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ORIGINAL ARTICLE

Radiating cylindrical gravitational collapse with structure scalars in $f(R)$ gravity

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Abstract In this paper, we discuss dynamical properties of dissipative collapsing cylindrical self-gravitating systems with account of $f(R) = R + \gamma R^2 + \beta_1 R^3$ gravity model. In this perspective, we see effects of higher curvature terms in the formulations of structure scalars already obtained from the orthogonal decomposition of Weyl curvature scalar in general relativity. We compute mass function by generalizing Misner-Sharp formalism and discuss the contribution of relaxation time in the radiating collapsing process. The contribution of scalar functions in the modeling of static anisotropic as well as isotropic fluid configurations are explored. We conclude that all static anisotropic cylindrical solutions of $f(R)$ field equations can be written explicitly by means of triplet of these scalar functions.

Keywords Dissipative systems · Relativistic systems · Modified gravity

1 Introduction

The general relativistic anisotropic as well as isotropic stellar bodies has always been a subject of great interest for astrophysicists. Among widely consented constraints in the analysis of compacts systems, one is that the pressure of some relativistic systems, like neutron star, is isotropic in nature. However, after the leading work of Bowers and Liang (1974) there is an immense literature for the analysis of spherically compact system with anisotropic fluid distribution. Herrera and Santos (1997) showed the relevance of anisotropic pressure and their importance in the study of collapsing compact stars with and without dissipation effects. Mak and Harko (2003) found some exact models of spherically symmetric stars coupled with anisotropic fluid distribution. Di Prisco et al. (2007) explored that anisotropic pressure increases the active gravitational mass of the collapsing spherical stellar body. Cipolletta and Giambó (2012) examined the contribution of pressure anisotropy on the spherical gravitational collapse in cosmos.

Penrose and Hawking (1979) found Weyl curvature scalar as a key figure for discussing energy density inhomogeneities of spherical stars. Herrera et al. (1998) determined inhomogeneity factor that may lead to naked singularities. Mena et al. (2000) investigated the contribution of irregularities and shearing motion on the final phases of the collapsing dust cloud. Herrera et al. (2004) found density irregularity constraint for a radiating star by means of a relation that correspond to anisotropic pressure, shear and Weyl scalars. Sharif and Bhatti extended their results and found inhomogeneity parameters for conformally flat (Sharif and Bhatti 2014a), non-tilted (Sharif and Bhatti 2014b) and tilted charged (Sharif and Bhatti 2014c) plane relativistic systems.

Fátima et al. (1991) studied the dynamics of cylindrical collapsing systems with non-adiabatic matter distribution. Wang (2003) explored the cylindrical collapse and found constraint under which collapse may lead to black holes. Herrera and Santos (2005) concluded that cylindrical system with collapsing source will always emit gravitational radiations from the source. Di Prisco et al. (2009) obtained analytical models for cylindrical collapsing system coupled with anisotropic fluid which may help to study the phenomenon of gravitational radiation during collapsing process. Ziaie et al. (2011) studied stellar collapse with

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$f(R)=\alpha R^n$ corrections and found that for some specific values of energy density as well as α and n, collapse may lead to a naked singularity.

Debnath et al. (2012) explored the nature of singularities in the collapse of stellar system coupled with isotropic matter distribution. Capozziello et al. (2012) studied perfect fluid collapse through Jeans investigation in the presence of $f(R)$ corrections. Sebastiani et al. (2013) investigated the evolution of Nariai black holes by means of cosmological patch scheme with a specific $f(R)$ formalism and concluded that evolutionary stages of collapsing system is controlled by particular choice of $f(R)$ model. Brito et al. (2013) found some exact cylindrically symmetric models with anisotropic pressure in the presence of cosmological constant. Guha and Banerji (2014) analyzed the contribution of several physical parameters to determine the possible solution of cylindrical collapse. We have explored the collapse of spherical (Sharif and Yousaf 2013) as well as cylindrical (Sharif and Yousaf 2014d) and restricted axial (Sharif and Yousaf 2014c) systems in $f(R)$ gravity and found that matter variables along with $f(R)$ corrections in the field equations occupy key role in the dynamics of collapsing system.

Bel (1961) was the first who performed orthogonal decomposition of the curvature tensor. Herrera et al. (2009) put forward this notion and introduced scalar functions, X_T, X_{TF}, Y_T, Y_{TF}, and relate them with the Weyl tensor. Herrera et al. (2010a, 2010b) discussed the formation and evolution of stellar dissipative stars by evaluating five distinct structure scalars. Sharif and Bhatti extended their results in the presence of electromagnetic field for planar (Sharif and Bhatti 2012a) and cylindrical (Sharif and Bhatti 2012b) celestial bodies. Herrera et al. (2012) studied dynamical properties of cylindrically symmetric metric and analyzed thermo-inertial effects in collapsing mechanism by evaluating eight distinct structure scalars. Recently, we have analyzed the effects of polynomial (Sharif and Yousaf 2014a) and generalized Carrol-Duvvuri-Trodden-Turner (Sharif and Yousaf 2014b) $f(R)$ models on energy density irregularities in relativistic self-gravitating fluids by evaluating modified structure scalars.

In the present paper, we explore the effects of fluid variables as well as $f(R)$ corrections on the structure and evolution of radiating cylindrical compact object. The paper is outlined as follows. Section 2 is devoted to describe the basic formalism for our study. In Sect. 3, we evaluate eight distinct $f(R)$ structure scalars as well as conservations laws. We also develop two important equations relating Weyl scalar, matter variables and scalar quantities. Section 4 is devoted to calculate generalized mass function and dynamical-transport equation. We also discuss their role on the evolutionary phases of gravitational collapse. In Sect. 5, we present static anisotropic as well as isotropic cylindrical models in terms of these scalars functions. Finally, we provide a summary of the results.

2 Radiating anisotropic fluid cylinders

In $f(R)$ gravity, the gravitational portion of the Einstein-Hilbert action is modified as

$$S_{f(R)}=\frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R), \tag{1}$$

where κ is the coupling constant and $f(R)$ is a non-linear generic function of the Ricci curvature. Notice that limit, i.e., $f(R)\rightarrow R$ provides the usual Einstein-Hilbert. Upon variation of the above equation with respect to $g_{\alpha\beta}$, the metric $f(R)$ field equations are obtained as

$$\begin{aligned}&f_R(R)R_{\alpha\beta}-\frac{1}{2}f(R)g_{\alpha\beta}+(g_{\alpha\beta}\Box-\nabla_\alpha\nabla_\beta)f_R(R)\\&\quad=\kappa T_{\alpha\beta},\end{aligned} \tag{2}$$

where $\Box$ is the d'Alembert operator while ∇_α symbolizes for covariant derivative. We can write Eq. (2) in the fabric of Einstein field equations as

$$G_{\alpha\beta}=\frac{\kappa}{f_R}(\overset{(D)}{T_{\alpha\beta}}+T_{\alpha\beta}), \tag{3}$$

where

$$\overset{(D)}{T_{\alpha\beta}}=\frac{1}{\kappa}\left\{\nabla_\alpha\nabla_\beta f_R-\Box f_R g_{\alpha\beta}+(f-Rf_R)\frac{g_{\alpha\beta}}{2}\right\}$$

describes gravitational interaction due to $f(R)$ gravity which disappears identically in $f(R)\rightarrow R$ limit. We model the system with general non-rotating cylindrically symmetric spacetime (Herrera et al. 2012)

$$\begin{aligned}ds^2=&-A^2(t,r)(dt^2-dr^2)+B^2(t,r)dz^2\\&+C^2(t,r)d\phi^2,\end{aligned} \tag{4}$$

filled with radiating anisotropic fluid. In order to achieve cylindrical symmetry of the above spacetime, we require validity of the following relations

$$-\infty\le t\le\infty,\qquad 0\le r,\qquad -\infty<z<\infty,$$
$$0\le\phi\le 2\pi.$$

We also suppose that $C=0$ at null radial distance, r, which shows non-singular axis. The fluid distribution is bounded by a timelike hypersurface Σ described by the following energy-momentum tensor

$$T_{\alpha\beta}=(\mu+P)V_\alpha V_\beta+q_\alpha V_\beta+Pg_{\alpha\beta}+q_\beta V_\alpha+\Pi_{\alpha\beta}, \tag{5}$$

where μ, P_r, P_z, P_ϕ, q_α are fluid energy density, principal stresses and heat conducting vector, respectively, while $\Pi_{\alpha\beta}=(P_\phi-P_r)(K_\alpha K_\beta-\frac{h_{\alpha\beta}}{3})+(P_z-P_r)(S_\alpha S_\beta-\frac{h_{\alpha\beta}}{3})$. The quantities, S_α, K_α and L_α, are unitary vectors configuring canonical orthonormal tetrad. Under comoving coordinate system, the vectors $V_\alpha=(-A,0,0,0)$, $L_\alpha=$

$(0, A, 0, 0)$, $S_\alpha = (0, 0, B, 0)$ and $K_\alpha = (0, 0, 0, C)$ satisfy the following equations

$$S^\alpha S_\alpha = L^\alpha L_\alpha = K_\alpha K^\alpha = 1, \qquad V^\alpha V_\alpha = -1,$$
$$V^\alpha S_\alpha = V^\alpha L_\alpha = K_\alpha V^\alpha = K_\alpha S^\alpha = 0.$$

The metric $f(R)$ field equations for the spacetime (4) are given as follows

$$\frac{\dot{C}\dot{B}}{BC} - \frac{C''}{C} - \frac{B'C'}{BC} - \frac{B''}{B} + \alpha = \frac{\kappa}{f_R}\left[\mu A^2 + \frac{A^2}{\kappa}\left\{\frac{f_R''}{A^2} - \frac{A' f_R'}{A^3} - \frac{\dot{A}\dot{f}_R}{A^3} - \gamma\right\}\right], \tag{6}$$

$$\left(\frac{C'}{C} + \frac{B'}{B}\right)\frac{\dot{A}}{A} - \frac{\dot{C}'}{C} + \left(\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\right)\frac{A'}{A} - \frac{\dot{B}'}{B} = \frac{\kappa}{f_R}\left[-qA^2 + \frac{1}{\kappa}\left(\dot{f}_R' - \frac{A'}{A}\dot{f}_R - \frac{\dot{A}}{A}f_R'\right)\right], \tag{7}$$

$$\frac{B'C'}{BC} - \frac{\dot{B}\dot{C}}{BC} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \alpha = \frac{\kappa}{f_R}\left[P_r A^2 + \frac{A^2}{\kappa}\left\{\frac{\ddot{f}_R}{A^2} - \frac{\dot{A}\dot{f}_R}{A^3} - \frac{A' f_R'}{A^3} + \gamma\right\}\right], \tag{8}$$

$$\left(\frac{B}{A}\right)^2\left[\beta - \frac{\ddot{C}}{C} + \frac{C''}{C}\right] = \frac{\kappa}{f_R}\left[P_z B^2 + \frac{B^2}{\kappa}\left\{\delta - \frac{1}{A^2}\frac{C' f_R'}{C}\right\}\right], \tag{9}$$

$$\left(\frac{C}{A}\right)^2\left[\beta - \frac{\ddot{B}}{B} + \frac{B''}{B}\right] = \frac{\kappa}{f_R}\left[P_\phi C^2 + \frac{C^2}{\kappa}\left\{\delta - \frac{1}{A^2}\frac{B' f_R'}{B}\right\}\right], \tag{10}$$

where

$$\alpha = \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{A'}{A}\left(\frac{B'}{B} + \frac{C'}{C}\right),$$
$$\beta = \frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A} - \frac{A'^2}{A^2} + \frac{A''}{A},$$
$$\gamma = \frac{f - Rf_R}{2} - \left(\frac{B'}{B} + \frac{C'}{C}\right)\frac{f_R'}{A^2} + \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\frac{\dot{f}_R}{A^2},$$
$$\delta = \frac{f - Rf_R}{2} + \frac{1}{A^2}\left(\ddot{f}_R - f_R'' + \frac{\dot{C}\dot{f}_R}{C}\right),$$

where prime and dot symbolize for $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial t}$ operators, respectively. The tensor $\sigma_{\alpha\beta}$ controlling the shearing motion of the fluid can be written in terms of σ_s and σ_k scalars as

$$\sigma_{\alpha\beta} = \left(K_\alpha K_\beta - \frac{1}{3}h_{\alpha\beta}\right)\sigma_k + \left(S_\alpha S_\beta - \frac{1}{3}h_{\alpha\beta}\right)\sigma_s,$$
$$\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \left(\sigma_k^2 + \sigma_s^2 - \sigma_k\sigma_s\right)\frac{2}{3},$$

where

$$\sigma_k = -\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)\frac{1}{A}, \qquad \sigma_s = -\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{1}{A}.$$

The Weyl tensor $C^\rho_{\alpha\beta\mu}$ can be splitted into its magnetic and electric parts that can be written with the help of unitary vectors S_α, K_α and some scalars functions E_s, E_k and H as

$$H_{\alpha\beta} = (K_\beta S_\alpha + K_\alpha S_\beta)H,$$
$$E_{\alpha\beta} = \left(S_\alpha S_\beta - \frac{h_{\alpha\beta}}{3}\right)E_s + \left(K_\alpha K_\beta - \frac{h_{\alpha\beta}}{3}\right)E_k, \tag{11}$$

where

$$H = -\frac{C_{0313}}{C^2 A^2},$$
$$E_s = -\frac{1}{A^2}\left(C_{0101} - \frac{C_{0202}}{B^2}\right), \tag{12}$$
$$E_k = -\frac{1}{A^2}\left(C_{0101} - \frac{C_{0303}}{C^2}\right).$$

The components of the Weyl tensor C_{0202}, C_{0101}, C_{0303}, C_{0313} are given in a recent paper (Herrera et al. 2012).

3 Modified structure scalars

The orthogonal decomposition of the Riemann curvature tensor provides three tensors, i.e., $X_{\alpha\beta}$, $Y_{\alpha\beta}$ and $Z_{\alpha\beta}$ which are further divided into trace (denoted with subscript T) and traceless (denoted by X_s, X_k, Y_s, Y_k, Z_H, Z_q) parts known as structure scalars. The three tensors are given as follows (Herrera et al. 2010a, 2010b)

$$Z_{\alpha\beta} = H_{\alpha\beta} + \frac{\kappa}{2f_R}\left(q - \frac{\overset{(D)}{T}_{01}}{A^2}\right)^\rho \epsilon_{\alpha\beta\rho}, \tag{13}$$

$$Y_{\alpha\beta} = \left(S_\alpha S_\beta - \frac{h_{\alpha\beta}}{3}\right)Y_s + \left(K_\alpha K_\beta - \frac{h_{\alpha\beta}}{3}\right)Y_k + \frac{h_{\alpha\beta}}{3}Y_T, \tag{14}$$

$$X_{\alpha\beta} = \left(S_\alpha S_\beta - \frac{h_{\alpha\beta}}{3}\right)X_s + \left(K_\alpha K_\beta - \frac{h_{\alpha\beta}}{3}\right)X_k + \frac{h_{\alpha\beta}}{3}X_T. \tag{15}$$

The $f(R)$ structure scalars for cylindrical symmetry by means of fluid variables can be written using Eqs. (6)–(10) and (12) as

$$Y_T = \frac{\kappa}{2f_R}\left(\mu + P_z + P_r + P_\phi + \frac{\overset{(D)}{T}_{00}}{A^2} + \frac{\overset{(D)}{T}_{11}}{A^2} + \frac{\overset{(D)}{T}_{22}}{B^2} + \frac{\overset{(D)}{T}_{33}}{C^2}\right), \tag{16}$$

$$X_T = \frac{\kappa}{f_R}\left(\mu + \frac{\overset{(D)}{T}_{00}}{A^2}\right),$$

$$Y_s = E_s - \frac{\kappa}{2f_R}\left(P_z - P_r + \frac{\overset{(D)}{T}_{22}}{B^2} - \frac{\overset{(D)}{T}_{11}}{A^2}\right), \tag{17}$$

$$Y_k = E_k - \frac{\kappa}{2f_R}\left(P_\phi - P_r + \frac{\overset{(D)}{T}_{33}}{C^2} - \frac{\overset{(D)}{T}_{11}}{A^2}\right),$$

$$Z_q = \frac{\kappa}{2f_R}\left(q - \frac{\overset{(D)}{T}_{01}}{A^2}\right), \tag{18}$$

$$X_s = -E_s - \frac{\kappa}{2f_R}\left(P_z - P_r + \frac{\overset{(D)}{T}_{22}}{B^2} - \frac{\overset{(D)}{T}_{11}}{A^2}\right), \tag{19}$$

$$Z_H = 2H,$$

$$X_k = -E_k - \frac{\kappa}{2}\left(P_\phi - P_r + \frac{\overset{(D)}{T}_{33}}{C^2} - \frac{\overset{(D)}{T}_{11}}{A^2}\right). \tag{20}$$

This clearly indicates that scalar functions associated in the modeling of collapsing cylindrical celestial body in $f(R)$ gravity are eight in number. These $f(R)$ structure scalars in spherical relativistic system were found to be five in number in our previous paper (Sharif and Yousaf 2014a). All structure scalars of general relativity (Herrera et al. 2012) can be retrieved, when we take $f(R) = R$ in the above equations.

The law of conservation of effective and usual energy-momentum tensors gives

$$\mu^* + (\mu + P_r)\Theta + q^\alpha a_\alpha + q^\alpha_{;\alpha} + \Pi^{\alpha\beta}\sigma_{\alpha\beta} + \frac{1}{3}\Pi^\alpha_\alpha\Theta + D_0 = 0, \tag{21}$$

$$h^{\alpha\beta}\left(\Pi^\mu_{\beta;\mu} + P_{r;\beta} + q^*_\beta\right) + a^\alpha(\mu + P_r) + \frac{4}{3}q^\alpha\Theta + q^\mu\sigma^\alpha_\mu + D_1 = 0, \tag{22}$$

while D_0 and D_1 are $f(R)$ dark source terms mentioned in Appendix. Equation (22) can be rewritten in the following manner as

$$P_r^\dagger + a(\mu + P_r) + q^* - \frac{1}{A}\left[(P_z - P_r)\frac{B'}{B} + (P_\phi - P_r)\frac{C'}{C}\right] - \frac{q}{3}(\sigma_s - 4\Theta + \sigma_k) + D_1 = 0, \tag{23}$$

where the operators $g^\dagger = g_{,\alpha}L^\alpha$ and $g^* = g_{,\alpha}V^\alpha$. The couple of important equations required to discuss the dynamics of relativistic system can be obtained by using Eqs. (6)–(10) and (16)–(23). These equations were computed in general relativity by Herrera et al. (2012) which in modified gravity turns out to be

$$\begin{aligned}
&\frac{\kappa}{f_R}\left(2\mu + P_z + P_r + P_\phi + \frac{2\overset{(D)}{T}_{00}}{A^2} + \frac{\overset{(D)}{T}_{11}}{A^2} + \frac{\overset{(D)}{T}_{22}}{B^2} + \frac{\overset{(D)}{T}_{33}}{C^2}\right)^\dagger \\
&\quad + \frac{3\kappa}{f_R}\left(\mu + P_r + \frac{\overset{(D)}{T}_{00}}{A^2} + \frac{\overset{(D)}{T}_{11}}{A^2}\right)a \\
&\quad - \frac{2\kappa}{f_R}\left(q - \frac{\overset{(D)}{T}_{01}}{A^2}\right)(\sigma_s - \Theta + \sigma_k) + \frac{3\kappa}{f_R}\left(q - \frac{\overset{(D)}{T}_{01}}{A^2}\right)^* \\
&= 3(X_k - Y_k)\frac{C'}{AC} - (Y_s + Y_k - X_s - X_k)^\dagger \\
&\quad + 3(X_s - Y_s)\frac{B'}{AB} - 6H(\sigma_s - \sigma_k),
\end{aligned} \tag{24}$$

$$\begin{aligned}
&-\frac{\kappa}{f_R}\left(\mu - P_r - P_z + 2P_\phi + \frac{\overset{(D)}{T}_{00}}{A^2} - \frac{\overset{(D)}{T}_{11}}{A^2} - \frac{\overset{(D)}{T}_{22}}{B^2} + \frac{2\overset{(D)}{T}_{22}}{B^2}\right)^\dagger \\
&\quad - \frac{\kappa}{f_R}(\sigma_s - \Theta - 2\sigma_k)\left(q - \frac{\overset{(D)}{T}_{01}}{A^2}\right) \\
&\quad - \frac{3\kappa}{f_R}\left(P_\phi - P_r - \frac{\overset{(D)}{T}_{11}}{A^2} + \frac{\overset{(D)}{T}_{33}}{C^2}\right)\frac{C'}{AC} \\
&= 6H^* + (2Y_s - 2X_s - Y_k + X_k)^\dagger + 3(Y_s - X_s)\frac{B'}{AB} \\
&\quad + 3a(X_k - X_s - Y_k + Y_s) + 6H(\Theta - \sigma_k).
\end{aligned} \tag{25}$$

These equations peculiarly relates fluid parameters, $f(R)$ dark source terms with modified structure scalars.

4 Viable $f(R)$ model, modified dynamical-transport equation and mass function

The $f(R)$ gravity can be considered as a way to explore several unknown aspects of gravitational physics at large scales. In this realm, one needs to take well-consistent $f(R)$ model satisfying experimental tests and conform to cosmological constraints. Thus, the choice of $f(R)$ model holds significant importance in $f(R)$ gravity. Here, we are interested to investigate the effects of R^2 model alongwith cubic corrections given as (Nojiri and Odintsov 2011; Capozziello and De Laurentis 2011)

$$f(R) = R + \gamma R^2 + \beta_1 R^3, \tag{26}$$

where γ and β_1 are constants. It is well-known that the inclusion of quadratic corrections is an attempt to renormalize general relativity thereby representing straightforward modification. The existence of such terms holds potential relevance in cosmology as they assist to understand dynamical analysis of celestial system with self-consistent inflationary universe. These corrections can be regarded as beginning approximations in particular dark energy models in extended theories of gravity. Amendola et al. (2007) presented viability conditions of $f(R)$ dark energy models. Another important issue is the stability of the $f(R)$ models which are well discussed in the literature (Faraoni and Nadeau 2005; Böhmer et al. 2007). This $f(R)$ model is a viable cosmological model with account of modified gravity that has broadly been examined in the last decade in several cosmological and astrophysical scenarios. More recent, Huang (2014) discussed inflationary mechanism of cosmos with polynomial $f(R)$ model.

Capozziello et al. (2011) derived modified Lané-Emden equation in metric $f(R)$ gravity through Newtonian limit. They obtained density and pressure relation to analyze the hydrostatic phases of stellar systems. Astashenok et al. (2013) studied effects of several $f(R)$ models on the evolution of compact objects and found that $f(R)$ theory likely to host comparatively huge and massive matter distributions in cosmos. Astashenok et al. (2013) discussed dynamical collapse of neutron stars in the presence of strong gravitational fields related to power law corrections. Astashenok et al. (2015a) explored the existence of several compact objects like neutron stars with quark cores through numerical technique by evaluating modified Tolman-Oppenheimer-Volkoff equations equipped with several realistic equations of state. They found that comparatively with general relativity, more supermassive celestial system can obtained through correction coming out from cubic $f(R)$ model. The same authors extended their results and studied the consequences of intensive magnetic field on the evolutionary stages of compact objects (e.g. neutron stars) with cubic and quadratic $f(R)$ models through numerical approach (Astashenok et al. 2015b).

Now, we discuss the role of dissipative parameters in gravitational collapse within the realm of extra degrees of freedom allowed due to R^2 models with cubic corrections. In this perspective, we compute dynamical-transport equation and mass function for cylindrical self-gravitating system. The variation of areal radius in cylindrical metric with proper time gives its velocity and is found to be

$$U=\frac{\dot{C}}{A}=C^*<0 \quad \text{(for collapsing system)}, \tag{27}$$

which after using Eqs. (8) and (26) provides

$$U^*=\frac{C'}{A}a-\frac{\kappa C}{1+2\gamma R+3\beta_1 R^2}\left(P_r+\frac{2\gamma}{\kappa}\phi_{\gamma 1}+\frac{6\beta}{\kappa}\phi_{\beta 1}\right)-\frac{C}{A^2}\left[\frac{\ddot{B}}{B}+\frac{\dot{B}}{B}\left(\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right)-\frac{B'}{B}\left(\frac{C'}{C}+\frac{A'}{A}\right)\right], \tag{28}$$

where $\phi_{\gamma i}$ and $\phi_{\beta i}$ are mentioned in the Appendix. This equation can be recast in terms of Riemann curvature tensor as

$$U^*=\frac{C'}{A}a-\frac{\kappa C}{1+2\gamma R+3\beta_1 R^2}\left(P_r+\frac{2\gamma}{\kappa}\phi_{\gamma 1}+\frac{6\beta}{\kappa}\phi_{\beta 1}\right)-\frac{C}{B^2}\left(\frac{R_{2323}}{C^2}-\frac{R_{0202}}{A^2}\right).$$

Using the value of a from above equation in Eq. (23), we have

$$\begin{aligned}U^*(\mu+P_r)=&-\left[\left\{\frac{D_3C'}{A}+\frac{\kappa(\mu+P_r)}{1+2\gamma R+3\beta_1 R^2}\times\left(P_r+\frac{2\gamma}{\kappa}\phi_{\gamma 1}+\frac{6\beta}{\kappa}\phi_{\beta 1}\right)\right\}\right.\\&\left.-\frac{A}{B^2}(\mu+P_r)\left(\frac{R_{0202}}{A^2}-\frac{R_{2323}}{C^2}\right)\frac{C'}{C}\right]\\&+\left[-q^*+\frac{1}{3}(\sigma_k+\sigma_s-4\Theta)q\right]\frac{C'}{A}-\frac{C'}{A}\\&\times\left[P_r^\dagger-\frac{1}{A}\left(\frac{B'}{B}(P_z-P_r)+\frac{C'}{C}(P_\phi-P_r)\right)\right],\end{aligned} \tag{29}$$

where D_3 indicates dark source terms incorporating quadratic and cubic $f(R)$ corrections and can be evaluated by using Eq. (26) and Eq. (A.2). We see that the above equation contains terms of four types. First is on the left hand side which is a product of time derivative of fluid velocity and inertial mass (density), while other three terms are on the other side in three different square brackets. The first and second terms encapsulate effects of effective gravitational force and dissipative phenomena, respectively while the last term is entitled as hydrodynamic force since it includes pressure and anisotropic gradient entities. Thus the above equation embodies Newtonian configuration of the type

Acceleration × Mass density = Force.

Now we proceed to evaluate mass function in $f(R)$ gravity with cylindrically symmetric background. In this context, the combination of structure parameters can be written as

$$\frac{B^2}{3}(2Y_s+Y_T-X_T+X_s+Y_k-X_k)=\frac{R_{0202}}{A^2}-\frac{R_{2323}}{C^2},$$

using Eqs. (16)–(20), this yields

$$\begin{aligned}\frac{R_{0202}}{A^2}-\frac{R_{2323}}{C^2}&=\frac{B^2}{3}(E_s-2E_k)+\frac{\kappa B^2}{3(1+2\gamma R+3\beta_1 R^2)}\\&\times\left[2P_r-P_z-\frac{\mu}{2}+\frac{P_\phi}{2}+\frac{\gamma}{\kappa}(-\phi_{\gamma 0}-2\phi_{\gamma 2}+4\phi_{\gamma 1}+\phi_{\gamma 3})+\frac{3\beta}{\kappa}(\phi_{\beta 3}-\phi_{\beta 0}-2\phi_{\beta 2}+4\phi_{\beta 1})\right].\end{aligned} \tag{30}$$

Using the above relation, we obtain

$$\frac{\kappa C}{1+2\gamma R+3\beta_1 R^2}\left(P_r+\frac{2\gamma}{\kappa}\phi_{\gamma 1}+\frac{6\beta}{\kappa}\phi_{\beta 1}\right) -\frac{C}{B^2}\left(\frac{R_{0202}}{A^2}-\frac{R_{2323}}{C^2}\right) =\left(P_r+\frac{2\gamma}{\kappa}\phi_{\gamma 1}+\frac{6\beta}{\kappa}\phi_{\beta 1}\right)\frac{\kappa C}{2(1+2\gamma R+3\beta_1 R^2)} +\left\{\mu-P_\phi-P_r+2P_z+\frac{2\gamma}{\kappa}\times(\phi_{\gamma 0}-\phi_{\gamma 1}+2\phi_{\gamma 2}-\phi_{\gamma 3}) +\frac{2\beta}{\kappa}(\phi_{\beta 0}-\phi_{\beta 1}+2\phi_{\beta 2}-\phi_{\beta 3})\right\} \times\frac{\kappa C}{6(1+2\gamma R+3\beta_1 R^2)}-\frac{C}{3}(E_s-2E_k). \tag{31}$$

In order to evaluate the expression for mass function, we use the general relativity technique mentioned in Di Prisco et al. (2009) for spherically symmetric case. This provides the possible generalization of Misner-Sharp mass function for cylindrically symmetric relativistic system with $f(R)$ corrections and is found to be

$$m=\frac{\kappa C^3}{6(1+2\gamma R+3\beta_1 R^2)}\left\{\mu-P_r+2P_z-P_\phi +\frac{6\beta}{\kappa}(\phi_{\beta 0}-\phi_{\beta 1}+2\phi_{\beta 2}-\phi_{\beta 3})+\frac{2\gamma}{\kappa}(\phi_{\gamma 0}-\phi_{\gamma 1} +2\phi_{\gamma 2}-\phi_{\gamma 3})\right\}-\frac{C^3}{3}(E_s-2E_k), \tag{32}$$

which after using Eqs. (17)–(20) can be written as

$$\frac{3m}{C^3}=\frac{\kappa}{2(1+2\gamma R+3\beta_1 R^2)}\left\{\mu+P_\phi-2P_r +P_z+\frac{2\gamma}{\kappa}(\phi_{\gamma 0}-2\phi_{\gamma 1}+\phi_{\gamma 2}+\phi_{\gamma 3}) +\frac{6\beta}{\kappa}(\phi_{\beta 0}-2\phi_{\beta 1}+\phi_{\beta 2}+\phi_{\beta 3})\right\}-(Y_s-2Y_k), \tag{33}$$

$$\frac{3m}{C^3}=\frac{\kappa}{2(1+2\gamma R+3\beta_1 R^2)}\left\{\mu+3P_z-3P_\phi +\frac{2\gamma}{\kappa}(\phi_{\gamma 0}-3\phi_{\gamma 3}+2\phi_{\gamma 2}) +\frac{6\beta}{\kappa}(\phi_{\beta 0}-3\phi_{\beta 3}+2\phi_{\beta 2})\right\}+(X_s-2X_k). \tag{34}$$

The alternative relation of mass function in terms of $f(R)$ scalars functions can be obtained with the help of Eqs. (17)–(20) and (23)–(25) as follows

$$(X_s+3Y_s+X_k-3Y_k)^\dagger -\left[\frac{\kappa}{1+2\gamma R+3\beta_1 R^2}\left(\mu+\frac{2\gamma}{\kappa}\phi_{\gamma 0}+\frac{6\beta}{\kappa}\phi_{\beta 0}\right)\right]^\dagger =\frac{\kappa}{1+2\gamma R+3\beta_1 R^2}\left(q-\frac{2\gamma}{\kappa}\phi_{\gamma q}-\frac{2\beta}{\kappa}\phi_{\beta q}\right) \times(\sigma_k-\Theta-2\sigma_s)-(Y_s+X_s)\frac{3B'}{AB} +(Y_k-X_k)\frac{3C'}{AC}-6H^*+6H(\sigma_s-\Theta) -3a(Y_s-Y_k-X_s+X_k). \tag{35}$$

Making use of Eqs. (33) and (34), we obtain

$$\frac{6m}{C^3}=\frac{\kappa}{1+2\gamma R+3\beta_1 R^2}\left(\mu+\frac{2\gamma}{\kappa}\phi_{\gamma 0}+\frac{6\beta}{\kappa}\phi_{\beta 0}\right) -X_k-3Y_s-X_s+3Y_k.$$

Next, we apply an operator † on the above equation and use Eq. (35), it follows that

$$\left(\frac{6m}{C^3}\right)^\dagger=6H^*+\frac{3}{A}(Y_s+X_s)\frac{B'}{B}-6H(\sigma_s-\Theta) +\frac{3}{A}(X_k-Y_k)\frac{C'}{C}+\left(q-\frac{2\gamma}{\kappa}\phi_{\gamma q}-\frac{2\beta}{\kappa}\phi_{\beta q}\right) \times\frac{\kappa(2\sigma_s-\sigma_k+\Theta)}{1+2\gamma R+3\beta_1 R^2} +3a(X_k+Y_s-X_s-Y_k),$$

whose integration provides

$$m=\frac{C^3}{2}\int\left[(Y_s+X_s)\frac{B'}{B}-2AH(\sigma_s-\Theta) +2AH^*+(X_k-Y_k)\frac{C'}{C}+\left(q-\frac{2\gamma}{\kappa}\phi_{\gamma q}-\frac{2\beta}{\kappa}\phi_{\beta q}\right) \times\frac{A\kappa(2\sigma_s-\sigma_k+\Theta)}{3(1+2\gamma R+3\beta_1 R^2)}(2\sigma_s+\Theta-\sigma_k) +aA(Y_s-X_s-Y_k+X_k)\right]dr+\frac{C^3\lambda(t)}{6}, \tag{36}$$

where λ is an integration function. It is obvious that the $f(R)$ dark source quantities, modified structure scalars and other physical variables affect the mass function. Feeding the values of X_s, Y_k, Y_s, X_k from Eqs. (17)–(20) in the above relation, we obtain

$$m=\frac{C^3}{2}\int\left[-\frac{\kappa}{1+2\gamma R+3\beta_1 R^2} \times\left\{P_z-P_r+\frac{2\gamma}{\kappa}(\phi_{\gamma 2}-\phi_{\gamma 1})+\frac{6\beta}{\kappa}(\phi_{\beta 2}-\phi_{\beta 1})\right\}\frac{B'}{B} -2AH(\sigma_s-\Theta)+2AH^*-2E_k\frac{C'}{C} +\frac{\kappa A}{3(1+2\gamma R+3\beta_1 R^2)}\left(q+\frac{2\gamma}{\kappa}\phi_{\gamma q}+\frac{2\beta}{\kappa}\phi_{\beta q}\right) \times(2\sigma_s+\Theta-\sigma_k) +aA(Y_s-X_s-Y_k+X_k)\right]dr+\frac{C^3\lambda(t)}{6}. \tag{37}$$

This yields that how Weyl tensor and fluid parameters like, dissipative quantities, pressure anisotropy, shear, expansion and modified structure scalars affect the existence of mass of cylindrical self-gravitating body with $f(R)$ corrections. The above relation describes the contribution of modified structure scalars (which were originally obtained through orthogonal splitting of the Riemann curvature tensor) in developing the cylindrical dissipative self-gravitating system. Here we take an account R^2 model with cubic corrections in Ricci scalar. The R^2 terms are controlled by the parameter γ while the cubic corrections are ensured through the existence of β_1. These may spark the tendency of $f(R)$ models to develop gravitational attraction at early times. It is worthy to stress that significant massive compact objects can be found with $f(R)$ extra degrees of freedom coming out from cubic $f(R)$ higher curvature terms (Astashenok et al. 2015a). This provides the realistic signature of the presence of more massive and huge self-gravitating stellar systems which do have direct correspondence with the observational cosmology.

In the analysis of collapsing phenomenon of celestial objects, the study of radiating variables in the fluid configurations occupies enticing interest. In this context, effects of thermal conduction on the dynamical analysis of a spherical collapsing stars were first investigated by Herrera et al. (1997) in general relativity. Herrera extended their results for shear-free anisotropic (Herrera 2006) and isotropic (Herrera 2002) collapsing self-gravitating systems and discovered thermoinertial effects associated with the inertia of thermal energy and hyperbolic character of the transport equation. Herrera et al. (2006) described decreasing behavior of inertial mass density of the conformally flat shear-free spherical relativistic star and discussed bouncing phenomenon through numerical technique. To understand the contribution of relaxation time and radiating phenomena in the evolutionary stages of collapsing system, we develop transport equation from casual radiating theory (Müller 1967; Israel 1976; Israel and Stewart 1976, 1979)

$$q^{\alpha}+\tau h^{\alpha\beta}V^{\gamma}q_{\beta;\gamma} =-\frac{1}{2}\xi q^{\alpha}K^{2}\left(\frac{\tau V^{\beta}}{\xi K^{2}}\right)_{;\beta}-\xi h^{\alpha\beta}(K_{,\beta}+Ka),$$

where τ, ξ and K indicate relaxation time, thermal conductivity and temperature, respectively. The choice of $\tau=0$ in the above equation results the Eckart-Landau (Eckart 1940) equation. The only one independent component is

$$\tau q^{*}+q=-\xi\left(K^{\dagger}+Ka\right)-\frac{1}{2}\xi K^{2}q\left(\frac{\tau}{\xi K^{2}}\right)^{*}-\frac{1}{2}q\tau\Theta. \tag{38}$$

Substituting Eqs. (31), (32) and (38) in (29), it follows that

$$\begin{aligned}&(\mu+P_r)\left\{1-\frac{\xi K}{\tau(\mu+P_r)}\right\}U^{*}\\ &=-\frac{(\mu+P_r)}{C^2}\left\{\frac{\kappa C^3}{2(1+2\gamma R+3\beta_1R^2)}\right.\\ &\quad\left.\times\left(P_r+\frac{2\gamma}{\kappa}\phi_{\gamma1}+\frac{6\beta}{\kappa}\phi_{\beta1}\right)+m\right\}\left\{1-\frac{\xi K}{\tau(\mu+P_r)}\right\}\\ &\quad+\frac{C'}{A}\left[-P_r^{\dagger}+\frac{1}{A}\left\{(P_\phi-P_r)\frac{C'}{C}\right.\right.\\ &\quad\left.\left.+(P_z-P_r)\frac{B'}{B}\right\}-D_3\right]+\frac{C'}{A}\left[\frac{q}{\tau}+\frac{\xi qK^2}{2\tau}\left(\frac{\tau}{\xi K^2}\right)^{*}\right.\\ &\quad\left.+\frac{\xi K^{\dagger}}{\tau}+\frac{(\sigma_s+\sigma_k)}{3q}-\frac{5}{6}q\Theta\right],\end{aligned}$$

which can be rewritten as

$$\begin{aligned}&(\mu+P_r)(1-\mathfrak{a})U^{*}\\ &=F_{grav}(1-\mathfrak{a})+\frac{C'}{A}\left[-P_r^{\dagger}+\frac{1}{A}\left\{(P_\phi-P_r)\frac{C'}{C}\right.\right.\\ &\quad\left.\left.+(P_z-P_r)\frac{B'}{B}\right\}-D_3\right]+\frac{C'}{A}\left[\frac{q}{\tau}+\frac{\xi qK^2}{2\tau}\left(\frac{\tau}{\xi K^2}\right)^{*}\right.\\ &\quad\left.+\frac{\xi K^{\dagger}}{\tau}+\frac{(\sigma_s+\sigma_k)}{3q}-\frac{5}{6}q\Theta\right],\end{aligned} \tag{39}$$

where

$$\begin{aligned}F_{grav}&=-\frac{(\mu+P_r)}{C^2}\left\{\frac{\kappa C^3}{2(1+2\gamma R+3\beta_1R^2)}\right.\\ &\quad\left.\times\left(P_r+\frac{2\gamma}{\kappa}\phi_{\gamma1}+\frac{6\beta}{\kappa}\phi_{\beta1}\right)+m\right\},\\ \mathfrak{a}&=\frac{\xi K}{\tau(\mu+P_r)}.\end{aligned}$$

This provides the thermoinertial contribution on the inertial mass (density) in $f(R)$ gravity. It is noticed that the above obtained relation invokes corrections to the general relativity expression (Herrera et al. 2012) thereby modifying the dynamics of cylindrical gravitational collapse. F_{grav} encapsulates the terms of quadratic as well as cubic Ricci scalar corrections controlled by γ and β_1 parameters, respectively. If we take $\beta_1=0$, then the above expression describes the gravitational force alongwith dynamics permitted due to γR^2 gravity. This describes the evolution of gravitational collapse in Starobinsky model (Starobinsky 1980) which is viable inflationary model compatible with temperature anisotropies in cosmic microwave background. Apart from this, if one takes $\gamma=0$, then the above equation describes the thermoinertial effects associated to the inertia of the thermal energy of the transport equation with cubic correction in metric $f(R)$ gravity. It is well-known that Astashenok et al. (2013) for sufficiently large and negative values of β_1, one can achieve even more massive compact objects thereby indicating interesting results about the dynamics of self-gravitating objects. Moreover, for R^2 gravity alongwith cubic correction, one can get even more stable stellar distributions in cosmos.

5 Static anisotropic cylinders

Here, we explore the importance of modified structure functions in the likely solutions of metric $f(R)$ field equations. We shall denote integration functions/constants with λ_i's. We consider the case of static anisotropic cylindrical system for which the field equations (6)–(10) lead to

$$\frac{B'}{B}\left(\frac{A'}{A}-\frac{B'}{B}\right)+\frac{A'C'}{AC}-\frac{C''}{C}-\frac{B''}{B} = \frac{\kappa}{f_R}\left[\mu A^2+\frac{A^2}{\kappa}\left\{\frac{Rf_R-f}{2}+\frac{f_R''}{A^2} + \left(\frac{B'}{B}-\frac{A'}{A}+\frac{C'}{C}\right)\frac{f_R'}{A^2}\right\}\right], \quad (40)$$

$$\frac{B'}{B}\left(\frac{A'}{A}+\frac{B'}{B}\right)+\frac{A'C'}{AC} = \frac{\kappa}{f_R}\left[P_rA^2+\frac{A^2}{\kappa}\left\{-\left(\frac{B'}{B}+\frac{A'}{A}+\frac{C'}{C}\right)\frac{f_R'}{A^2} + \frac{f-Rf_R}{2}\right\}\right], \quad (41)$$

$$\left(\frac{C''}{C}+\frac{A''}{A}-\frac{A'^2}{A^2}\right)\frac{B^2}{A^2} = \frac{\kappa}{f_R}\left[P_zB^2+\frac{B^2}{\kappa}\left\{\frac{f-Rf_R}{2}-\left(f_R''+\frac{C'f_R'}{C}\right)\frac{1}{A^2}\right\}\right], \quad (42)$$

$$\left(\frac{B''}{B}+\frac{A''}{A}-\frac{A'^2}{A^2}\right)\frac{C^2}{A^2} = \frac{\kappa}{f_R}\left[P_\phi C^2+\frac{C^2}{\kappa}\left\{\frac{f-Rf_R}{2}-\left(f_R''+\frac{B'f_R'}{B}\right)\frac{1}{A^2}\right\}\right]. \quad (43)$$

Let us define some auxiliary entities, $\eta_1=\frac{A'}{A}$, $\eta_2=\frac{B'}{B}$, $\eta_3=\frac{C'}{C}$, $\eta_4=\frac{f_R'}{f_R}$ so that the above equations as well as scalars E_s and E_k can be recast as

$$-\eta_2'-\eta_2^2-\eta_3'-\eta_3^2+\eta_1\eta_2+\eta_1\eta_3-\eta_2\eta_3 = \frac{\kappa A^2}{f_R}\left(\mu+\frac{\overset{(D)}{\mathcal{T}_{00}}}{A^2}\right), \quad (44)$$

$$\eta_1\eta_2+\eta_1\eta_3+\eta_2\eta_3=\frac{\kappa A^2}{f_R}\left(P_r+\frac{\overset{(D)}{\mathcal{T}_{11}}}{A^2}\right), \quad (45)$$

$$\eta_1'+\eta_3'+\eta_3^2=\frac{\kappa A^2}{f_R}\left(P_z+\frac{\overset{(D)}{\mathcal{T}_{22}}}{A^2}\right),$$
$$\eta_1'+\eta_2'+\eta_2^2=\frac{\kappa A^2}{f_R}\left(P_\phi+\frac{\overset{(D)}{\mathcal{T}_{33}}}{A^2}\right), \quad (46)$$

$$E_s=\frac{1}{2A^2}\left(-\eta_1'+\eta_3'+\eta_3^2+\eta_1\eta_2-\eta_2\eta_3-\eta_2\eta_3\right), \quad (47)$$

$$E_k=\frac{1}{2A^2}\left(-\eta_1'+\eta_2'+\eta_2^2-\eta_1\eta_2+\eta_1\eta_3-\eta_2\eta_3\right), \quad (48)$$

where terms $\overset{(D)}{\mathcal{T}_{\alpha\beta}}$ stand for dark source terms evaluated for static cylindrical system.

One of the $f(R)$ structure scalars can be found by specific combination of all the above equations as

$$\eta_1'+\eta_1\eta_2+\eta_1\eta_3=Y_TA^2. \quad (49)$$

Equations (45) and (46) provide

$$\eta_1'+\eta_2'+\eta_2^2-\eta_1\eta_2-\eta_1\eta_3-\eta_2\eta_3 = \frac{\kappa}{f_RA^2}\left(P_\phi-P_r+\frac{\overset{(D)}{\mathcal{T}_{33}}}{C^2}-\frac{\overset{(D)}{\mathcal{T}_{11}}}{A^2}\right), \quad (50)$$

$$\eta_1'+\eta_3'+\eta_3^2-\eta_1\eta_2-\eta_1\eta_3-\eta_2\eta_3 = \frac{\kappa A^2}{f_R}\left(P_z-P_r+\frac{\overset{(D)}{\mathcal{T}_{22}}}{B^2}-\frac{\overset{(D)}{\mathcal{T}_{11}}}{A^2}\right), \quad (51)$$

$$\eta_2'+\eta_2^2-\eta_3'+\eta_3^2=\frac{\kappa}{f_R}\left(P_\phi-P_z+\frac{\overset{(D)}{\mathcal{T}_{33}}}{C^2}-\frac{\overset{(D)}{\mathcal{T}_{22}}}{B^2}\right). \quad (52)$$

Using Eqs. (17), (18), (47), (48), (50) and (51), we obtain

$$\eta_1'-\eta_1\eta_2=-Y_sA^2, \qquad \eta_1'+\eta_1\eta_3=-Y_kA^2. \quad (53)$$

The first of above equations, on integration, yields

$$A=\lambda_1e^{\int B(\int\frac{-Y_sA^2}{B}dr)dr},$$

which implies that $B=B(A)$ or $\eta_1=\eta_1(\eta_2)$, $\forall Y_s$. Further, the second of Eq. (53) gives

$$A=\lambda_2e^{\int C(\int\frac{-Y_kA^2}{C}dr)dr},$$

which yields $C=C(A)$ or $\eta_1=\eta_1(\eta_3)$, $\forall Y_k$. Thus, we can write any of the auxiliary variable in terms of the others. For instance, one can find the expressions of η_2 and η_3 (depending upon η_1) for some values of right hand side of Eq. (52). Equations (18) and (20) give

$$\frac{\kappa}{f_R}\left(P_\phi-P_r+\frac{\overset{(D)}{\mathcal{T}_{33}}}{C^2}-\frac{\overset{(D)}{\mathcal{T}_{11}}}{A^2}\right)=-(X_k+Y_k). \quad (54)$$

Equations (17) and (19) yield

$$\frac{\kappa}{f_R}\left(P_z-P_r+\frac{\overset{(D)}{\mathcal{T}_{22}}}{B^2}-\frac{\overset{(D)}{\mathcal{T}_{11}}}{A^2}\right)=-(X_s+Y_s). \quad (55)$$

This suggests that any static cylindrical anisotropic solution in $f(R)$ gravity can be determined through a triplet of structure scalars (Y_k, Y_s, X_s) or (Y_k, Y_s, X_k).

Now, we investigate models for static cylindrical celestial body coupled with perfect fluid. Thus we take $P_\phi=P_z=P_r=P$ and the corresponding field equations (40)–(43) reduce to

$$\frac{B'}{B}\left(\frac{A'}{A}-\frac{B'}{B}\right)+\frac{A'C'}{AC}-\frac{C''}{C}-\frac{B''}{B}$$
$$=\frac{\kappa}{f_R}\left[\mu A^2+\frac{A^2}{\kappa}\left\{\frac{Rf_R-f}{2}+\frac{f''_R}{A^2}\right.\right.$$
$$\left.\left.+\left(\frac{B'}{B}-\frac{A'}{A}+\frac{C'}{C}\right)\frac{f'_R}{A^2}\right\}\right], \tag{56}$$

$$\frac{B'}{B}\left(\frac{A'}{A}+\frac{B'}{B}\right)+\frac{A'C'}{AC}$$
$$=\frac{\kappa}{f_R}\left[PA^2+\frac{A^2}{\kappa}\left\{-\left(\frac{B'}{B}+\frac{A'}{A}+\frac{C'}{C}\right)\frac{f'_R}{A^2}\right.\right.$$
$$\left.\left.+\frac{f-Rf_R}{2}\right\}\right], \tag{57}$$

$$\left(\frac{C''}{C}+\frac{A''}{A}-\frac{A'^2}{A^2}\right)\frac{B^2}{A^2}$$
$$=\frac{\kappa}{f_R}\left[PB^2+\frac{B^2}{\kappa}\left\{\frac{f-Rf_R}{2}-\left(f''_R+\frac{C'f'_R}{C}\right)\frac{1}{A^2}\right\}\right], \tag{58}$$

$$\left(\frac{B''}{B}+\frac{A''}{A}-\frac{A'^2}{A^2}\right)\frac{C^2}{A^2}$$
$$=\frac{\kappa}{f_R}\left[PC^2+\frac{C^2}{\kappa}\left\{\frac{f-Rf_R}{2}-\left(f''_R+\frac{B'f'_R}{B}\right)\frac{1}{A^2}\right\}\right]. \tag{59}$$

Equations (58) and (59) yield

$$\frac{C''}{C}+\frac{C'f'_R}{Cf_R}=\frac{B''}{B}+\frac{B'f'_R}{Bf_R}, \tag{60}$$

which can be rewritten in the form of Ricatti equation as

$$\eta'_3+\eta_3^2+\eta_3\eta_4=\eta'_2+\eta_2^2+\eta_2\eta_4. \tag{61}$$

The general solution of the above equation is expressed as

$$\eta_3=\eta_2+\frac{1}{k(r)}, \tag{62}$$

where

$$k(r)=e^{\int(2\eta_2+\eta_4)dr}\left[\int e^{\int-(2\eta_2+\eta_4)dr}dr+\lambda_3\right]. \tag{63}$$

Using the value of k in Eq. (62), we obtain the following form of metric function

$$C=B\gamma\exp\left[B^2 f_R\left(\int\frac{dr}{B^2 f_R}+\lambda_3\right)\right], \tag{64}$$

where we have used regularity constraint, i.e., $C(t,0)=0$.

6 Conclusion

In this paper, we have computed set of leading relations regulating the structure and evolution of cylindrical collapsing relativistic body and highlighted the effects of $f(R)$ structure scalars in building these equations. For this purpose, we have coupled the cylindrically symmetric spacetime with imperfect matter configuration by invoking $f(R)$ corrections. The modified form of structure scalars are investigated from the general relativity generic formula of scalar functions (Herrera et al. 2012). We have also developed relationship between Weyl scalar and matter parameters for the usual and effective fluid distributions. In order to analyze the effects of relaxation time in the evolution of collapsing system, we have computed the dynamical-transport equation. We have also analyzed the role of $f(R)$ scalar functions for anisotropic as well as isotropic cylindrical solutions. The main results are summarized as follows.

1. Unlike spherical system, we have found eight modified scalar functions $(X_T, Y_T, X_s, Y_s, X_k, Y_k, Z_H, Z_q)$ along with two shear scalars (σ_s, σ_k) and three Weyl scalars (H, E_s, E_k) that are required to discuss the dynamics of cylindrically symmetric metric.
2. The $f(R)$ structure scalars, X_T, Z_H, Z_q, have a very clear interpretation as these measure energy density, heat radiation and magnetic portion of curvature effects arising from Weyl tensor along with $f(R)$ dark source effects.
3. In general relativity, the structure scalars, Y_s, Y_k and Y_T, hold fundamental importance in governing the contribution of shear and expansion scalars in the dynamics of compact objects. We have evaluated these scalars by invoking $f(R)$ corrections and noticed that dark source terms tend to disturb the contribution of these scalars in the evolution of the stellar system.
4. Equation (37) shows that $f(R)$ high curvature quantities try to complicate the expression of mass function due to its non-attractive nature.
5. In the static case of cylinders, we have found that any possible static solutions can be expressed by a set of (Y_k, X_k, Y_s) and (Y_k, X_s, Y_s) structure functions. We have also obtained particular constraints required for isotropic cylindrical models obeying regular conditions.
6. It is seen from Eq. (39) that both F_{grav} and effective inertial mass density are multiplied by the same factor $(1-\mathfrak{a})$. This indicates the null nature of inertial mass as $\mathfrak{a}\to 1$. Thus the effective gravitational attraction on dissipative relativistic fluid declines at the same rate as the effective mass density. This sparks the validity of equivalence principle.
7. The system would receive gravitational attraction, if during fluctuations, $\mathfrak{a}$ imparts negative value to the right hand side of Eq. (39).
8. Let us now study the evolution of cylindrical collapse by analyzing the value fluctuations of the factor $\mathfrak{a}$. Suppose that in some region of cylinder, the entity $\mathfrak{a}$ first increases and then approaches towards 1. This leads to

small value of effective inertial mass which eventually diminishes F_{grav} thereby giving opposite sign to the right hand side of Eq. (39). Since such kind of system development is accompanied with small values of the inertial mass density, therefore this entails vigorous bouncing off of that cylindrical part of the collapsing system. Thus the cylindrical system with dissipative source begins to emit gravitational radiations. These results are well-consistent with (Herrera et al. 2006) under the limit $\gamma=\beta_1=0$.

9. It is worthy to stress that all results of general relativity can be obtained by taking $f(R)\rightarrow R$ (Herrera et al. 2012).

It is worthy to stress that our investigation provides dynamical behavior of self-gravitating cylindrical system with general and $f(R)=R+\gamma R^2+\beta_1 R^3$ gravity models. The obtained results may correspond to early-time inflationary cosmos where extra curvature terms appear into dynamics. This paper provides the dynamical collapse process and can be taken as a toy model of localized systems. Finally, we remark that this approach can be extended by considering strange stars (Staykov et al. 2014), compact stars (Astashenok et al. 2015b) or its interpretation as scalar-tensor stars (Fiziev 2014).

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Appendix

The $f(R)$ corrections in Eqs. (21) and (22) are

$$\begin{aligned}D_0=&\frac{1}{\kappa}\Bigg[\bigg\{\bigg(\dot{f}_R\frac{A'}{A}+f'_R\frac{\dot{A}}{A}-\dot{f}'_R\bigg)\frac{1}{A^4}\bigg\}_{,1}\\&+\frac{1}{A^2}\bigg\{\bigg(\frac{Rf_R-f}{2}\bigg)+\frac{f'_R}{A^2}\bigg(\frac{B'}{B}-\frac{A'}{A}+\frac{C'}{C}\bigg)\\&-\bigg(\frac{\dot{C}}{C}+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\bigg)\frac{\dot{f}_R}{A^2}+\frac{f''_R}{A^2}\bigg\}_{,0}\\&+\frac{\dot{A}}{A}\bigg\{\frac{\ddot{f}_R}{A^2}+\frac{f''_R}{A^2}-2\frac{A'f'_R}{A^3}-2\frac{\dot{A}\dot{f}_R}{A^3}\bigg\}\frac{1}{A^2}\\&+\frac{\dot{B}}{B}\bigg\{\frac{\ddot{f}_R}{A^2}-\bigg(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\bigg)\frac{\dot{f}_R}{A^2}+\frac{f'_R}{B^2}\bigg(\frac{B'}{B}-\frac{A'}{A}\bigg)\bigg\}\frac{1}{A^2}\\&+\bigg(\frac{\dot{A}}{A}f'_R-\dot{f}'_R+\frac{A'}{A}\dot{f}_R\bigg)\bigg(\frac{4A'}{A}+\frac{C'}{C}+\frac{B'}{B}\bigg)\frac{1}{A^4}\\&+\frac{2}{A^2}\bigg\{-\bigg(\frac{A'}{A}-\frac{C'}{C}\bigg)\frac{f'_R}{B^2}+\frac{\ddot{f}_R}{A^2}\\&-\frac{\dot{f}_R}{A^2}\bigg(\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\bigg)\bigg\}\frac{\dot{C}}{C}\Bigg],\end{aligned}\tag{A.1}$$

$$\begin{aligned}D_1=&\frac{1}{\kappa}\Bigg[\bigg\{\frac{1}{A^4}\bigg(\frac{A'}{A}\dot{f}_R-\dot{f}'_R+\frac{\dot{A}}{A}f'_R\bigg)\bigg\}_{,0}\\&+\frac{1}{A^2}\bigg\{\frac{\ddot{f}_R}{A^2}+\frac{f-Rf_R}{2}-\frac{f'_R}{B^2}\bigg(\frac{B'}{B}+\frac{A'}{A}+\frac{C'}{C}\bigg)\\&-\frac{\dot{f}_R}{A^2}\bigg(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\bigg)\bigg\}_{,1}+\frac{1}{A^2}\bigg\{\frac{f''_R}{A^2}-\bigg(\frac{A'}{A}+\frac{B'}{B}\bigg)\\&\times\frac{f'_R}{B^2}+\bigg(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\bigg)\frac{\dot{f}_R}{A^2}\bigg\}\frac{B'}{B}+\bigg\{\frac{f''_R}{A^2}+\frac{\ddot{f}_R}{A^2}-2\frac{A'f'_R}{A^3}\\&-2\frac{\dot{A}\dot{f}_R}{A^3}\bigg\}\frac{A'}{A^3}+\frac{1}{A^2}\bigg\{\frac{f''_R}{A^2}-\frac{f'_R}{A^2}\bigg(\frac{A'}{A}+\frac{C'}{C}\bigg)\\&-\frac{\dot{f}_R}{A^2}\bigg(\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\bigg)\bigg\}\frac{C'}{C}-\frac{1}{A^4}\bigg(\dot{f}'_R-\frac{\dot{A}}{A}f'_R-\frac{A'}{A}\dot{f}_R\bigg)\\&\times\bigg(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}+\frac{4\dot{A}}{A}\bigg)\Bigg].\end{aligned}\tag{A.2}$$

The quantities $\phi_{\gamma i}$ and $\phi_{\beta i}$ mentioned in Eqs. (28) and (30) are

$$\begin{aligned}\phi_{\gamma0}=&\frac{R^2}{4}+\frac{R''}{A^2}-\frac{R'}{A^2}\bigg(\frac{B'}{B}-\frac{A'}{A}+\frac{C'}{C}\bigg)\\&-\frac{\dot{R}}{A^2}\bigg(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\bigg),\end{aligned}$$

$$\begin{aligned}\phi_{\gamma1}=&-\frac{R^2}{4}+\frac{\ddot{R}}{A^2}+\frac{\dot{R}}{A^2}\bigg(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\bigg)\\&-\frac{R'}{A^2}\bigg(\frac{B'}{B}+\frac{C'}{C}-\frac{A'}{A}\bigg),\end{aligned}$$

$$\phi_{\gamma2}=-\frac{R^2}{4}+\frac{1}{A^2}\bigg(\ddot{R}-R''+\frac{\dot{C}\dot{R}}{C}-\frac{C'R'}{C}\bigg),$$

$$\phi_{\gamma3}=-\frac{R^2}{4}+\frac{1}{A^2}\bigg(\ddot{R}-R''+\frac{\dot{C}\dot{R}}{C}-\frac{B'R'}{B}\bigg),$$

$$\begin{aligned}\phi_{\beta0}=&\frac{R^3}{6}+\frac{RR''+R'^2}{A^2}-\frac{RR'}{A^2}\bigg(\frac{B'}{B}-\frac{A'}{A}+\frac{C'}{C}\bigg)\\&-\frac{R\dot{R}}{A^2}\bigg(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\bigg),\end{aligned}$$

$$\begin{aligned}\phi_{\beta1}=&-\frac{R^3}{6}+\frac{R\ddot{R}+\dot{R}^2}{A^2}+\frac{R\dot{R}}{A^2}\bigg(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\bigg)\\&-\frac{RR'}{A^2}\bigg(\frac{B'}{B}+\frac{C'}{C}-\frac{A'}{A}\bigg),\end{aligned}$$

$$\begin{aligned}\phi_{\beta2}=&-\frac{R^3}{6}+\frac{1}{A^2}\bigg(\dot{R}^2+R\ddot{R}-RR''-R'^2\\&+\frac{R\dot{C}\dot{R}}{C}-\frac{RC'R'}{C}\bigg),\end{aligned}$$

$$\begin{aligned}\phi_{\beta3}=&-\frac{R^3}{6}+\frac{1}{A^2}\bigg(\dot{R}^2+R\ddot{R}-RR''-R'^2\\&+\frac{R\dot{C}\dot{R}}{C}-\frac{RB'R'}{B}\bigg).\end{aligned}$$

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