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On dynamical instability of spherical star in $f(R, T)$ gravity

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Abstract This work is based on stability analysis of spherically symmetric collapsing star surrounding in locally anisotropic environment in $f(R, T)$ gravity, where R is Ricci scalar and T corresponds to the trace of energy momentum tensor. Field equations and dynamical equations are presented in the context of $f(R, T)$ gravity. Perturbation scheme is employed on dynamical equations to find the collapse equation. Furthermore, condition on adiabatic index Γ is constructed for Newtonian and post-Newtonian eras to address instability problem. Some constraints on physical quantities are imposed to maintain stable stellar configuration. The results in this work are in accordance with $f(R)$ gravity for specific case.

Keywords Collapse · $f(R, T)$ gravity · Dynamical equations · Instability range · Adiabatic index

1 Introduction

The final outcome of stellar collapse and investigations regarding stability of compact objects is emerging as key issue in astrophysics and gravitational theories. A star collapses when it exhaust all its fuel and gravitational force dominates the outward drawn pressure (Joshi and Malafa-

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rina 2011). However, the final outcome of evolution depends on the size of compact object undergoing collapse. The life cycle of a massive stars assuming mass of the order 10–20 solar masses is nominal in comparison to the stars having relatively less mass i.e., ≈ 1 solar mass. Also, due to high energy dissipation during collapse more massive star tends to be more unstable because of massive radiation transport (Hansen and Kawaler 1994; Kippenhahn and Weigert 1990).

The celestial objects are of interest only if they are stable against fluctuations. Chandrasekhar (1964) worked out the dynamical instability of a spherical star, he established instability range in the form of adiabatic index Γ as $\Gamma \geq \frac{4}{3} + n\frac{M}{r}$, where M is the mass and r stands for radius of the star. Stability analysis in General Relativity (GR) associated with expansion free condition, isotropy, local anisotropy, dissipation etc was presented by Herrera and his collaborators (Chan et al. 1989, 1993, 1994). It was observed that dissipative effects and slight change in isotropy alters subsequent evolution considerably (Chan et al. 2000; Herrera and Santos 2003, 2004; Herrera et al. 1989, 2012). Sharif and Abbas presented the dynamical analysis of charged cylindrical gravitational collapse for non-adiabatic and perfect fluid (Sharif and Abbas 2011a, 2011b). Sharif and Azam (2013a, 2013b) worked out stability problem for cylindrically symmetric thin-shell wormholes.

The limitations of GR on large scales urge astrophysicists towards modified gravity explorations, they made enormous advancements to analyze collapse and stability of gravitating objects in modified theories of gravity. Among many modified theories of gravity, $f(R)$ gravity presents one of the elementary modification in Einstein-Hilbert (EH) action by including higher order curvature terms to incorporate dark source candidates. People (Cembranos et al. 2012; Ghosh and Maharaj 2012) have discussed gravitational col-

lapse in modified gravity theories, Cembranos et al. investigated collapse of self-gravitating dust particles (Cembranos et al. 2012). The null dust non-static exact solutions have been established in $f(R)$ gravity, constrained by constant curvature describing anti de-Sitter background evolution (Ghosh and Maharaj 2012).

Some valuable prospects of gravitational collapse for $f(R)$ theory of gravity are worked out in (Sharif and Kausar 2011, 2012; Kausar 2013, 2014, 2014) considering observational situations such as clustering spectrum, cosmic microwave background and weak lensing (Carroll et al. 2006; Bean et al. 2007; Song et al. 2007; Schmidt 2008), concluding that inclusion of higher order curvature terms enhances the stability range. The $f(R)$ model in the presence of electromagnetic field assist in slowing down the collapsing phenomenon (Kausar and Noureen 2014). Spherically symmetric collapse of $f(R)$ gravity models is studied in Borisov et al. (2012) with the help of one-dimensional numerical simulations including non-linear coupling of scalar field. Sebastiani et al. (2013) investigate the instabilities appearing in extremal Schwarzschild de-Sitter background in context of modified gravity theories. Recently, instability range of anisotropic, non-dissipative spherical collapse is established in $f(T)$ theory (Sharif and Rani 2014). Sharif and Abbas (2013a, 2013b) examined the dynamics of charged radiating and shearfree dissipative collapse framed in Gauss-Bonnet gravity theory.

Capozziello et al. (2012) analyzed the collapse and dynamics of collisionless self-gravitating systems by considering coupled collisionless Boltzmann and Poisson equations in the context of $f(R)$ gravity. In order to analyze the collapse and dynamics in context of $f(R)$ gravity, the authors (Capozziello et al. 2012) considered the coupled collisionless Boltzmann and Poisson equations. Initially, the system is taken to be in static equilibrium, the variation in potentials with the time transition is measured with the help of linearly perturbed field equations. Furthermore, Jeans wave number and Jeans mass limit is discussed on obtention of the dispersion relation from Fourier analyzed field equations. The Jeans instability criterion in $f(R)$ is presented by using the dispersion relations and numerical estimation of Jeans length in weak field limit. The numerically solved dispersion relation is utilized for the study of interstellar medium (ISM) and its properties.

In 2011, Harko et al. (2011) introduced $f(R, T)$ gravity theory as another extension of GR based on matter and geometry coupling. In $f(R, T)$, Einstein-Hilbert (EH) action is modified in a way that matter Lagrangian includes an arbitrary function of Ricci scalar R and trace of energy momentum tensor T. This theory can also be conceived as generalization of $f(R)$ theory in which T is included in the action so that quantum effects or existence of some exotic

matter can be taken into account. The action in $f(R, T)$ is written as (Harko et al. 2011)

$$\int dx^4 \sqrt{-g} \left[\frac{f(R, T)}{16\pi G} + \mathcal{L}_{(m)} \right], \tag{1.1}$$

where $\mathcal{L}_{(m)}$ corresponds to matter Lagrangian, and g stands for the metric tensor. Different choices of $\mathcal{L}_{(m)}$ can be considered, each choice implies a set of field equations for particular form of fluid.

Shabani and Farhoudi (2014) used dynamical system approach to study the consequences of $f(R, T)$ gravity models with the help of various cosmological parameters such as Hubble parameter, its inverse, weight function, deceleration, snap parameters, jerk and equation of state parameter. They explained cosmological and solar system implications of $f(R, T)$ models and weak field limit. Sharif and Zubair (2012a, 2012b, 2013a, 2013b) studied the laws of thermodynamics, energy conditions and anisotropic universe models in context of $f(R, T)$ gravity. Chakraborty (2013) formulate field equations for homogeneous and isotropic cosmological models and analyzed energy conditions for perfect fluid in $f(R, T)$ gravity. Recently, dynamical instability of locally isotropic spherically symmetric self-gravitating object is studied in Sharif and Yousaf (2014) framed in $f(R, T)$ theory of gravity.

We aimed to find out stability range of the model under consideration in the presence of anisotropic fluid. The perturbation approach is used to analyze gravitational collapse in $f(R, T)$ gravity. The manuscript is arranged as: Einstein's field equations and dynamical equations for $f(R, T)$ gravity are furnished in Sect. 2 that leads to the collapse equation. Section 3 covers the discussion of stability in terms of adiabatic index and factors affecting stability of compact objects in both Newtonian and post-Newtonian limits. Section 4 contains conclusion followed by Appendix.

2 Dynamical equations in $f(R, T)$

The three dimensional timelike spherical boundary surface Σ is chosen pertaining two regions termed as interior and exterior spacetimes. The line element for region inside Σ is given by

$$ds_-^2 = A^2(t, r)dt^2 - B^2(t, r)dr^2 - C^2(t, r)$$
$$\times \left(d\theta^2 + \sin^2\theta d\phi^2 \right). \tag{2.2}$$

The domain beyond the boundary is exterior region having line element (Sharif and Kausar 2012)

$$ds_+^2 = \left(1 - \frac{2M}{r}\right)dv^2 + 2dr\,dv$$
$$- r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right), \tag{2.3}$$

where v corresponds to retarded time, M is the total mass. It is assumed that gravitational Lagrangian depends only on the components of metric tensor and so corresponding energy momentum tensor for usual matter is given by (Landau and Lifshitz 2002)

$$T_{uv}^{(m)} = g_{uv}\mathcal{L}_{(m)} - \frac{2\partial\mathcal{L}_{(m)}}{\partial g^{uv}}. \tag{2.4}$$

The variation of action (1.1) with the metric g_{uv} constitutes the field equations in $f(R,T)$ gravity as

$$R_{uv}f_R(R,T) - \frac{1}{2}g_{uv}f(R,T) + (g_{uv}\Box - \nabla_u\nabla_v)f_R(R,T)$$
$$= 8\pi G T_{uv}^{(m)} - f_T(R,T)T_{uv}^{(m)} - f_T(R,T)\Theta_{uv}, \tag{2.5}$$

where $f_R(R,T)$ and $f_T(R,T)$ denote derivatives of $f(R,T)$ with respect to R and T respectively; $\Box = g^{\mu\nu}\nabla_\nu\nabla_\mu$ is the d'Alembert operator, ∇_μ is the covariant derivative associated with the Levi-Civita connection of the metric tensor and $\Theta_{\mu\nu}$ is defined by

$$\Theta_{\mu\nu} = \frac{g^{\alpha\beta}\delta T_{\alpha\beta}}{\delta g^{\mu\nu}} = -2T_{\mu\nu} + g_{\mu\nu}\mathcal{L}_m - 2g^{\alpha\beta}\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}.$$

In this study we choose $\mathcal{L}_m = \rho$ so that $\Theta_{uv} = -2T_{uv}^{(m)} + g_{uv}\mathcal{L}_{(m)}$. Hence Eq. (2.5) takes the form

$$G_{uv} = \frac{1}{f_R}\left[(f_T + 1)T_{uv}^{(m)} - \rho g_{uv}f_T + \frac{f - Rf_R}{2}g_{uv}\right.$$
$$\left. + (\nabla_u\nabla_v - g_{uv}\Box)f_R\right]. \tag{2.6}$$

Here $T_{uv}^{(m)}$ represents energy momentum tensor for usual matter describing anisotropic fluid, given by

$$T_{uv}^{(m)} = (\rho + p_\perp)V_uV_v - p_\perp g_{uv} + (p_r - p_\perp)\chi_u\chi_v, \tag{2.7}$$

where ρ is energy density, V_u stands for four-velocity of the fluid, χ_u is the corresponding radial four vector, p_r and $p_\perp$ denote radial and tangential pressure respectively. These physical quantities satisfy following identities

$$V^u = A^{-1}\delta_0^u, \quad V^uV_u = 1,$$
$$\chi^u = B^{-1}\delta_1^u, \quad \chi^u\chi_u = -1. \tag{2.8}$$

Components of field equations are

$$G_{00} = \frac{1}{f_R}\left[\rho + \frac{f - Rf_R}{2} + \frac{f_R''}{B^2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C}\right)\right.$$
$$\left. - \frac{f_R'}{B^2}\left(\frac{B'}{B} - \frac{2C'}{C}\right)\right], \tag{2.9}$$

$$G_{01} = \frac{1}{f_R}\left[\dot{f}_R' - \frac{A'}{A}\dot{f}_R - \frac{\dot{B}}{B}f_R'\right], \tag{2.10}$$

$$G_{11} = \frac{1}{f_R}\left[p_r + (\rho + p_r)f_T - \frac{f - Rf_R}{2} + \frac{\ddot{f}_R}{A^2}\right.$$
$$\left. - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} - \frac{2\dot{C}}{C}\right) - \frac{f_R'}{B^2}\left(\frac{A'}{A} + \frac{2C'}{C}\right)\right], \tag{2.11}$$

$$G_{22} = \frac{1}{f_R}\left[p_\perp + (\rho + p_\perp)f_T - \frac{f - Rf_R}{2} + \frac{\ddot{f}_R}{A^2} - \frac{f_R''}{B^2}\right.$$
$$\left. - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) - \frac{f_R'}{B^2}\left(\frac{A'}{A} - \frac{B'}{B} + \frac{C'}{C}\right)\right], \tag{2.12}$$

where dot and prime indicates the derivatives with respect to "r" and "t". Conservation laws play fundamental role in establishment of stability range. In $f(R,T)$ gravity the energy momentum tensor induce non-vanishing divergence. Here, we have taken into account the conservation of full field equations i.e., Einstein tensor to describe fluid evolution. Bianchi identities are

$$G_{;v}^{uv}V_u = 0, \qquad G_{;v}^{uv}\chi_u = 0, \tag{2.13}$$

implying

$$\dot{\rho} + \rho\left\{[1 + f_T]\left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C}\right) - \frac{\dot{f}_R}{f_R}\right\}$$
$$+ [1 + f_T]\left\{p_r\frac{\dot{B}}{B} + 2p_\perp\frac{\dot{C}}{C}\right\} + Z_1(r,t) = 0, \tag{2.14}$$

$$(\rho + p_r)f_T' + (1 + f_T)\left\{p_r' + \rho\frac{A'}{A} + p_r\left(\frac{A'}{A} + 2\frac{C'}{C} - \frac{f_R'}{f_R}\right)\right.$$
$$\left. - 2p_\perp\frac{C'}{C}\right\} + f_T\left(\rho' - \frac{f_R'}{f_R}\right) + Z_2(r,t) = 0, \tag{2.15}$$

where $Z_1(r,t)$ and $Z_2(r,t)$ are given in Appendix as Eqs. (A.1) and (A.2) respectively. Conservation equations assist in describing variation from equilibrium position leading to collapse process. The variation of physical parameters of gravitating system with the passage of time can be observed by using perturbation scheme as presented in the following section.

3 $f(R,T)$ model and perturbation scheme

The gravitational field equations depicts a set of highly complicated non-linear differential equations whose general solution is still undetermined. We employ perturbation approach to discover the effects of $f(R,T)$ model on the evolution of locally anisotropic spherical star. Implementation

of linear perturbation on evolution equations leads to the pressure to density ratio i.e., adiabatic index describing instability range. The dynamics of evolution can be anticipated either by following Eulerian or Lagrangian pattern i.e., fixed or co-moving coordinates respectively. We have chosen co-moving coordinates, all quantities are considered to be in static equilibrium initially and then with the time transition, the perturbed metric and material variables has both the time and radial dependence.

In our discussion, we consider a particular model of the action (1.1) which is defined as (Sharif and Zubair 2012b)

$$f(R, T) = R + \alpha R^2 + \lambda T,$$

where α is any real number, λ is a coupling parameter and λT represents correction to $f(R)$ gravity. Assuming $0 < \varepsilon \ll 1$, $0 < \xi \ll 1$ and $0 < \eta \ll 1$, the perturbed form of quantities alongwith their initial form can be arranged as (Chan et al. 2000; Herrera and Santos 2003, 2004; Herrera et al. 1989, 2012)

$$A(t, r) = A_0(r) + \varepsilon D(t) a(r), \tag{3.16}$$

$$B(t, r) = B_0(r) + \varepsilon D(t) b(r), \tag{3.17}$$

$$C(t, r) = r B(t, r) \left[1 + \varepsilon D(t) \bar{c}(r) \right], \tag{3.18}$$

$$\rho(t, r) = \rho_0(r) + \varepsilon \bar{\rho}(t, r), \tag{3.19}$$

$$p_r(t, r) = p_{r0}(r) + \varepsilon \bar{p}_r(t, r), \tag{3.20}$$

$$p_\perp(t, r) = p_{\perp 0}(r) + \varepsilon \bar{p_\perp}(t, r), \tag{3.21}$$

$$m(t, r) = m_0(r) + \varepsilon \bar{m}(t, r), \tag{3.22}$$

$$R(t, r) = R_0(r) + \xi D_1(t) e_1(r), \tag{3.23}$$

$$T(t, r) = T_0(r) + \eta D_2(t) e_2(r), \tag{3.24}$$

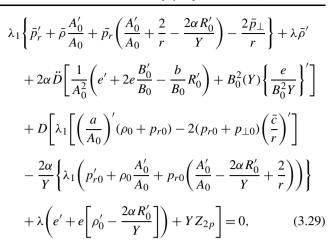$$f(R, T) = \left[R_0(r) + \alpha R_0^2(r) + \lambda T_0 \right] + \xi D_1(t) e_1(r)$$
$$\times \left[1 + 2\alpha R_0(r) \right] + \eta D_2(t) e_2(r), \tag{3.25}$$

$$f_R = 1 + 2\alpha R_0(r) + \xi 2\alpha D_1(t) e_1(r), \tag{3.26}$$

$$f_T = \lambda. \tag{3.27}$$

We take Schwarzschild coordinate $C_0(r) = r$ and apply perturbation scheme on dynamical equations i.e., Eqs. (2.14) and (2.15), we found that

$$\dot{\bar{\rho}} + \left[\frac{2e\rho_0}{Y} + \lambda_1 \left\{ \frac{2\bar{c}}{r}(\rho_0 + 2p_{\perp 0}) + \frac{b}{B_0}(\rho_0 + p_{r0}) \right\} \right.$$
$$\left. + Y Z_{1p} \right] \dot{D} = 0, \tag{3.28}$$

$$\lambda_1 \left\{ \bar{p}_r' + \bar{\rho} \frac{A_0'}{A_0} + \bar{p}_r \left(\frac{A_0'}{A_0} + \frac{2}{r} - \frac{2\alpha R_0'}{Y} \right) - \frac{2\bar{p}_\perp}{r} \right\} + \lambda \bar{\rho}'$$

$$+ 2\alpha \ddot{D} \left[\frac{1}{A_0^2} \left(e' + 2e \frac{B_0'}{B_0} - \frac{b}{B_0} R_0' \right) + B_0^2(Y) \left\{ \frac{e}{B_0^2 Y} \right\}' \right]$$

$$+ D \left[\lambda_1 \left[\left(\frac{a}{A_0} \right)' (\rho_0 + p_{r0}) - 2(p_{r0} + p_{\perp 0}) \left(\frac{\bar{c}}{r} \right)' \right] \right.$$

$$\left. - \frac{2\alpha}{Y} \left\{ \lambda_1 \left(p_{r0}' + \rho_0 \frac{A_0'}{A_0} + p_{r0} \left(\frac{A_0'}{A_0} - \frac{2\alpha R_0'}{Y} + \frac{2}{r} \right) \right) \right\} \right]$$

$$+ \lambda \left(e' + e \left[\rho_0' - \frac{2\alpha R_0'}{Y} \right] \right) + Y Z_{2p} = 0, \tag{3.29}$$

where $Y = 1 + 2\alpha R_0$, $\lambda_1 = \lambda + 1$ and Z_{1p}, Z_{2p} are provided in Appendix. It is assumed that the perturbed quantities $D_1 = D_2 = D$ and $e_1 = e_2 = e$.

Eliminating $\dot{\bar{\rho}}$ from Eq. (3.28) and integrating it with respect to time provides $\bar{\rho}$ as

$$\bar{\rho} = - \left[\frac{2e\rho_0}{Y} + \lambda_1 \left\{ \frac{2\bar{c}}{r}(\rho_0 + 2p_{\perp 0}) + \frac{b}{B_0}(\rho_0 + p_{r0}) \right\} \right.$$
$$\left. + Y Z_{1p} \right] D. \tag{3.30}$$

The Harrison-Wheeler type equation of state associating $\bar{\rho}$ and $\bar{p}_r$ depicts/describes second law of thermodynamics, written as

$$\bar{p}_r = \Gamma \frac{p_{r0}}{\rho_0 + p_{r0}} \bar{\rho}. \tag{3.31}$$

Inserting of $\bar{\rho}$ in above relation, we have

$$\bar{p}_r = -\Gamma \frac{p_{r0}}{\rho_0 + p_{r0}} \left[\frac{2e\rho_0}{Y} + \lambda_1 \frac{2\bar{c}}{r}(\rho_0 + 2p_{\perp 0}) + Y Z_{1p} \right] D$$
$$- \Gamma p_{r0} \frac{b}{B_0} D. \tag{3.32}$$

The expression for $\bar{p}_\perp$ can be found by applying perturbation on field equation (2.12) and eliminating $\bar{p}_\perp$, that turns out to be

$$\bar{p}_\perp = \left\{ \frac{Y\bar{c}}{r} - 2\alpha e \right\} \frac{\ddot{D}}{A_0^2} - \frac{\lambda \bar{\rho}}{\lambda_1} + \left\{ \left(p_{\perp 0} - \frac{\lambda}{\lambda_1} \rho_0 \right) \frac{2\alpha e}{Y} \right.$$
$$\left. + \frac{Z_3}{\lambda_1} \right\} D, \tag{3.33}$$

where Z_3 corresponds to effective part of the field equation and is given in Appendix as Eq. (A.5).

Substituting $\bar{\rho}$, $\bar{p}_r$ and $\bar{p_\perp}$ from Eqs. (3.30), (3.32) and (3.33) in second Bianchi identity (3.29), we obtain

$$\ddot{D}\left[\frac{2\alpha}{A_0^2 Y}\left\{e' + 2e\frac{B_0'}{B_0} - \frac{b}{B_0}R_0'\right\} - 2\alpha B_0^2\left\{\frac{e}{B_0^2 Y}\right\}'\right]$$

$$+ \frac{1}{A_0^2}\left\{\frac{Y\bar{c}}{r} - 2\alpha e\right\} + D\left[\frac{1}{Y}\left\{\lambda_1\left((\rho_0 + p_{r0})\left(\frac{a}{A_0}\right)'\right.\right.\right.$$

$$- 2(\rho_0 + p_{\perp 0})\left(\frac{\bar{c}}{r}\right)'\right) - \frac{2\alpha}{Y}\left\{\lambda\left(e' - \rho_0' - \frac{2\alpha R_0'}{Y}\right)\right.$$

$$+ \lambda_1\left(e'p_{r0} + e\left[p_{r0}' + \rho_0\frac{A_0'}{A_0} + p_{r0}\left(\frac{A_0'}{A_0} + \frac{2}{r}\right)\right.\right.$$

$$\left.\left.\left.- \frac{2\alpha R_0'}{Y}\right)\right]\right)\right\} - \left(\lambda + \lambda_1\Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}\right)\left\{\rho_0\frac{2e}{Y}\right.$$

$$+ \lambda_1\left\{\frac{2\bar{c}}{r}(\rho_0 + 2p_{\perp 0}) + \frac{b}{B_0}(\rho_0 + p_{r0})\right\} + YZ_{1p}\right\}_{,1}$$

$$+ \left\{\frac{A_0'}{A_0} + \frac{2}{r}\frac{\lambda}{\lambda_1} + \Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}\left(\frac{A_0'}{A_0} + \frac{2}{r} - \frac{2\alpha R_0'}{Y}\right)\right.$$

$$+ \lambda_1\left(\Gamma\frac{p_{r0}}{\rho_0 + p_{r0}}\right)'\right\}\left\{\frac{2e\rho_0}{Y} + \lambda_1\left\{\frac{2\bar{c}}{r}(\rho_0 + 2p_{\perp 0})\right.\right.$$

$$\left.\left.+ \frac{b}{B_0}(\rho_0 + p_{r0})\right\} + (Y)Z_{1p}\right\} + \frac{2}{r}\frac{1}{\lambda_1}Z_3\right\} + Z_{2p}\right]$$

$$= 0. \tag{3.34}$$

An ordinary differential equation is retrieved from perturbed form of Ricci scalar, written as

$$\ddot{D}(t) - Z_4(r)D(t) = 0. \tag{3.35}$$

Z_4 is given in [Appendix](#) as Eq. (A.6), all terms of Z_4 are presumed in a way that all the terms remain positive. The solution of Eq. (3.35) takes form

$$D(t) = -e^{\sqrt{Z_4}t}. \tag{3.36}$$

Following subsections provide description of limiting cases appear in stability problem in $f(R, T)$ gravity.

3.1 Newtonian regime

In this approximation, we take $\rho_0 \gg p_{r0}$, $\rho_0 \gg p_{\perp 0}$ and $A_0 = 1$, $B_0 = 1$. Inserting these assumptions along with Eq. (3.36) in Eq. (3.35), we arrive at following stability condition

$$\Gamma < \frac{Z_4 Z_6 + Z_7 + \lambda\rho_0(Z_5 + YZ_{1p(N)})_{,1}}{\lambda_1 p_{r0}Z_5' + \{p_{r0}(\frac{2\alpha R_0'}{Y} - \frac{2}{r})\}Z_5}$$

$$+ \frac{XZ_5 - \frac{2}{r\lambda_1}Z_{3(N)} + YZ_{2p(N)}}{\lambda_1 p_{r0}Z_5' + \{p_{r0}(\frac{2\alpha R_0'}{Y} - \frac{2}{r})\}Z_5}, \tag{3.37}$$

where

$$X = \left(\lambda\rho_0' + \frac{2\lambda}{r\lambda_1}\right), \qquad Z_5 = \frac{2e}{Y} + \lambda_1\left(\frac{2\bar{c}}{r} + b\right),$$

$$Z_6 = -2\alpha^2 bR_0' + Y\frac{\bar{c}}{r},$$

$$Z_7 = \lambda_1\left[\rho_0 a' + 2(p_{r0} + p_{\perp 0})\left(\frac{\bar{c}}{r}\right)'\right]$$

$$+ \frac{2\alpha}{Y}\left[\lambda\left(\frac{2\alpha R_0'}{Y} - \rho_0' + e'\right) + \lambda_1\left\{p_{r0} + e\left[p_{r0}'\right.\right.\right.$$

$$\left.\left.\left.+ p_{r0}\left(\frac{2}{r} - \frac{2\alpha R_0'}{Y}\right)\right]\right\}\right]\right],$$

where $Z_{1p(N)}$ and $Z_{2p(N)}$ are terms of Z_{1p} and Z_{2p} corresponding to Newtonian era. The case when $\alpha \to 0$ and $\lambda \to 0$ corresponds to GR corrections and Γ reads

$$\Gamma < \frac{\frac{\bar{c}}{r}Z_4 + \rho_0 a' + 2(p_{r0} + p_{\perp 0})(\frac{\bar{c}}{r})' + e' - \frac{2}{r^2}(\frac{\bar{c}''}{r} + a'')}{2p_{r0}e' + p_{r0}\frac{4}{r}} \tag{3.38}$$

These results reduce in $f(R)$ gravity for $\lambda \to 0$. The self-gravitating body remains stable in Newtonian regime until the inequality for Γ holds, for which following constraints must be fulfilled.

$$2\alpha^2 bR_0' < Y\frac{\bar{c}}{r}, \qquad 2\alpha R_0' < Y, \qquad \frac{2\alpha R_0'}{Y} > \rho_0' - e'.$$

3.2 Post Newtonian regime

In this approximation we take, $A_0 = 1 - \frac{m_0}{r}$ and $B_0 = 1 + \frac{m_0}{r}$ implying

$$\frac{A_0'}{A_0} = \frac{m_0}{r(r - m_0)}, \qquad \frac{B_0'}{B_0} = \frac{-m_0}{r(r + m_0)},$$

substitution of above assumptions in Eq. (3.34) yields

$$\Gamma < \frac{Z_4 Z_8 + Z_9 + \lambda\rho_0(Z_{10} + YZ_{1p(PN)})_{,1}}{\lambda_1 p_{r0}Z_{10}' + \{p_{r0}(\frac{m_0}{r(r-m_0)} + \frac{2\alpha R_0'}{Y} + \frac{2}{r})\}Z_{10}}$$

$$+ \frac{Z_{11}Z_{10} - \frac{2}{r\lambda_1}Z_{3(PN)} + YZ_{2p(PN)}}{\lambda_1 p_{r0}Z_{10}' + \{p_{r0}(\frac{m_0}{r(r-m_0)} + \frac{2\alpha R_0'}{Y} + \frac{2}{r})\}Z_{10}}, \tag{3.39}$$

where

$$Z_8 = \frac{2\alpha r^2}{(r - m_0)^2}\left\{e' - \frac{r}{r + m_0}\left(bR_0' + 2e\frac{m_0}{r}\right)\right\}$$

$$+ Y\left[\frac{r^2}{(r - m_0)^2}\left\{2\alpha e - Y\frac{\bar{c}}{r}\right\} - \frac{2\alpha(r + m_0)^2}{r^2}\right.$$

$$\left.\times\left\{\frac{er^2}{Y(r + m_0)^2}\right\}'\right]$$

$$Z_9 = \lambda_1 \left\{ \rho_0 \left(\frac{ar}{r - m_0} \right)' - 2(p_{r0} + p_{\perp 0}) \left(\frac{\bar{c}}{r} \right)' \right\}$$

$$- \frac{2\alpha}{Y} \left[(\lambda_1 p_{r0} + \lambda) e' + e \left\{ \lambda_1 \left(p_{r0}' + \frac{\rho_0 m_0}{r(r - m_0)} \right) \right. \right.$$

$$\left. \left. + p_{r0} \left(\frac{2}{r} - \frac{2\alpha R_0'}{Y} \right) \right\} - \lambda \left(\rho_0' - \frac{2\alpha R_0'}{Y} \right) \right]$$

$$Z_{10} = \frac{2e}{Y} + \lambda_1 \left(2\frac{\bar{c}}{r} + \frac{br}{r + m_0} \right),$$

$$Z_{11} = \left(\frac{m_0}{r(r - m_0)} + \frac{2\alpha R_0'}{Y} + \frac{2\lambda}{\lambda_1 r} + \lambda \rho_0' \right)$$

$Z_{1p(PN)}$ and $Z_{2p(PN)}$ are terms of Z_{1p} and Z_{2p} corresponding to post-Newtonian era. Again inequality holds for positive definite terms restricting physical quantities as under
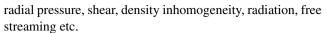
$$\frac{r}{r + m_0} \left(bR_0' + \frac{2em_0}{r} \right) < e',$$

$$2\alpha e - Y\frac{\bar{c}}{r} > \frac{(r^2 - m_0^2)^2}{r^4} \left\{ \frac{er^2}{Y(r + m_0)^2} \right\}'$$

The restrictions imposed on these quantities have significant impact on stellar structure and must be fulfilled to achieve stability of considered model.

4 Summary and discussion

Modified theory can be considered as an effective candidate to explain the issue of cosmic acceleration. In this setting, $f(R, T)$ gravity provides an alternative way without introducing an exotic energy component. The matter geometry coupling results in existence of extra force due to non-geodesic motion of test particles. This work is devoted to examine the impact of $f(R, T)$ model on dynamical instability of locally anisotropic spherical star in $f(R, T)$ gravity. The model $f(R, T) = R + \alpha R^2 + \lambda T$ provides a viable substitute to the exotic matter in our universe (Sharif and Zubair 2012b). The local anisotropy in matter configuration contribute high dissipation of energy via heat flow, radiation transport, shearing stresses etc. that affect stability range largely.

The highly complicated non-linear differential equations corresponding to set of field equations can not be solved generally, that is why we have employed perturbation approach to incorporate this problem. Perturbed dynamical equations constitute collapse equation that describe non-static spherical star. Perturbed second Bianchi identity reveals that instability range of gravitating objects has dependence on adiabatic index Γ. Moreover, the index Γ constitute impressions of various factors such as tangential and radial pressure, shear, density inhomogeneity, radiation, free streaming etc.

It is worth noticing that in comparison to $f(R)$ gravity the dynamical equations of $f(R, T)$ gravity possess extra term of T that depict the more generalized modification of GR. Further, adding the terms of trace T in EH action includes the description of quantum effects or so-called exotic matter. The adiabatic index Γ admits the positive definite terms to maintain stable stellar configuration for both Newtonian and post-Newtonian eras. Inclusion of trace of energy momentum tenor in action (1.1) results in positive addition to Γ and so enhances the stability of stars by slowing down the subsequent collapse.

It is concluded that our results are in agreement with the Γ configuration found in (Sharif and Kausar 2012) for $\lambda \to 0$ presenting $f(R)$ gravity. Local isotropy of the model can be retrieved by setting $p_r = p_\perp = p$ and the results are well consistent with the findings of (Sharif and Yousaf 2014). The terms corresponding to GR corrections can be found by assuming vanishing values of both α and λ.

The extension of present work to discuss stability and expansion free condition in $f(R, T)$ is in process. Furthermore, it is worth stressing that the pattern devised in (Capozziello et al. 2012) can be generalized for various modified gravity theories (Capozziello et al. 2008; Bogdanos et al. 2010; De Laurentis and Capozziello 2011; Roshan and Abbassi 2014). To address instability problem in an alternate and adequate way, the work on Jeans instability criterion can be extended for $f(R, T)$ theory. We are planning to work out this problem (Jeans analysis in $f(R, T)$) in our forthcoming article incorporating the comparison with interstellar medium.

Appendix

$$Z_1(r, t) = f_R A^2 \left[\left\{ \frac{1}{f_R A^2} \left(\frac{f - Rf_R}{2} - \frac{\dot{f}_R}{A^2} \left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C} \right) \right. \right. \right.$$

$$\left. \left. - \frac{f_R'}{B^2} \left(\frac{B'}{B} - \frac{2C'}{C} \right) + \frac{f_R''}{B^2} \right) \right\}_{,0} + \left\{ \frac{1}{f_R A^2 B^2} \right.$$

$$\left. \times \left(\dot{f}_R' - \frac{A'}{A} \dot{f}_R - \frac{\dot{B}}{B} f_R' \right) \right\}_{,1} \right] - \frac{\dot{f}_R}{A^2} \left\{ \left(\frac{\dot{B}}{B} \right)^2 \right.$$

$$\left. + 2 \left(\frac{\dot{C}}{C} \right)^2 + \frac{3\dot{A}}{A} \left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C} \right) \right\} + \frac{\ddot{f}_R}{A^2} \left(\frac{\dot{B}}{B} + \frac{2\dot{C}}{C} \right)$$

$$+ \frac{\dot{A}}{A} (f - Rf_R) - \frac{2f_R'}{B^2} \left\{ \frac{\dot{A}}{A} \left(\frac{B'}{B} - \frac{C'}{C} \right) \right.$$

$$+ \frac{\dot{B}}{B}\left(\frac{2A'}{A} + \frac{B'}{B} + \frac{C'}{C}\right) + \frac{\dot{C}}{C}\left(\frac{A'}{A} - \frac{3C'}{C}\right)\Big\}$$

$$+ \frac{f_R''}{B^2}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{1}{B^2}\left(\dot{f}_R' - \frac{A'}{A}\dot{f}_R\right)$$

$$\times \left(\frac{3A'}{A} + \frac{B'}{B} + \frac{2C'}{C}\right), \tag{A.1}$$

$$Z_2(r,t) = f_R B^2\Bigg[\Bigg\{\frac{1}{f_R A^2 B^2}\left(\dot{f}_R' - \frac{A'}{A}\dot{f}_R - \frac{\dot{B}}{B}f_R'\right)\Bigg\}_{,0}$$

$$+ \Bigg\{\frac{1}{f_R B^2}\left(\frac{R f_R - f}{2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} - \frac{2\dot{C}}{C}\right)\right.$$

$$\left.- \frac{f_R'}{B^2}\left(\frac{A'}{A} + \frac{2C'}{C}\right) + \frac{\ddot{f}_R}{A^2}\right)\Bigg\}_{,1}\Bigg] + (R f_R - f)\frac{B'}{B}$$

$$- \frac{\dot{f}_R}{A^2}\Bigg\{\frac{A'}{A}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{B'}{B}\left(\frac{\dot{A}}{A} - \frac{2\dot{C}}{C}\right)$$

$$+ \frac{2C'}{C}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\Bigg\} + \left(\frac{\dot{A}}{A} + \frac{3\dot{B}}{B} + \frac{2\dot{C}}{C}\right)$$

$$\times \left(\dot{f}_R' - \frac{A'}{A}\dot{f}_R - \frac{\dot{B}}{B}f_R'\right)\frac{1}{A^2}$$

$$- \frac{f_R'}{B^2}\Bigg\{\frac{A'}{A}\left(\frac{A'}{A} + \frac{3B'}{B}\right) + \frac{2C'}{C}\left(\frac{3B'}{B} + \frac{C'}{C}\right)\Bigg\}$$

$$+ \frac{\ddot{f}_R}{A^2}\left(\frac{A'}{A} + \frac{2B'}{B}\right) + \frac{f_R''}{B^2}\left(\frac{A'}{A} + \frac{2C'}{C}\right). \tag{A.2}$$

$$Z_{1p} = 2\alpha A_0^2\Bigg[\frac{1}{A_0^2 B_0^2 Y}\Bigg\{e' - e\frac{A_0'}{A_0} - \frac{b}{B_0}R_0'\Bigg\}\Bigg]_{,1} + \frac{1}{Y}\Bigg[e$$

$$- [\lambda T_0 - \alpha R_0^2]\left(\frac{a}{A_0} + \frac{e}{Y}\right) + \frac{2\alpha}{B_0^2}\Bigg\{\left(\frac{B_0'}{B_0} - \frac{2}{r}\right)$$

$$\times \left(e' - 2R_0'\left(\frac{a}{A_0} + \frac{b}{B_0}\right) + \frac{2\alpha e}{Y}R_0'\right)$$

$$+ R_0''\left(\frac{2a}{A_0} + \frac{b}{B_0}\right) - 2R_0'\left(\frac{b}{B_0}\left(\frac{2A_0'}{A_0} + \frac{B_0'}{B_0} + \frac{1}{r}\right)\right.$$

$$\left.+ \frac{\bar{c}}{r}\left(\frac{A_0'}{A_0} - \frac{3}{r}\right)\right) + \left(e' - e\frac{A_0'}{A_0}\right)$$

$$\times \left(\frac{3A_0'}{A_0} + \frac{B_0'}{B_0} + \frac{2}{r}\right)\Bigg\}\Bigg] \tag{A.3}$$

$$Z_{2p} = B_0^2 Y\Bigg[\frac{1}{B_0^2 Y}\Bigg\{e - \frac{2\alpha}{B_0^2}\Bigg\{\left(\frac{A_0'}{A_0} + \frac{2}{r}\right)$$

$$\times \left(e - \left[\frac{2\alpha e}{Y} + \frac{4b}{B_0}\right]R_0'\right) + R_0'\left[\left(\frac{a}{A_0}\right)' + \left(\frac{\bar{c}}{r}\right)'\right]\Bigg\}$$

$$- [\lambda T_0 - \alpha R_0^2]\left(\frac{b}{B_0} + \frac{e}{Y}\right)\Bigg\}\Bigg]_{,1} + b B_0 Y\Bigg[\frac{1}{B_0^2 Y}$$

$$\times \Bigg\{\lambda T_0 - \alpha R_0^2 - \frac{4\alpha}{B_0^2}\left(\frac{A_0'}{A_0} + \frac{2}{r}\right)R_0'\Bigg\}\Bigg]_{,1}$$

$$+ \frac{2\alpha}{B_0^2}\Bigg[R_0''\Bigg\{\left(\frac{a}{A_0}\right)' - 2\left(\frac{A_0'}{A_0} + \frac{2}{r}\right)\left(\frac{b}{B_0} + \frac{e}{Y}\right)$$

$$+ \left(\frac{\bar{c}}{r}\right)'\Bigg\} - R_0'\Bigg\{\frac{A_0'}{A_0}\Bigg[2\left(\frac{a}{A_0}\right)' + 3\left(\frac{b}{B_0}\right)'\Bigg]$$

$$+ 3\frac{B_0'}{B_0}\Bigg[\left(\frac{a}{A_0}\right)' + 2\left(\frac{\bar{c}}{r}\right)'\Bigg] + \frac{2}{r}\Bigg[\left(3\frac{b}{B_0}\right)'$$

$$+ 2\left(\frac{\bar{c}}{r}\right)'\Bigg]\Bigg\} + \left(\frac{2b}{B_0}R_0' - e\right)\Bigg\{3\frac{B_0'}{B_0}\left(\frac{A_0'}{A_0} + \frac{2}{r}\right)$$

$$+ \left(\frac{A_0'}{A_0}\right)^2 + \frac{2}{r^2}\Bigg\}\Bigg] + e\frac{B_0'}{B_0}$$

$$- [\lambda T_0 - \alpha R_0^2]\left(\frac{b}{B_0} + \frac{2e}{Y}\frac{B_0'}{B_0}\right) \tag{A.4}$$

$$Z_3 = \frac{Y}{B_0^2}\Bigg[\frac{a''}{A_0} + \frac{\bar{c}''}{r} - \frac{A_0''}{A_0}\left(\frac{a}{A_0} + \frac{2b}{B_0}\right) + \frac{A_0'}{A_0}{}'$$

$$\times \Bigg\{\frac{2b}{B_0}\left(\frac{B_0'}{B_0} - \frac{1}{r}\right) + \frac{\bar{c}}{r} - \left(\frac{b}{B_0}\right)'\Bigg\} + \frac{B_0'}{B_0}\Bigg\{\frac{2b B_0'}{r B_0}$$

$$- \left(\frac{a}{A_0}\right)' - \left(\frac{\bar{c}}{r}\right)'\Bigg\} + \frac{1}{r}\Bigg\{\left(\frac{a}{A_0}\right)' - \left(\frac{b}{B_0}\right)'\Bigg\}\Bigg]$$

$$- \frac{2\alpha e}{Y}\Bigg\{\frac{\lambda T_0 - \alpha R_0^2}{2} - \frac{2\alpha}{B_0^2}\left(R_0'\left(\frac{A_0'}{A_0} - \frac{B_0'}{B_0} + \frac{1}{r}\right)\right.$$

$$\left.- R_0''\right)\Bigg\} - \frac{2\alpha}{B_0^2}\Bigg\{e'' + \frac{2b}{B_0}R_0''$$

$$+ \left(\frac{A_0'}{A_0} - \frac{B_0'}{B_0} + \frac{1}{r}\right)\left(\frac{2b}{B_0}R_0' - e'\right)\Bigg\} \tag{A.5}$$

$$Z_4 = -\frac{r A_0^2 B_0}{br + 2B_0\bar{c}}\Bigg[\frac{e}{2} - \frac{2\bar{c}}{r^3} - \frac{1}{A_0 B_0^2}\Bigg\{A_0''\Bigg[\frac{a}{A_0} + \frac{2b}{B_0}\Bigg]$$

$$- \frac{1}{B_0}\left(a'B_0' + a'' + A_0'b' - A_0'B_0'\Bigg[\frac{a}{A_0} + \frac{3b}{B_0}\Bigg]\right)$$

$$+ \frac{2}{r}\Bigg\{a' + \bar{c}'A_0' - A_0'\Bigg[\frac{a}{A_0} + \frac{2b}{B_0} + \frac{\bar{c}}{r}\Bigg]\Bigg\} + \frac{A_0}{r}$$

$$\times \Bigg\{\bar{c}'' - \frac{b'}{B_0} - \frac{B_0'\bar{c}'}{B_0} + \frac{3b}{B_0} + \frac{\bar{c}}{r}\Bigg\}$$

$$\times \Bigg\{\bar{c}'' - \frac{b'}{B_0} - \frac{B_0'\bar{c}'}{B_0} + \frac{2}{r^2}\Bigg[\bar{c}' - \frac{b}{B_0}\frac{\bar{c}}{r}\Bigg]\Bigg\}\Bigg] = 0. \tag{A.6}$$

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