in the literature, is related to the black hole surface gravity. Therefore, the Hawking temperature becomes an important concept to investigate a black hole physics. Since then, in the framework of standard Einstein general relativity, the Hawking radiation as a tunnelling process of the particles from various black holes has been studied, extensively, in the literature in both $3+1$ and $2+1$ dimensional (Chen et al. 2008, 2009; Zhang and Zhao 2006; Li and Ren 2008; Li et al. 2006; Gecim and Sucu 2013; Qi 2013). On the other hand, Kerner and Mann extended the tunnelling process to include the Dirac particle emission from a $3+1$ dimensional black hole (Kerner and Mann 2006, 2008a). Also, Ren and Li considered the Dirac particles' tunnelling process to investigate the Hawking radiation for the $2+1$-dimensional BTZ black hole using the tunneling method (Li and Ren 2008). The particle tunnelling process in all these studies give useful information about the mathematical and physical properties of the black holes. In the similar way, the Hawking radiation is

✉ Y. Sucu
ysucu@akdeniz.edu.tr

G. Gecim
ganimgecim@akdeniz.edu.tr

[1] Department of Physics, Faculty of Science, Akdeniz University, 07058 Antalya, Turkey

used to discuss the properties of a black hole in the context of modified gravity theories (Mirekhtiary and Sakalli 2014; Slavov and Yazadjiev 2012; Zeng et al. 2008). As an example, Gecim and Sucu discussed Hawking radiations for both Dirac and scalar particles from the New-type black hole in the framework of $2+1$ dimensional New Massive Gravity theory (Gecim and Sucu 2013). However, according to the method, both particles probe the black hole in same way. Also, in the context of modified gravity theories, Qi investigates the fermion tunnelling radiation from the static Lifshitz black hole in $2+1$ dimensional New Massive Gravity theory, and from New Class Black Holes in $3+1$ Einstein-Gauss-Bonnet Gravity (Qi 2013).

The $(2+1)$ dimensional gravitational models provide a suitable area to investigate the quantum effects of the gravity (Carlip 1998; Deser et al. 1984; Witten 1988, 1989). Among these, Topologically Massive Gravity as an interesting modified three-dimensional gravitation theory is formed by adding a Cern-Simons term to the standard Einstein-Hilbert action (Deser et al. 1982). With this term, the gravity theory has gained both physical and mathematical interesting properties. However, in contrast to other gravitational theories, the graviton becomes a massive particle (Aliev and Nutku 1996; Carlip et al. 2008; Clement 1992, 2009).

The warped AdS_3 black holes for the solution of the Topological massive gravity is given by the following metric (Moussa et al. 2003).

$$ds^2 = N(r)^2dt^2 - \frac{1}{N(r)^2F(r)^2}dr^2 - F(r)^2\left[d\phi + N^{\phi}(r)dt\right]^2 \tag{1}$$

The abbreviations used in here are as follows;

$$F(r)^2 = r^2 + 4\omega r + 3\omega^2 + \frac{r_0^2}{3}$$

$$N(r)^2 = \frac{r^2 - r_0^2}{F(r)^2}, \qquad N^{\phi}(r) = -\frac{2r + 3\omega}{F(r)^2}$$

The Warped-AdS_3 Black holes have two horizon at $r = \mp r_0$. The parameters ω and r_0 are related to the physical parameters of the black hole, mass and angular momentum (Moussa et al. 2003). For the metric, the surface gravity is calculated by classical (standard) method as,

$$\kappa = \frac{1}{2}\left[F(r)\frac{\partial}{\partial r}\left[N^2(r)\right]\right]_{r=r_0} \tag{2}$$

and thus,

$$\kappa = \sqrt{3}\left(\frac{r_0}{2r_0 + 3\omega}\right)$$

The Hawking temperature, T_H, is defined in terms of the surface gravity as $T_H = \frac{\hbar\kappa}{2\pi}$ and, for the black hole, it is given as follows

$$T_H = \frac{\hbar\sqrt{3}}{2\pi}\left(\frac{r_0}{2r_0 + 3\omega}\right).$$

The Warped-AdS_3 Black hole becomes extremal at $r_0 = 0$. According to (2) the surface gravity becomes zero in the extremal case, hence the Hawking temperature of the extremal black hole is zero. Additionally, in the extremal case, the black hole has a double horizon at $r = 0$. Moreover, this result does not depend on parameter ω. In an even more special case ($\omega = r_0 = 0$), the metric (1) is reduced to the horizonless metric that is characterized as the ground state or 'vacuum' of the black-hole (Moussa et al. 2003).

To understand the quantum mechanical properties of the black hole, we find the probability of tunnelling and Hawking temperature by using the solutions of the relativistic quantum mechanical wave equation for the scalar and Dirac particles.

The organization of this work are follows. In Sect. 2, we write the Dirac equation in Warped-AdS_3 Black holes background, and calculate the tunnelling possibility of the Dirac particle by using the semi-classical method. Also, we find Hawking temperature. In Sect. 3, Klein-Gordon equation is rewritten in Warped-AdS_3 Black hole spacetime. The tunnelling probability of scalar particles from the black hole and their Hawking temperature is also calculated. Finally, we evaluate and summarize the results.

2 Tunnelling of Dirac particles

To investigate tunnelling the Dirac particles from Warped-AdS_3 Black hole, we write Dirac equation in $(2+1)$ dimensional spacetime in the following representation (Sucu and Unal 2007),

$$\left\{i\overline{\sigma}^{\mu}(x)\left[\partial_{\mu} - \Gamma_{\mu}(x)\right]\right\}\Psi(x) = \frac{m_0}{\hbar}\Psi(x). \tag{3}$$

In this representation; Dirac spinor, $\Psi(x)$, has only two components corresponding positive and negative energy eigenstates which has only one spin polarization. $\overline{\sigma}^{\mu}(x)$ are the spacetime depended Dirac matrices and they are written in terms of constant Dirac matrices, $\overline{\sigma}^i$, by using triads, $e^{\mu}_{(i)}(x)$, as follows

$$\overline{\sigma}^{\mu}(x) = e^{\mu}_{(i)}(x)\overline{\sigma}^i, \tag{4}$$

where $\overline{\sigma}^i$ are Dirac matrices in a flat spacetime and given as

$$\overline{\sigma}^i = \left(\overline{\sigma}^0, \overline{\sigma}^1, \overline{\sigma}^2\right) \tag{5}$$

with

$$\overline{\sigma}^0 = \sigma^3, \qquad \overline{\sigma}^1 = i\sigma^1, \qquad \overline{\sigma}^2 = i\sigma^2, \tag{6}$$

where σ^1, σ^2 and σ^3 Pauli matrices, and $\Gamma_{\mu}(x)$ are the spin affine connection by the following definition,

$$\Gamma_{\mu}(x) = \frac{1}{4}g_{\lambda\alpha}\left(e^i_{\nu,\mu}e^{\alpha}_i - \Gamma^{\alpha}_{\nu\mu}\right)s^{\lambda\nu}(x). \tag{7}$$

Here, $\Gamma^{\alpha}_{\nu\mu}$ is Christoffell symbol, and $g_{\mu\nu}(x)$ is spacetime depended metric tensor and it is given in term of triads as follows,

$$g_{\mu\nu}(x) = e^{(i)}_{\mu}(x) e^{(j)}_{\nu}(x) \eta_{(i)(j)}, \tag{8}$$

where μ and ν are curved spacetime indices running from 0 to 2. i and j are flat spacetime indices running from 0 to 2 and $\eta_{(i)(j)}$ is the metric of $(2+1)$ dimensional Minkowski spacetime, with signature $(1, -1, -1)$, and $s^{\lambda\nu}(x)$ is a spin operator given by

$$s^{\lambda\nu}(x) = \frac{1}{2}\left[\overline{\sigma}^{\lambda}(x), \overline{\sigma}^{\nu}(x)\right]. \tag{9}$$

From Eqs. (1) and (8), the triads of $e^{\alpha}_{(i)}$ are written as;

$$e^{\mu}_{(0)} = \left(\frac{1}{N}, 0, -\frac{N^{\phi}}{N}\right)$$
$$e^{\mu}_{(1)} = (0, FN, 0)$$
$$e^{\mu}_{(2)} = \left(0, 0, \frac{1}{F}\right)$$

The tunnelling probability for the classically forbidden trajectory from inside to outside of the black hole horizon is given by

$$\Gamma = e^{-\frac{2}{\hbar}\operatorname{Im} S} \tag{10}$$

where S is the classical action function of a particle trajectory (Kerner and Mann 2006; Li and Ren 2008; Di Criscienzo and Vanzo 2008; Volovik 1992, 1999, 2003). Therefore, in order to discuss tunneling probability, one needs to calculate the imaginary part of a classical action function, S, in regards to the tunnelling probability. To investigate the tunnelling probability of a Dirac particle from the black hole, we use the following ansatz for the wave function in Eq. (3);

$$\Psi(x) = \exp\left(\frac{i}{\hbar} S(t, r, \phi)\right) \begin{pmatrix} A(t, r, \phi) \\ B(t, r, \phi) \end{pmatrix} \tag{11}$$

where $A(t, r, \phi)$ and $B(t, r, \phi)$ are functions of space-time (Li and Ren 2008; Di Criscienzo and Vanzo 2008). To apply the Hamilton-Jacobi method, we insert Eq. (11) in the Dirac equation given by Eq. (3). Dividing by the exponential term and neglecting the terms with $\hbar$, we derive the following two coupled differential equations.

$$\begin{aligned} &A\left[m_0 N(r) + \frac{\partial S}{\partial t} - N^{\phi}(r)\frac{\partial S}{\partial \phi}\right] \\ &\quad + B\left[iF(r)N(r)^2\frac{\partial S}{\partial r} + \frac{N(r)}{F(r)}\frac{\partial S}{\partial \phi}\right] = 0 \\ &A\left[iF(r)N(r)^2\frac{\partial S}{\partial r} - \frac{N(r)}{F(r)}\frac{\partial S}{\partial \phi}\right] \\ &\quad + B\left[m_0 N(r) - \frac{\partial S}{\partial t} + N^{\phi}(r)\frac{\partial S}{\partial \phi}\right] = 0. \end{aligned} \tag{12}$$

These two equations have nontrivial solutions for $A(t, r, \phi)$ and $B(t, r, \phi)$ when the determinant of the coefficient matrix is vanished. Accordingly,

$$\begin{aligned} &F(r)^2\left(\frac{\partial S}{\partial t}\right)^2 - 2F(r)^2 N^{\phi}(r)\left(\frac{\partial S}{\partial t}\right)\left(\frac{\partial S}{\partial \phi}\right) \\ &\quad + \left(F(r)^2 N^{\phi}(r)^2 - N(r)^2\right)\left(\frac{\partial S}{\partial \phi}\right)^2 \\ &\quad - N(r)^4 F(r)^4\left(\frac{\partial S}{\partial r}\right)^2 - N(r)^2 F(r)^2 m_0^2 = 0. \end{aligned} \tag{13}$$

As (∂_t) and (∂_ϕ) are two killing vectors we can separate $S(t, r, \phi)$ to the variables as follows

$$S(t, r, \phi) = -Et + j\phi + K(r) + C, \tag{14}$$

where E and j are the energy and angular momentum of a Dirac particle, respectively, and C is a complex constant. Inserting Eq. (14) in Eq. (13) and integrating the radial function, $K(r)$, by using a contour that has a semicircle around the pole at the horizon $r = r_0$ we get

$$\begin{aligned} K_{\pm}(r) &= \pm \int \frac{\sqrt{(E + jN^{\phi}(r))^2 - N(r)^2(m_0^2 + \frac{j^2}{F(r)^2})}}{F(r)N(r)^2} dr \\ &= \pm \frac{i\pi\sqrt{3}(E - j\Omega_+)}{6r_0}(2r_0 + 3\omega) \end{aligned} \tag{15}$$

where $K_+(r)$ is outgoing and $K_-(r)$ is incoming solutions of radial part and the $K_{\pm}(r)$ values stem from the first order poles of the complex integral. Here, $\Omega_+ = -N^{\phi}(r_0) = \frac{3}{2r_0+3\omega}$ is the angular velocity of the outer event horizon of the black hole. The total imaginary part of the action is $\operatorname{Im} S(t, r, \phi) = \operatorname{Im} K_{\pm}(r) = \operatorname{Im} K_+(r) - \operatorname{Im} K_-(r)$ (Gecim and Sucu 2013; Li 2010). Hence, the two kind probabilities of crossing the outer horizon, from outside to inside or from inside to outside, are given by

$$\begin{aligned} P_{out} &= \exp\left[-\frac{2}{\hbar}\operatorname{Im} K_+(r)\right] \\ P_{in} &= \exp\left[-\frac{2}{\hbar}\operatorname{Im} K_-(r)\right]. \end{aligned} \tag{16}$$

From Eq. (15), we find that $\operatorname{Im} K_+(r) = -\operatorname{Im} K_-(r)$. And, the tunneling probability of the Dirac particle from the outer event horizon is given by Kerner and Mann (2006, 2008b), Di Criscienzo and Vanzo (2008),

$$\Gamma = \frac{P_{out}}{P_{in}} = \exp\left[-\frac{2\pi(E - j\Omega_+)(2r_0 + 3\omega)}{\hbar\sqrt{3}r_0}\right] \tag{17}$$

If one expands the classical action in terms of the particle energy, the Hawking temperature is obtained at the lowest order (linear order). So, we can write

$$\Gamma = e^{-\frac{2}{\hbar}\operatorname{Im} S} = e^{-\beta(E - j\Omega_+)} \tag{18}$$

where β is the inverse temperature of the outer horizon. Where, the Hawking temperature is given as follows

$$T_H = \frac{\hbar\sqrt{3}}{2\pi}\left(\frac{r_0}{2r_0+3\omega}\right), \tag{19}$$

where ω is a parameter in terms of r_0. This result is consistent with the result of the classical gravity.

The Warped-AdS$_3$ Black hole metric given in Eq. (1) shows different characteristic properties for various values of the ω parameter, both mathematically and physically. As mentioned in the previous section, the Warped-AdS$_3$ Black hole has two horizons located at $r = \pm r_0$, and hence their perimeters are given as $A_\pm = 2\pi r_\pm = \frac{2\pi|2r_0 \pm 3\omega|}{\sqrt{3}}$. In particular, if $\omega \neq \mp\frac{2r_0}{3}$, the metric can be extended analytically through the these horizons by the Kruskal coordinates. And, if $\omega > 0$, the regions of $F(r)^2 < 0$ are safely hidden behind the horizon for the observer located at infinity. Furthermore, for $\omega = \frac{2r_0}{3}$, the Warped-AdS$_3$ Black hole has only one horizon located at $r = r_0$, and also it can be extended through this horizon by the Kruskal coordinates (Moussa et al. 2003). So, the $\frac{2r_0}{3}$ value of the ω parameter becomes a critical value for the Warped-AdS$_3$ Black hole metric. From these points of view, under the condition of $\omega > 0$, it is interesting to note that these values of the parameter ω play a critical role on Hawking temperature of the Warped-AdS$_3$ Black hole, given in Eq. (19). From this Hawking temperature expression, we see that the Hawking temperature increases where $\omega < \frac{2r_0}{3}$ while it decreases in the case $\omega > \frac{2r_0}{3}$. This behavior of the temperature can be simply explained by the black hole instability. In this perspective, the angular velocities of the outer and the inner horizons are given as $\Omega_+ = \frac{3}{2r_0+3\omega}$ and $\Omega_- = \frac{3}{3\omega-2r_0}$, respectively. In the case where $\omega < \frac{2r_0}{3}$, the angular velocity of the outer horizon becomes positive ($\Omega_+ > 0$) while the angular velocity of the inner horizon becomes negative ($\Omega_- < 0$). Furthermore, the angular momentum of the black hole (Moussa et al. 2003),

$$J = \frac{\pi}{\kappa}\left(\omega^2 - \frac{5}{9}r_0^2\right),$$

becomes increasingly negative. Then, the black hole is said that emit gravitational radiation and to extract angular momentum, as in the rotating fluid stars where the *CFS* mechanism applies (Chandrasekhar 1970; Friedman and Schutz 1978; Campbell 1970). This leads to loses in both an energy and angular momentum. Perhaps, the gravitational radiation gives a contribution to the Hawking radiation.

3 Tunnelling of scalar particles

The scalar field $\Psi(t, r, \phi)$ is represented by the Klein-Gordon equation. In the curved space-time, the Klein-Gordon equation is given as follows,

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left[\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right]\Psi(t,r,\phi) = \frac{m_0^2}{\hbar^2}\Psi(t,r,\phi), \tag{20}$$

where m_0 is mass of a scalar particle, $\hbar$ is Planck's constant, and g is the determinant of the metric tensor given in Eq. (1). To study the quantum tunnelling of scalar particles from the Warped-AdS$_3$ Black hole, we assume an ansatz for the solution in a form that is similar to Eqs. (11) as,

$$\Psi(t,r,\phi) = A\exp\left(\frac{i}{\hbar}S(t,r,\phi)\right), \tag{21}$$

where A is a constant and $S(t, r, \phi)$ is the classical action term for the outgoing trajectory. Now substituting Eq. (21) into Eq. (20) and ignoring the small terms of $\hbar$ as multiplicator via semi-classical approximation, we obtain the Hamilton-Jacobi equation in the following way

$$\begin{aligned} &F(r)^2\left(\frac{\partial S}{\partial t}\right)^2 - 2F(r)^2N^\phi(r)\left(\frac{\partial S}{\partial t}\right)\left(\frac{\partial S}{\partial \phi}\right) \\ &\quad + \left(F(r)^2N^\phi(r)^2 - N(r)^2\right)\left(\frac{\partial S}{\partial \phi}\right)^2 \\ &\quad - N(r)^4F(r)^4\left(\frac{\partial S}{\partial r}\right)^2 + N(r)^2F(r)^2m_0^2 = 0. \end{aligned} \tag{22}$$

Because of ∂_t and ∂_ϕ are Killing vectors of the Warped-AdS$_3$ black hole, we can assume the following separation of variables for the classical action as a solution of Eq. (22),

$$S(t,r,\phi) = -Et + j\phi + W(r) + C.$$

Here E and j are the energy and angular momentum of the scalar particle, respectively, and C is a complex constant. We are only considering radial trajectories $W(r)$. Using this assumption in Eq. (22), after some simplification, we get

$$\begin{aligned} W_\pm(r) &= \pm\int\frac{\sqrt{(E + jN^\phi(r))^2 + N(r)^2(m_0^2 - \frac{j^2}{F(r)^2})}}{F(r)N(r)^2}dr \\ &= \pm\frac{i\pi\sqrt{3}(E - j\Omega_+)}{6r_0}(2r_0 + 3\omega) \end{aligned} \tag{23}$$

Here '+' and '−' are representing the outgoing and incoming trajectories of the tunnelling scalar particles, respectively, and the $W_\pm(r)$ values stem from the first order poles, as are the complex integral of the $K_\pm(r)$. The tunnelling probabilities of crossing the horizon from inside to outside and outside to inside given by Eq. (17). This means that the probability of the scalar particle tunnelling from inside to outside the horizon is

$$\begin{aligned} \Gamma &= \exp\left[-\frac{4}{\hbar}\operatorname{Im}W_+(r)\right] \\ &= \exp\left[-\frac{2\pi(E - j\Omega_+)(2r_0 + 3\omega)}{\hbar\sqrt{3}r_0}\right], \end{aligned}$$

which is the same result for both Dirac particles: particle and anti-particle. Accordingly, the hawking temperature is also the same,

$$T = \frac{\hbar\sqrt{3}}{2\pi}\left(\frac{r_0}{2r_0 + 3\omega}\right).$$

4 Summary and conclusion

In this study, we have studied Hawking radiation of fermion and scalar particles as a quantum tunnelling effect from the Warped-AdS$_3$ Black holes. By using Hamilton-Jacobi method, we have derived the tunneling probability of the relativistic particles (fermions and scalar) from the Warped-AdS$_3$ Black holes. Subsequently, using the obtained these particle tunnelling probabilities, we have calculated the Hawking temperature for the black hole.

The temperature increases when the parameter ω is $\omega < \frac{2r_0}{3}$. The situation is reasonable, because the angular momentum becomes negative for values for which the black hole becomes unstable when the $\omega < \frac{2r_0}{3}$. This causes the black hole to radiate gravitational waves similar to the rotating starts where the *CFS* mechanism applies.

All of these results show that the classical surface gravity is in accordance with the Hawking temperature calculated from the imaginary part of the complex integral with the first order pole in the Hamilton-Jacobi method. These results are consistent with previous work. Therefore, each particle, no matter what their spins are, probes a black hole in the same way (Kerner and Mann 2008a; Gecim and Sucu 2013).

Acknowledgements The Authors thanks Dr. Timur Sahin for useful discussion. This work was supported by Akdeniz University, Scientific Research Projects Unit.

References

Aliev, A.N., Nutku, Y.: Class. Quantum Gravity **13**, L29 (1996)
Arzano, M., Medved, A.J.M., Vagenas, E.C.: J. High Energy Phys. **0509**, 037 (2005)
Barut, A.O., Duru, I.H.: Phys. Rev. D **36**, 3705 (1987)
Bekenstein, J.D.: Phys. Rev. D **7**, 2333 (1973)
Brill, D.R., Wheeler, J.A.: Rev. Mod. Phys. **29**, 465 (1957)
Campbell, W.B.: Phys. Rev. D **2**, 2123 (1970)
Carlip, S.: Quantum Gravity in 2+1 Dimensions. Cambridge University Press, Cambridge (1998)
Carlip, S., Deser, S., Waldron, A., Wise, D.K.: Phys. Lett. B **666**, 272 (2008)
Carter, B.: Nature **238**, 71 (1972)
Chandrasekhar, S.: Phys. Rev. Lett. **24**, 611 (1970)
Chandrasekhar, S.: Proc. R. Soc. Lond., Ser. A **349**, 571 (1976)
Chen, D.Y., Jian, Q.Q., Zu, X.T.: Class. Quantum Gravity **25**, 205022 (2008)
Chen, D.Y., Yang, H., Zu, X.T.: Phys. Lett. B **681**, 463 (2009)
Clement, G.: Class. Quantum Gravity **9**, 2635 (1992)
Clement, G.: Class. Quantum Gravity **26**, 165002 (2009)
Deser, S., Jackiw, R., Templeton, S.: Phys. Rev. Lett. **48**, 975 (1982)
Deser, S., Jackiw, R., 't Hooft, G.: Ann. Phys. **152**, 220 (1984)
Di Criscienzo, R., Vanzo, L.: Europhys. Lett. **82**, 60001 (2008)
Friedman, J.L., Schutz, B.F.: Astrophys. J. **222**, 281 (1978)
Gecim, G., Sucu, Y.: J. Cosmol. Astropart. Phys. **02**, 023 (2013)
Greif, J.M.: Junior thesis, Princeton University (1969, unpublished)
Hawking, S.W.: Nature **248**, 30 (1974)
Hawking, S.W.: Commun. Math. Phys. **43**, 199 (1975)
Hawking, S.W.: Phys. Rev. D **13**, 191 (1976)
Kerner, R., Mann, R.B.: Phys. Rev. D **73**, 104010 (2006)
Kerner, R., Mann, R.B.: Class. Quantum Gravity **25**, 095014 (2008a)
Kerner, R., Mann, R.B.: Phys. Lett. B **665**, 277 (2008b)
Kraus, P., Wilczek, F.: Nucl. Phys. B **433**, 403 (1995a)
Kraus, P., Wilczek, F.: Nucl. Phys. B **437**, 231 (1995b)
Li, H.-L.: Europhys. Lett. **92**, 20003 (2010)
Li, R., Ren, J.R.: Phys. Lett. B **661**, 370 (2008)
Li, H.L., Yang, S.Z., Jiang, Q.Q., Qi, D.J.: Phys. Lett. B **641**, 139 (2006)
Mirekhtiary, S.F., Sakalli, I.: Commun. Theor. Phys. **61**, 558 (2014)
Moussa, K.A., Clement, G., Leygnac, C.: Class. Quantum Gravity **20**, L277 (2003)
Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
Parker, L.: Phys. Rev. Lett. **21**, 562 (1968)
Parker, L.: Phys. Rev. **183**, 1057 (1969)
Parker, L.: Phys. Rev. D **3**, 346 (1971)
Qi, D.-J.: Int. J. Theor. Phys. **52**, 345 (2013)
Shankaranarayanan, S., Srinivasan, K., Padmanabhan, T.: Mod. Phys. Lett. A **16**, 571 (2001)
Slavov, P.I., Yazadjiev, S.S.: Phys. Rev. D **86**, 084042 (2012)
Srinivasan, K., Padmanabhan, T.: Phys. Rev. D **60**, 024007 (1999)
Sucu, Y., Unal, N.: Class. Quantum Gravity **21**, 1443 (2004)
Sucu, Y., Unal, N.: J. Math. Phys. **48**, 052503 (2007)
Vagenas, E.C.: Phys. Lett. B **503**, 399 (2001)
Vagenas, E.C.: Nuovo Cimento B **117**, 899 (2002)
Volovik, G.E.: Exotic Properties of Superfluid 3He. World Scientific, Singapore (1992)
Volovik, G.E.: JETP Lett. **69**, 705 (1999)
Volovik, G.E.: The Universe in a Helium Droplet. Clarendon Press, Oxford (2003)
Witten, E.: Nucl. Phys. B **311**, 46 (1988)
Witten, E.: Commun. Math. Phys. **121**, 351 (1989)
Zeng, X.X., Hao, X.Z., Yang, S.Z.: Int. J. Mod. Phys. D **17**, 2079 (2008)
Zhang, J., Zhao, Z.: Phys. Lett. B **638**, 110 (2006)