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ORIGINAL ARTICLE

Bianchi type-V string cosmological models in $f(R, T)$ gravity

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Abstract Spatially homogeneous and anisotropic Bianchi type-V space-time is considered in the presence of cosmic strings in a modified theory of gravity proposed by Harko et al. (Phys. Rev. D 84:024020, 2011). Solving the field equations we have presented geometric and Takabayasi strings in this theory. Some physical and kinematical properties of the models are also discussed.

Keywords $f(R, T)$ gravity · String models · Bianchi type-V metric

1 Introduction:

In recent years there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which must be formed during a phase transition in the early universe. The investigation of cosmic strings and physical processes near such strings has received considerable attention because of the fact that they give rise to density perturbations leading to the formation of galaxies. It is interesting to study the gravitational effects that arise from strings because the stress energy of a string can be coupled to the gravitational field.

The general relativistic treatment of strings was initiated by Letelier (1979, 1983), Stachel (1980) and Vilenkin (1985). Bianchi type-I and Kantowski-Sachs cosmologies have been studied by Letelier (1983). Subsequently Kroni et al. (1990) and Wang (2003) have discussed the Bianchi types-II, -VI, -VII, and -IX cosmological models for a choice of strings in general relativity. Rao et al. (2008a, 2008b) have obtained Bianchi types-II, -VI, -VII, and -IX string cosmological models in Saez-Ballester and self creation theories of gravitation respectively. Rao and Vijaya Santhi (2012b) have obtained Bianchi types-II, -VI, -VII, and -IX string cosmological models in Brans-Dicke theory of gravitation.

In the light of the recent discovery of the accelerated expansion of the universe (Reiss et al. 1998, Perlmutter et al. 1999), modified theories of gravity are attracting more and more attention of cosmologists because of the fact that these theories may serve as the possible candidates for explaining the late time acceleration of the universe. Noteworthy among them are $f(R)$ theory of gravity (Caroll et al. 2004), $f(R, T)$ gravity (Harko et al. 2011) and scalar tensor theories of gravity proposed by Brans and Dicke (1961) and Saez and Ballester (1986). Reddy (2003a, 2003b, 2005a, 2005b), Reddy and Rao (2006a, 2006b) and Reddy et al. (2006) are some of the workers who have studied string cosmological models in modified theories of gravitation. Recently Naidu et al. (2013) and Reddy et al. (2013) have investigated Bianchi type-V and Kaluza Klein bulk viscous string cosmological models, respectively, in $f(R, T)$ gravity.

Inspired by the above investigations and discussion, we, in this paper present spatially homogeneous and anisotropic Bianchi type-V string cosmological models in $f(R, T)$ gravity. This work is organized as follows: in Sect. 2 a brief review of $f(R, T)$ gravity is presented. In Sect. 3, we derive field equations of $f(R, T)$ gravity with the help of Bianchi type-V metric. Solving the field equations we obtain geometric and P-string models in $f(R, T)$ gravity in Sect. 4 and some physical and kinematical properties are discussed in Sect. 5. Finally summary and concluding remarks are included

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2 Brief review of $f(R,T)$ gravity

In $f(R,T)$ gravity proposed by Harko et al. (2011), gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor T_{ij}. The field equations of this theory are derived from the Hilbert-Einstein type variational principle by taking the action

$$S = \frac{1}{16\pi}\int \left[f(R,T) + L_m\right]\sqrt{-g}d^4x \tag{1}$$

where L_m is the matter Lagrangian density.

Stress energy tensor of matter is defined as

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta^{ij}} \tag{2}$$

and the trace by $T = g^{ij}T_{ij}$ respectively. By assuming that L_m of matter depends only on the metric tensor components g_{ij}, we have obtained the field equations of $f(R,T)$ gravity as

$$\begin{aligned} f_R(R,T)R_{ij} &- \frac{1}{2}f(R,T)g_{ij} \\ &+ \left(g_{ij}\nabla_k\nabla^k - \nabla_i\nabla_j\right)f_R(R,T) \\ &= 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \end{aligned} \tag{3}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \tag{4}$$

here

$$f_R = \frac{\partial f(R,T)}{\partial R}, \qquad f_T = \frac{\partial f(R,T)}{\partial T}$$

and ∇^i is the covariant derivative. It may be noted that when $f(R,T) = f(R)$, Eq. (3) yields the field equations of $f(R)$ gravity.

The problem of perfect fluid described by an energy density ρ, pressure p and four velocities u^i are complicated since there is no unique definition of the matter Lagrangian. However, here, we assume that stress energy tensor of matter is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{5}$$

and the matter Lagrangian can be taken as $L_m = -p$ and we have

$$u^i\nabla_j u_i = 0, u^i u_i = 1 \tag{6}$$

Now with the use of Eq. (5) we obtain, for the variation of stress energy of perfect fluid, the expression

$$\Theta_{ij} = -2T_{ij} - p g_{ij} \tag{7}$$

Generally, the field equations also depend through the tensor θ_{ij}, on the physical nature of the matter field. Hence in the case of $f(R,T)$ gravity, depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of $f(R,T)$. Assuming

$$f(R,T) = R + 2f(T) \tag{8}$$

as a first choice where $f(T)$ is an arbitrary function of the trace of stress energy tensor of matter, we get the gravitational field equations of $f(R,T)$ gravity from Eq. (3) as

$$\begin{aligned} R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} &+ 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} \\ &+ f(T)g_{ij} \end{aligned} \tag{9}$$

where the prime denotes differentiation with respect to the argument. If the matter source is perfect fluid then the field equations of $f(R,T)$ gravity, in view of Eq. (7), become

$$\begin{aligned} R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} &+ 2f'(T)T_{ij} \\ &+ \left[2pf'(T) + f(T)\right]g_{ij} \end{aligned} \tag{10}$$

3 Metric and field equations

We consider the spatially homogeneous and anisotropic Bianchi type-V space time described by the line element

$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}\left(B^2dy^2 + C^2dz^2\right) \tag{11}$$

where A, B, C are functions of cosmic time t only.

We consider the energy momentum tensor for cosmic strings as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{12}$$

where ρ is the energy density of the string cloud, u^i is the four-velocity, x^i is the direction of the string and λ is the string tension density.

Also we have

$$g^{ij}u_i u_j = -x^i x_j = -1 \quad \text{and} \quad u^i x_i = 0 \tag{13}$$

$$\rho = \rho_p + \lambda \tag{14}$$

where ρ_p being the rest energy of particles attached to the string. As pointed out by Letelier (1983), λ may be positive or negative.

Now using the commoving coordinate system and choosing (Harko et al. 2011)

$$f(T) = \mu T \tag{15}$$

where μ is a constant, the field Eq. (10) with the help of Eqs. (12)–(15) for the metric (11) take the form

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{1}{A^2}=\lambda(8\pi+3\mu)+\mu\rho \tag{16}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{1}{A^2}=\lambda\mu+\mu\rho \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{1}{A^2}=\lambda\mu+\mu\rho \tag{18}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{C}}{C}\frac{\dot{A}}{A}-\frac{3}{A^2}=\rho(8\pi+7\mu)+\mu\lambda \tag{19}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{20}$$

where an overhead dot indicates differentiation with respect to t. Now we define some important physical and kinematical parameters. The average scale factor a and the spatial volume are defined as

$$V=a^3=ABC \tag{21}$$

The generalized mean Hubble parameter H is given by

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{22}$$

Here $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$, and $H_3=\frac{\dot{C}}{C}$ are defined as the directional Hubble parameters in the directions of x, y and z axes respectively.

The mean anisotropy parameter Δ is given by

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{23}$$

The expansion scalar θ and shear scalar σ^2 are defined as follows

$$\theta=u_i;\quad i=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{24}$$

$$\begin{aligned}\sigma^2&=\frac{1}{2}\sigma_{ij}\sigma^{ij}\\&=\frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2-\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{\dot{C}}{C}\frac{\dot{A}}{A}\right]\end{aligned} \tag{25}$$

The declaration parameter q is the measure of the cosmic acceleration expansion of the universe which is defined as

$$q=-\frac{a\ddot{a}}{a^{\cdot 2}} \tag{26}$$

The behavior of the universe models is determined by the sign of the q. The positive value of q suggests decelerating model which the negative value indicates inflation.

4 Solutions and the model

Now the field Eqs. (16)–(20) reduce to the following independent equations

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\frac{\dot{A}}{A}=0 \tag{27}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}=(4\pi+2\mu)(\lambda-\rho) \tag{28}$$

$$A^2=kBC \tag{29}$$

where k is the constant of integration which can be chosen as unity without loss of generality so that we have

$$A^2=BC \tag{30}$$

Now Eqs. (27)–(30) are a system of there independent equations in five unknowns A, B, C, λ and ρ. Hence to find a determinate solution we use the following physically plausible conditions

(i) The shear scalar σ^2 is proportional to scalar expansion θ so that we can take (Collins et al. 1980)

$$B=C^m \tag{31}$$

where $m\neq 0$ is a constant.

(ii) The equations of state for string models

$$\rho=\lambda\quad\text{(Letelier 1983)} \tag{32}$$

$$\lambda=(1+\omega)\lambda\quad\text{(Takabayasi 1976)} \tag{33}$$

Here condition (32) corresponds to geometric or Nambu string while (33) corresponds to p-string or Takabayasi string.

Case (i): Geometric string ($\rho=\lambda$)

Now using Eqs. (27), (30), (31) and (32) we obtain the metric coefficients as

$$\begin{aligned}A&=\left[(m+2)(c_1t+c_2)\right]^{\frac{m+1}{2(m+2)}}\\B&=\left[(m+2)(c_1t+c_2)\right]^{\frac{m}{(m+2)}}\\C&=\left[(m+2)(c_1t+c_2)\right]^{\frac{1}{(m+2)}}\end{aligned} \tag{34}$$

where $c_1\neq 0$ and c_2 are constants of integration and m satisfies

$$m^2+16m+7=0 \tag{35}$$

The cosmic string model, in this case, can be written (through a proper choice of coordinates and constants i.e. choosing $c_1=1$, $c_2=0$) with the help of metric (11), as

$$ds^2=-dt^2+\left\{(m+2)t\right\}^{\frac{m+1}{m+2}}dx^2$$

$$+e^{2x}\Big[\big\{(m+2)t\big\}^{\frac{2m}{m+2}}dy^2 + \big\{(m+2)t\big\}^{\frac{2}{m+2}}dz^2\Big] \tag{36}$$

5 Some physical properties of the model

Equation (36) represents an exact cosmological model which gives us the Nambu string in $f(R,T)$ gravity with the following physical and kinematical properties which plays a vital role in the discussion of cosmology.

The spatial volume is

$$V = t^{\frac{3(m+1)}{2(m+2)}} \tag{37}$$

The mean Hubble parameter is

$$H = \frac{m+1}{(m+2)t} \tag{38}$$

The average anisotropy parameter is

$$\Delta = 0 \tag{39}$$

The scalar parameter θ is given by

$$\theta = 3H = \frac{m+1}{(m+2)t} \tag{40}$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{(m-1)^2}{4(m+2)^2t^2} \tag{41}$$

The deceleration parameter q is given by

$$q = \frac{m-3}{(m+1)} \tag{42}$$

We use the results (36)–(42) to discuss the physical behavior of the string cosmological model presented here. It can be observed that the Nambu string obtained in $f(R,T)$ gravity has no initial singularity at $t=0$.The spatial volume increases as t increases and attains infinite volume as $t\to\infty$. This shows the spatial expansion from zero volume. The mean Hubble parameter, the scalar expansion and shear scalar diverge at $t=0$ and will vanish as $t\to\infty$.The average anisotropy parameter vanishes showing that the model will ultimately, become isotropic and will exhibit late time acceleration. The deceleration parameter is positive showing early deceleration. It may also be observed that when $m=1$, the model becomes isotropic and shear free. Also for this value of m, $q=-1$, showing accelerated expansion of the model which is in accordance with this recent observations. (Reiss et al. 1998; Perlmutter et al. 1999). It can be, also, seen that since $\frac{\sigma^2}{\theta^2}\neq 0$, the model remains anisotropic for $m\neq 1$. The physical quantities ρ and λ for this model are given by

$$\rho = \lambda = \frac{1}{8(\pi+\mu)}\left\{\frac{1}{t^2}\left[\frac{m^2+4m+1}{2m^2+8m+8}\right] - 3t^{-(\frac{m+1}{m+2})}\right\} \tag{43}$$

It can be seen that the rest energy density and the tension density of the string diverge for $t=0$ and will approach zero as $t\to\infty$.

Case (ii): P-string or Takabayasi string

Using the equations of the state given by Eq. (33) we obtain p-string or Takabayasi string in $f(R,T)$ gravity. The model, in this case is again given by Eq. (36) with the rest energy density and string tension density is given by

$$\rho = (1+\omega)\lambda = \frac{(1+\omega)}{8(\pi+\mu)+\omega(8\pi+7\mu)} \times \left\{\frac{1}{t^2}\left[\frac{m^2+4m+1}{2m^2+8m+8}\right] - 3t^{-(\frac{m+1}{m+2})}\right\} \tag{44}$$

It may be observed that this model also has same physical and kinematical properties as the geometric string. Also when $\omega=0$, we obtain the geometric string, from p-string with $\rho=\lambda$.

6 Conclusions

In this paper, we have considered Bianchi type-V metric in the presence of cosmic strings in $f(R,T)$ gravity. We have found exact string cosmological models solving the field equations of the theory which represent Nambu and Takabayasi strings in this theory. It is observed that these models are free from singularities. The models decelerate initially and attain late time acceleration which is in accordance with the present observations.

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