Astrophys Space Sci (2015) 355:389–397
DOI 10.1007/s10509-014-2174-6

ORIGINAL ARTICLE

Stability analysis of expansion-free charged planar geometry

M. Sharif · M. Zaeem Ul Haq Bhatti

Received: 28 August 2014 / Accepted: 16 October 2014 / Published online: 4 December 2014

Abstract This paper analyzes the stability of a collapsing matter distribution enclosed by plane symmetry in the presence of electromagnetic field. The field equations, matching conditions as well as conservation laws are formulated for non-static planar geometry. We apply perturbation to obtain the dynamical equation for Newtonian and post-Newtonian eras with expansion-free scenario. The role of electric charge with anisotropic matter configuration is studied in the stability regions. We conclude that this system becomes more stable as compared to the uncharged case.

Keywords Anisotropy · Electromagnetic field · Instability

1 Introduction

In order to understand physical behavior of the early stages of the universe, a particular attention is given to the relativistic cosmology. It is well-known that immediately after the big-bang, instinctive symmetry breaking has given rise to certain cosmological structures. Among them plane symmetric models are believed to play a vital role because they are treated as viable seeds for galaxy formation of the early universe. The physical properties of such models like radius, mass and red-shift generally depend on the matter profile being used. The plane symmetry is considered to be less restrictive than spherical symmetry and widely discussed in the literature (Anguige 2000; Nouri-Zonoz and Tavanfar 2001; Yazadjiev 2003; Saha and Shikin 2005). Pradhan et al. (2007) investigated a class of planar solutions and concluded that such solutions will be non-static if they have non-vanishing shear. Sharif and Yousaf (2012) explored several non-static plane symmetric models with anisotropic matter.

Zeldovich et al. (1993) pointed out the occurrence of magnetic fields for a variety of astrophysical phenomena on galactic scale. Since the electromagnetic field has a cosmological horizon, so it is essential to include it in the stress energy tensor of the early universe (Harrison 1973). Pradhan and Panday (2005) analyzed the behavior of magnetic field for inhomogeneous plane symmetric cosmological model with a variable cosmological parameter. Varela et al. (2010) discussed charged anisotropic matter field of spherical star with linear as well as non-linear equations of state. Sharif and Azam (2012) investigated charge effects on the dynamical instability of expansionfree spherical collapse. Consequently, the study of charged fluid structure is of great significance in relativistic astrophysics.

The instability issue of magnetized star has been interesting as only stable models are physically viable. Chandrasekhar (1964) found a variational principle to examine the stability of spherical solutions with isotropic source. This issue has been addressed by many authors after the pioneering work of Tayler (1973) who found that for purely toroidal magnetic fields, stars would be unstable. In scalar-tensor theory, Harada (1997) provided the stability analysis for spherically symmetric star and concluded that stability depends on the choice of coupling function. Seifert (2007) explored the stability of static spherical solutions through perturbations in three alternative theories of gravity. Yoshida et al. (2012) constructed magnetized stars which are

M. Sharif (✉) · M.Z.U.H. Bhatti
Department of Mathematics, University of the Punjab,
Quaid-e-Azam Campus, Lahore 54590, Pakistan
e-mail: msharif.math@pu.edu.pk

M.Z.U.H. Bhatti
e-mail: mzaeem.math@pu.edu.pk

assumed to be much stable as compared to existing relativistic models.

There are many phases during the evolution of the universe in which anisotropic pressure may arise such as mixture of two fluids, solid core, slow rotation, pion condensation and phase transition (Sawyer 1972; Letelier 1980; Herrera and Santos 1995, 1997). Many researchers investigated solutions of the field equations characterizing the inner gravitational field as anisotropic source (Consenza et al. 1981; Bayin 1982; Krori et al. 1984; Maharaj and Maartens 1989). Govinder et al. (1998) examined the effects of anisotropy in the dynamical behavior of a self-gravitating collapsing star. It is found that if a star before the collapsing process is isotropic in pressure then anisotropy may flourish at a later stage due to the presence of viscosity (Chan 2000; Pinheiro and Chan 2008). Sharma and Tikekar (2012) investigated the impact of anisotropy on various parameters of the collapsing star.

It is well-consistent to form a vacuum cavity within the matter distribution after the central explosion. Skripkin (1960) presented the first model which satisfies the expansion-free condition for non-dissipative isotropic matter. Since then it remains the area of interest of theoretical and astrophysical problems. In order to investigate physical models under expansion-free scenario, we require system evolution with inhomogeneous energy density and anisotropic pressure for spherical (Herrera et al. 2009) and plane (Sharif and Yousaf 2012) symmetric configurations. It is observed that expansion-free condition together with junction conditions rule out the Skripkin model. We would like to mention here that zero expansion is a sufficient but not a necessary condition for the appearance of cavities. The phenomenon of cavity formation under kinematical condition other than zero expansion has been discussed in spherical (Herrera et al. 2010) as well as plane symmetric models (Sharif and Bhatti 2014c) with anisotropic matter distribution.

Recently, Herrera et al. (2012) provided the instability ranges for spherical fluid configuration with vanishing expansion at both Newtonian (N) and post-Newtonian (pN) eras. We have investigated the dynamical instability of cylindrical as well as spherical stars with axial symmetry with and without expansion scalar (Sharif and Bhatti 2013a, 2013b, 2014a, 2014b). Sharif and Yousaf (2013a, 2013b, 2013c, 2013d, 2014) also studied N and pN regions for different symmetric backgrounds in $f(R)$ theory of gravity. Most recently, Sharif and Azam (2014) examined the role played by anisotropic stresses during the expansion-free collapse of plane symmetric spacetime.

We extend this study by including the effects of electromagnetic field and explore the stability of planar symmetry. The layout of this paper is as follows. In the next section, we write down the field equations with conservation laws and junction conditions. Section 3 is devoted for the perturbation technique to obtain the perturbed form of these equations. In Sect. 4, we develop our main dynamical equation by taking zero expansion scalar which is then used to identify the N and pN regions. In the last section, we furnish our final remarks.

2 Matter distribution and the field equations

The non-static plane symmetric spacetime for the interior region is (Sharif and Bhatti 2012)

$$ds_-^2 = B^2(t,z)\big(dx^2+dy^2\big) + C^2(t,z)dz^2 - A^2(t,z)dt^2. \tag{1}$$

The matter distribution is considered to be anisotropic in pressure (Herrera et al. 2012)

$$T^-_{\alpha\beta} = (P_\perp+\mu)V_\beta V_\alpha + (P_z - P_\perp)\chi_\alpha\chi_\beta + P_\perp g_{\alpha\beta}, \tag{2}$$

here μ, $P_\perp$, P_z, V_α and χ_α are the energy density, anisotropic stresses, four-velocity and unit four-vector in z-direction, respectively. The four-vectors in comoving coordinates are taken such that

$$\chi^\alpha = C^{-1}\delta^\alpha_3, \qquad V^\alpha = A^{-1}\delta^\alpha_0,$$
$$l^\alpha = A^{-1}\delta^\alpha_0 + C^{-1}\delta^\alpha_3,$$

satisfying $V^\alpha V_\alpha = -1$, $\chi^\alpha V_\alpha = 0$, $\chi^\alpha\chi_\alpha = 1$. The electromagnetic field is given as follows (Sharif and Bhatti 2012)

$$S_{\alpha\beta} = \frac{1}{4\pi}\left(F^\gamma_\alpha F_{\beta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta}\right),$$

where $F_{\alpha\beta} = -\phi_{\alpha,\beta} + \phi_{\beta,\alpha}$ is the strength field tensor while ϕ_β represents the four-potential. This electromagnetic field must obey the Maxwell field equations given by

$$F^{\alpha\beta}_{;\beta} = \mu_0 J^\alpha, \qquad F_{[\alpha\beta;\gamma]} = 0, \tag{3}$$

here J_α and $\mu_0 = 4\pi$ indicate four-current and magnetic permeability, respectively. In comoving coordinates, the four-potential and four-current are $\phi^\alpha = \phi\delta^\alpha_0$, $J^\alpha = \xi V^\alpha$, where ϕ, ξ are functions of t and z and represent the scalar potential and charge density, respectively.

The non-zero components of the Maxwell field equations can be obtained by using Eq. (3) as follows

$$\frac{\partial^2\phi}{\partial z^2} - \left(\frac{A'}{A} + \frac{C'}{C} - \frac{2B'}{B}\right)\frac{\partial\phi}{\partial z} = \xi\mu_0 AC^2, \tag{4}$$

$$\frac{\partial^2\phi}{\partial t\partial z} - \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{2\dot{B}}{B}\right)\frac{\partial\phi}{\partial z} = 0. \tag{5}$$

Here prime and dot show z and t differentiations, respectively. Integrating Eq. (4) with respect to z leads to

$$\phi' = \frac{\mu_0 s(z) AC}{B^2}, \quad \text{where } s(z) = \int_0^z \xi C B^2 dz, \tag{6}$$

which equivalently satisfies Eq. (5). Making use of this in the Maxwell field tensor, it leads to the non-zero components of Einstein–Maxwell field equations (i.e. $G_{\alpha\beta} = 8\pi(T_{\alpha\beta} + S_{\alpha\beta})$) as follows

$$\begin{aligned} 8\pi\mu A^2 + \left(\frac{\mu_0 s A}{B^2}\right)^2 &= \frac{\dot{B}}{B}\left(\frac{2\dot{C}}{C} + \frac{\dot{B}}{B}\right) \\ &\quad - \left(\frac{A}{C}\right)^2\left[\frac{2B''}{B} - \left(\frac{2C'}{C} - \frac{B'}{B}\right)\frac{B'}{B}\right], \end{aligned} \tag{7}$$

$$0 = -2\left(\frac{\dot{B}'}{B} - \frac{A'\dot{B}}{AB} - \frac{B'\dot{C}}{BC}\right), \tag{8}$$

$$\begin{aligned} 8\pi P_\perp B^2 + \left(\frac{\mu_0 s}{B}\right)^2 &= -\left(\frac{B}{A}\right)^2\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{B}\dot{C}}{BC}\right] \\ &\quad + \left(\frac{B}{C}\right)^2\left[\frac{A''}{A} + \frac{B''}{B} - \frac{A'}{A}\left(\frac{C'}{C} - \frac{B'}{B}\right) - \frac{B'C'}{BC}\right], \end{aligned} \tag{9}$$

$$\begin{aligned} 8\pi P_z C^2 - \left(\frac{\mu_0 s C}{B^2}\right)^2 &= -\left(\frac{C}{A}\right)^2\left[\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{2\dot{A}\dot{B}}{AB}\right] \\ &\quad + \left(\frac{B'}{B}\right)^2 + \frac{2A'B'}{AB}. \end{aligned} \tag{10}$$

Three kinematical quantities, i.e., expansion scalar, four-acceleration and shear tensor, describing the irrotational flow of the fluid distribution yield

$$\begin{aligned} &\Theta = \frac{1}{A}\left(2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \qquad a_3 = \frac{A'}{A}, \\ &a^2 = a^\alpha a_\alpha = \left(\frac{A'}{AC}\right)^2, \\ &\sigma^2 = \frac{1}{2}\sigma_{\alpha\beta}\sigma^{\alpha\beta} = \frac{1}{9}F^2 \quad \text{with } F = \frac{1}{A}\left(-\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \end{aligned} \tag{11}$$

The decomposition of the Weyl tensor ($C_{\alpha\mu\beta\nu}$) leads to its electric and magnetic components for which the magnetic part vanishes while the electric part is $E_{\alpha\beta} = C_{\alpha\mu\beta\nu}V^\mu V^\nu$, whose non-vanishing components are $E_{11} = \frac{1}{3}B^2\varepsilon = E_{22}$, $E_{33} = -\frac{2}{3}C^2\varepsilon$, where

$$\begin{aligned} \varepsilon = &-\frac{1}{2A^2}\left[\frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC}\right] \\ &- \frac{1}{2C^2}\left[\frac{A'B'}{AB} - \frac{B'^2}{B^2} + \frac{A'C'}{AC} - \frac{B'C'}{BC} - \frac{A''}{A} + \frac{B''}{B}\right]. \end{aligned}$$

It can be written in an alternative way by using projection tensor ($h_{\alpha\beta} = V_\alpha V_\beta + g_{\alpha\beta}$) and unit four-vector in z-direction as

$$E_{\alpha\beta} = \varepsilon\left(\chi_\alpha\chi_\beta - \frac{1}{3}h_{\alpha\beta}\right).$$

The mass function under the effects of electromagnetic field takes the form (Zannias 1990)

$$m(t,z) = \frac{B}{2}\left(\frac{s^2\mu_0^2}{B^2} + \frac{\dot{B}^2}{A^2} - \frac{B'^2}{C^2}\right). \tag{12}$$

In order to join different geometries of spacetime across the boundary, Darmois (1927) introduced matching conditions based on two fundamental forms. Here we explore these matching conditions for our planar geometry in the interior with a suitable exterior spacetime outside the hypersurface ($\Sigma^{(e)}$). The exterior spacetime is defined as follows (Chao-Guang 1995)

$$ds_+^2 = \left(\frac{2M(\nu)}{Z} - \frac{Q^2}{Z^2}\right)d\nu^2 - 2dZd\nu + Z^2\left(dX^2 + dY^2\right),$$

where ν, $M(\nu)$ and Q are the retarded time, total mass and charge, respectively. The continuity of first and second fundamental forms over $\Sigma^{(e)}$ yield

$$\begin{aligned} &M \overset{\Sigma^{(e)}}{=} m(t,z) \quad \Leftrightarrow \quad s(z) \overset{\Sigma^{(e)}}{=} Q, \\ &P_z - \frac{s^2}{2B^4}\left(\mu_0^2 - 1\right) \overset{\Sigma^{(e)}}{=} 0. \end{aligned} \tag{13}$$

It shows that the masses for interior and exterior regions will be equal if and only if they are filled with the same amount of charge while the pressure in z-direction with the contribution of electromagnetic field vanishes over the boundary. If we take boundary surface between the cavity and matter to be $\Sigma^{(i)}$, then the matching of Minkowskian geometry within the cavity to the matter distribution leads to

$$m \overset{\Sigma^{(i)}}{=} 0, \qquad P_z \overset{\Sigma^{(i)}}{=} 0.$$

The properties of expansion-free spherically symmetric self-gravitating fluid have also been discussed by Herrera et al. (2008). The conservation law, $T^{-\alpha\beta}_{;\beta} = 0$, has the following independent components

$$\dot{\mu} + (\mu + P_z)\frac{\dot{C}}{C} + 2(\mu + P_\perp)\frac{\dot{B}}{B} = 0, \tag{14}$$

$$P_z{}' + (\mu + P_z)\frac{A'}{A} + 2(P_z - P_\perp)\frac{B'}{B} - 4\pi\left(BE' + 2B'E\right)\frac{E}{B} = 0, \tag{15}$$

where $E = \frac{\mu_0 s}{4\pi B^2}$. It is found that only one dynamical equation has the contribution of electromagnetism.

3 Perturbation technique

In this section, we perturb the metric coefficient as well as matter variables upto first order in ε, where $0 < \varepsilon \ll 1$ is known as the perturbation parameter. We consider static state initially and then enters into the non-static phase with same time dependence. The perturbed equations help us to identify the N and pN eras during stability analysis through collapse equation. The perturbation scheme is defined as follows (Herrera et al. 2012; Sharif and Bhatti 2013b, 2014b; Sharif and Yousaf 2013a, 2013c; Chan et al. 1993, 1994)

$$P_z(t,z) = P_{z0}(z) + \varepsilon\bar{P}_z(t,z), \tag{16}$$

$$P_\perp(t,z) = P_{\perp 0}(z) + \varepsilon\bar{P}_\perp(t,z), \tag{17}$$

$$\mu(t,z) = \mu_0(z) + \varepsilon\bar{\mu}(t,z), \tag{18}$$

$$A(t,z) = A_0(z) + \varepsilon a(z)T(t), \tag{19}$$

$$B(t,z) = B_0(z) + \varepsilon b(z)T(t), \tag{20}$$

$$C(t,z) = C_0(z) + \varepsilon c(z)T(t), \tag{21}$$

$$E(t,z) = E_0(z) + \varepsilon e(z)T(t), \tag{22}$$

$$m(t,z) = m_0(z) + \varepsilon\bar{m}(t,z), \tag{23}$$

$$\Theta(t,z) = \varepsilon\bar{\Theta}(t,z). \tag{24}$$

The static part of the field equations is found by using Eqs. (7)–(10) and (16)–(22) with $B_0 = z$ as follows

$$8\pi A_0^2\left(2\pi E_0^2 + \mu_0\right) = \frac{A_0^2}{zC_0^2}\left(\frac{2C_0'}{C_0} - \frac{1}{z}\right), \tag{25}$$

$$8\pi\left(2\pi E_0^2 - P_{z0}\right) = \frac{1}{z}\left(\frac{1}{z} + \frac{2A_0'}{A_0}\right), \tag{26}$$

$$8\pi\left(2\pi E_0^2 + P_{\perp 0}\right) = \frac{1}{C_0^2}\left(\frac{A_0''}{A_0} - \frac{A_0'}{A_0}\left(\frac{C_0'}{C_0} - \frac{1}{z}\right) - \frac{C_0'}{zC_0}\right). \tag{27}$$

The corresponding non-static part after perturbation leads to

$$8\pi\left[\frac{2aT}{A_0}\left(2\pi E_0^2 + \mu_0\right) + 4\pi TeE_0 + \bar{\mu}\right] = \frac{TA_0^2}{C_0^2}\left[2\left(\frac{a}{A_0} - \frac{c}{C_0}\right)\left\{\frac{1}{z}\left(\frac{2C_0'}{C_0} - \frac{1}{z}\right)\right\} + \frac{1}{z}\left(\frac{2C_0'}{C_0} - \frac{1}{z}\right)\left(b' - \frac{b}{z}\right) - \frac{2}{z}\left(\frac{c}{C_0} - \frac{b}{z}\right)'\right],$$

$$0 = \left(-c' + \frac{bA_0'}{zA_0} + \frac{c}{zC_0}\right)\dot{T}, \tag{28}$$

$$8\pi C_0^2\left[\frac{2Tc}{C_0}\left(P_{z0} - 2\pi E_0^2\right) + \bar{P}_z - 4\pi E_0 eT\right] = \frac{2}{z}T\left(b' - \frac{b}{z}\right) + 2T\frac{A_0'}{A_0 z}\left(\frac{a'}{A_0'} + b' - \frac{a}{A_0} - \frac{b}{z}\right) - \frac{bC_0^2}{zA_0^2}\ddot{T}. \tag{29}$$

The mass function (12) has the following static and non-static parts

$$m_0 = 8\pi^2 z^3 E_0^2 - \frac{z}{2C_0^2}, \tag{30}$$

$$\bar{m} = 8\pi^2 z^2 E_0 T(2ez + 3bE_0) - \left\{z\left(b' - \frac{2c}{C_0}\right) - b\right\}\frac{T}{2C_0^2}. \tag{31}$$

The conservation laws obtained in Eqs. (14) and (15) have only one static part as follows

$$P_{z0}' + \frac{A_0'}{A_0}(\mu_0 + P_{z0}) + \frac{2}{z}(P_{z0} - P_{\perp 0}) - 4\pi\left(zE_0' + 2E_0\right)\frac{E_0}{z} = 0, \tag{32}$$

while the perturbed part turns out to be

$$\dot{\bar{\mu}} + (\mu_0 + P_{z0})\frac{\dot{T}c}{C_0} + \frac{2\dot{T}b}{z}(\mu_0 + P_{\perp 0}) = 0, \tag{33}$$

$$\bar{P}_z' + \frac{A_0'}{A_0}(\bar{\mu} + \bar{P}_z) + T\frac{A_0'}{A_0}(\mu_0 + P_{z0})\left(\frac{a'}{A_0'} - \frac{a}{A_0}\right) + \frac{2}{z}(\bar{P}_z - \bar{P}_\perp) + \frac{2T}{z}\left(b' - \frac{b}{z}\right)(P_{z0} - P_{\perp 0}) - 4\pi T\frac{E_0}{z}\left[\left(zE_0' + 2E_0\right)\left(\frac{e}{E_0} - \frac{b}{z}\right) + zE_0'\left(\frac{b}{z} + \frac{e'}{E_0'}\right) + 2E_0\left(b' + \frac{e}{E_0}\right)\right] = 0. \tag{34}$$

Integrating Eq. (33) with respect to t, it follows that

$$\bar{\mu} = -\left[(\mu_0 + P_{z0})\frac{c}{C_0} + \frac{2b}{z}(\mu_0 + P_{\perp 0})\right]T. \tag{35}$$

Using the perturbation scheme on the matching conditions (13), we find that the static as well as non-static parts of the radial pressure become zero over the boundary as

$$P_{z0} \overset{\Sigma^{(e)}}{=} 0, \qquad \bar{P}_z \overset{\Sigma^{(e)}}{=} 0. \tag{36}$$

Moreover, Eq. (29) with (36) yields

$$\ddot{T}-\alpha(z)T\overset{\Sigma^{(e)}}{=}0, \tag{37}$$

where

$$\alpha(z)=\frac{zA_0^2}{bC_0^2}\left[16\pi^2C_0E_0(cE_0+eC_0)+\frac{1}{z^2}\left(b'-\frac{b}{z}\right)+\frac{A_0'}{zA_0}\left(\frac{a'}{A_0}-\frac{a}{A_0}+b'-\frac{b}{z}\right)\right]. \tag{38}$$

Equation (37) is solved to obtain stable as well as unstable states of the collapsing planar geometry. Since our aim is to investigate the instability regions, thus we carry our systematic analysis only with its unstable part

$$T(t)\overset{\Sigma^{(e)}}{=}-\exp\left(\sqrt{\alpha(z)}t\right), \tag{39}$$

here α is chosen to be positive. This shows that the system goes in the collapsing state with $T(-\infty)=0$ at $t=-\infty$ keeping it in static state.

4 Dynamical instability and expansion-free condition

This section aims to investigate the dynamical equation for zero expansion which is used to build the instability ranges for N and pN regions. We consider a state in N regime such that the static part of energy density is much greater than the static parts of the principal stresses, i.e., $\mu_0\gg P_{z0}$, $\mu_0\gg P_{\perp 0}$ while in pN limit, we consider the metric coefficients such that

$$A_0=1-\frac{m_0}{z}+\frac{Q^2}{2z^2},\qquad C_0=1+\frac{m_0}{z}-\frac{Q^2}{2z^2},$$

where we have chosen $c=G=1$ in relativistic units, where c is the speed of light and $c\neq c(z)$. Using the value of C_0^2 from Eq. (30) in (25) and (26), it follows that

$$\frac{C_0'}{C_0}=\frac{4\pi z^3\mu_0-m_0+16\pi^2E_0^2z^3}{2z(8\pi^2z^3E_0^2-m_0)},\qquad \frac{A_0'}{A_0}=\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}. \tag{40}$$

Using $\frac{A_0'}{A_0}$ from Eq. (40) in (32), the dynamical equation in relativistic units turns out to be

$$P_{z0}'=-\left[\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}\right](\mu_0+P_{z0})+\frac{2}{z}(P_{\perp 0}-P_{z0})+\frac{4\pi E_0}{z}\left(zE_0'+2E_0\right), \tag{41}$$

and transforming the above equation in cgs units, we have

$$P_{z0}'=-\left[\frac{8\pi c^{-2}z^3P_{z0}+2m_0-32c^{-2}\pi^2E_0^2z^3}{2z(16Gc^{-4}\pi^2z^3E_0^2-2Gc^{-2}m_0)}\right]\times\left(\mu_0+c^{-2}P_{z0}\right)+\frac{2}{z}(P_{\perp 0}-P_{z0})+\frac{4\pi E_0}{z}\left(zE_0'+2E_0\right). \tag{42}$$

This equation can be expanded upto order c^{-4} to get the terms associated with c^0, c^{-2} and c^{-4} which correspond to N, pN and parameterized post-Newtonian (ppN) regions, respectively, as follows

$$\begin{aligned}P_{z0}'={}&\frac{2}{z}(P_{\perp 0}-P_{z0})+\frac{4\pi E_0}{z}\left(zE_0'+2E_0\right)\\&-\frac{G}{c^2z^3}\left(2G\mu_0m_0^2+P_{z0}m_0z+4\pi\mu_0P_{z0}z^4-16\pi^2E_0^2\mu_0z^4\right)\\&-\frac{G}{c^4z^4}\left(4G^2\mu_0m_0^2+2GP_{z0}m_0^2z+4\pi\mu_0P_{z0}z^4-32\pi^2GE_0^2m_0\mu_0z^4-16\pi^2E_0^2P_{z0}z^5\right).\end{aligned} \tag{43}$$

The expansion scalar can be perturbed to obtain its static and non-static parts using Eqs. (11) and (24) as

$$\bar{\Theta}=\left(\frac{c}{C_0}+\frac{2b}{z}\right)\frac{\dot{T}}{A_0}.$$

It is interesting to mention here that matter with vanishing expansion continues to change with the evolution of time without being compressed resulting a naked singularity by slowing down the apparent horizon formation. Thus, we specialize our study to the zero expansion ($\bar{\Theta}=0$) so that

$$\frac{c}{C_0}=-\frac{2b}{z}. \tag{44}$$

Inserting the above equation in Eq. (28) and then its integration leads to

$$b=h_1\frac{A_0}{z^2}, \tag{45}$$

where h_1 is the integration constant.

The perturbed energy density and pressure in the z-direction ($\bar{\mu}$ and $\bar{P}_z$) can be related through stiffness parameter Γ (which is chosen as constant in our stability analysis) through Harrison–Wheeler equation of state (Harrison et al. 1965)

$$\bar{P}_z=\Gamma\frac{P_{z0}}{\mu_0+P_{z0}}\bar{\mu}. \tag{46}$$

The non-static part of the energy density can be found from Eqs. (35) and (44) as

$$\bar{\mu}=(P_{z0}-P_{\perp 0})T\frac{2b}{z}, \tag{47}$$

describing the relation of perturbed energy density with the static anisotropic pressure. Substituting the above equation in Eq. (46), it leads to

$$\bar{P}_z=\Gamma\frac{2bP_{z0}(P_{z0}-P_{\perp 0})}{z(\mu_0+P_{z0})}T. \tag{48}$$

It is noted from the above equation that $\bar{P}_z$ belongs to ppN regime, thus we neglect these terms in our following analysis. From the static part of the second conservation law, we have

$$\frac{A_0'}{A_0}=-\frac{1}{\mu_0+P_{z0}}\left[P_{z0}'+\frac{2(P_{z0}-P_{\perp 0})}{z}+\frac{4\pi E_0}{z}\left(zE_0'+2E_0\right)\right]. \tag{49}$$

The main dynamical equation can be obtained from Eqs. (25), (34) and (39) to examine the instability regions under N and pN limits as

$$\begin{aligned}&\kappa(\mu_0+P_{z0})z\left(\frac{a}{A_0}\right)'+\frac{2z^2\alpha}{A_0^2B_0^2}\left(\frac{b}{z}+\frac{c}{C_0}\right)\\&-\frac{4z^2}{B_0^2C_0^2}\left[\frac{A_0''}{A_0}-\frac{C_0'}{zC_0}-\frac{A_0'}{A_0}\left(\frac{C_0'}{C_0}-\frac{1}{z}\right)\right]\\&-\frac{2z^2}{B_0^2C_0^2}\left[\frac{A_0''}{A_0}\left(\frac{a''}{A_0''}-\frac{a}{A_0}\right)-\frac{z}{C_0}\left(\frac{C_0}{z}\right)'\left(\frac{a}{A_0}\right)'\right.\\&\left.+\frac{2}{z}\left(\frac{c}{C_0}\right)'-\frac{A_0'}{A_0}\left(\frac{c}{C_0}-\frac{b}{z}\right)'+\frac{C_0'}{C_0}\left(\frac{b}{z}\right)'\right]\\&+\kappa^2eE_0+4\kappa b\left(P_{\perp 0}+2\pi E_0^2\right)+2\kappa z(P_{z0}-P_{\perp 0})\left(\frac{b}{z}\right)'\\&-4\pi\kappa E_0\left[\left(zE_0'+2E_0\right)\left(\frac{e}{E_0}-\frac{b}{z}\right)+bE_0'\right.\\&\left.+ze'+2e+2b'\right]=0.\end{aligned} \tag{50}$$

The expressions for $\frac{A_0''}{A_0B_0^2}$ and $(\frac{a}{A_0})'$ from Eqs. (27) and (29) turn out to be

$$\frac{A_0''}{A_0C_0^2}=8\pi\left(P_{\perp 0}+2\pi E_0^2\right)+\frac{A_0'}{A_0C_0^2}\left(\frac{C_0'}{C_0}-\frac{1}{z}\right)+\frac{C_0'}{zC_0^3}, \tag{51}$$

and

$$\begin{aligned}\left(\frac{a}{A_0}\right)'=\frac{z}{2}&\left[8\pi C_0^2\left\{\frac{2c}{C_0}\left(P_{z0}-2\pi E_0^2\right)-4\pi eE_0\right\}\right.\\&\left.+\frac{2\alpha bC_0^2}{zA_0}-\left(\frac{2A_0'}{A_0}+\frac{2}{z}\right)\left(\frac{b}{z}\right)'\right].\end{aligned} \tag{52}$$

Substituting Eqs. (45), (51) and (52) in (50) and discarding the ppN terms ($\bar{P}_z$ and $\bar{\mu}\frac{A_0'}{A_0}$), it follows that

$$\begin{aligned}&\frac{\kappa z^2}{2}(\mu_0+P_{z0})\left[\kappa C_0^2\left\{\frac{2c}{C_0}\left(P_{z0}-2\pi E_0^2\right)-4\pi eE_0\right\}\right.\\&\left.+\frac{2\alpha cC_0^2}{z^3}-2\left(\frac{cA_0}{z^3}\right)'\left\{\frac{A_0'}{A_0}-\frac{1}{z}\right\}\right]\\&+\frac{2\alpha}{A_0^2}\left(\frac{cA_0}{z^3}+\frac{c}{C_0}\right)-32\pi\left(P_{\perp 0}+2\pi E_0^2\right)-4\frac{A_0'}{A_0C_0^2}\\&+\frac{4}{C_0^2}\left\{\frac{C_0'}{zC_0}-\frac{A_0'}{A_0}\left(\frac{C_0'}{C_0}-\frac{1}{z}\right)\right\}-2\left(\frac{a''}{A_0''}-\frac{a}{A_0}\right)\\&\times\left[\frac{A_0'}{A_0C_0^2}\left(\frac{C_0'}{C_0}-\frac{1}{z}\right)+\frac{C_0'}{zC_0^3}+8\pi\left(P_{\perp 0}+2\pi E_0^2\right)\right]\\&+\frac{z^2}{C_0^3}\left(\frac{C_0}{z}\right)'\left[\kappa C_0^2\left\{\frac{2c}{C_0}\left(P_{z0}-2\pi E_0^2\right)-4\pi eE_0\right\}\right.\\&\left.+\frac{2\alpha cC_0^2}{z^3}-2\left(\frac{cA_0}{z^3}\right)'\left\{\frac{A_0'}{A_0}-\frac{1}{z}\right\}\right]\\&-\frac{2}{C_0^2}\left[\left(\frac{c}{C_0}\right)'\left(\frac{2}{z}-\frac{A_0'}{A_0}\right)+\left(\frac{cA_0}{z^3}\right)'\left(\frac{A_0'}{A_0}+\frac{C_0'}{C_0}\right)\right]\\&+\kappa^2eE_0+\frac{4\kappa cA_0}{z^2}\left(P_{\perp 0}+2\pi E_0^2\right)\\&+2\kappa z(P_{z0}-P_{\perp 0})\left(\frac{cA_0}{z^3}\right)'\\&-4\pi\kappa\left\{\frac{cA_0E_0'}{z}+ze'+2e\right.\\&\left.+2\left(\frac{cA_0}{z^2}\right)'\left(zE_0'+2E_0\right)\left(\frac{e}{E_0}-\frac{cA_0}{z^3}\right)\right\}=0.\end{aligned} \tag{53}$$

It is found that the terms in Eq. (53) generally constitute one static part of planar geometry and matter profile for the applicability of stability of collapsing fluid. Substituting the values of metric coefficient, $a=a_0+a_1z$, where a_0, a_1 are arbitrary positive constants, neglecting the ppN order terms and expanding upto pN order, we obtain

$$\begin{aligned}&\frac{\kappa z^2\mu_0}{2}\left[\kappa\left(1+\frac{2m_0}{z}-\frac{Q^2}{z^2}\right)\right.\\&\times\left\{2c\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\left(P_{z0}-2\pi E_0^2\right)-4\pi eE_0\right\}\end{aligned}$$

$$+\frac{2\alpha c}{z^3}\left(1+\frac{2m_0}{z}-\frac{Q^2}{z^2}\right)-2\left\{\frac{c}{z^3}\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right\}'$$
$$\times\left\{\frac{8\pi z^3P_{z0}+2m_0}{2z(16\pi^2z^3E_0^2-2m_0)}\right.$$
$$\left.-\frac{32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}-\frac{1}{z}\right\}\Bigg]$$
$$+2\alpha\left(1+\frac{2m_0}{z}-\frac{Q^2}{z^2}\right)\left(\frac{c}{z^3}\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right.$$
$$\left.+c\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right)-32\pi\left(P_{\perp0}+2\pi E_0^2\right)$$
$$-4\left(1-\frac{2m_0}{z}+\frac{Q^2}{z^2}\right)\left(\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}\right)$$
$$+4\left(1-\frac{2m_0}{z}+\frac{Q^2}{z^2}\right)\left\{\frac{4\pi z^3\mu_0-m_0}{2z^2(8\pi^2z^3E_0^2-m_0)}\right.$$
$$+\frac{16\pi^2E_0^2z^3}{2z^2(8\pi^2z^3E_0^2-m_0)}-\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}$$
$$\times\left(\left(\frac{4\pi z^3\mu_0-m_0}{2z(8\pi^2z^3E_0^2-m_0)}+\frac{16\pi^2E_0^2z^3}{2z(8\pi^2z^3E_0^2-m_0)}\right)\right.$$
$$\left.\left.-\frac{1}{z}\right)\right\}-2\left(a''\left(1+\frac{m_0}{z}-\frac{Q^2}{2z^2}\right)''\right.$$
$$\left.-a\left(1+\frac{m_0}{z}-\frac{Q^2}{2z^2}\right)\right)$$
$$\times\left[\left(\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}\right)\left(1-\frac{2m_0}{z}+\frac{Q^2}{z^2}\right)\right.$$
$$\times\left(\left(\frac{4\pi z^3\mu_0-m_0+16\pi^2E_0^2z^3}{2z(8\pi^2z^3E_0^2-m_0)}\right)-\frac{1}{z}\right)$$
$$+\left(\frac{4\pi z^3\mu_0-m_0+16\pi^2E_0^2z^3}{2z^2(8\pi^2z^3E_0^2-m_0)}\right)\left(1-\frac{2m_0}{z}+\frac{Q^2}{z^2}\right)$$
$$\left.+8\pi\left(P_{\perp0}+2\pi E_0^2\right)\right]+z^2\left(1-\frac{3m_0}{z}+\frac{3Q^2}{2z^2}\right)$$
$$\times\left(\frac{1}{z}+\frac{m_0}{z^2}-\frac{Q^2}{2z^3}\right)'\left[\kappa\left(1+\frac{2m_0}{z}-\frac{Q^2}{z^2}\right)\right.$$
$$\times\left\{2c\left(1+\frac{m_0}{z}-\frac{Q^2}{2z^2}\right)\left(P_{z0}-2\pi E_0^2\right)-4\pi eE_0\right\}$$
$$+\frac{2\alpha c}{z^3}\left(1+\frac{2m_0}{z}-\frac{Q^2}{z^2}\right)-2\left(\frac{c}{z^3}\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right)'$$
$$\times\left\{\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}-\frac{1}{z}\right\}\Bigg]$$
$$-\left(2-\frac{4m_0}{z}+\frac{2Q^2}{z^2}\right)\left[\left(c\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right)'\right.$$
$$\times\left(\frac{2}{z}-\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}\right.$$
$$\left.-\frac{-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}\right)+\left(\frac{c}{z^3}-\frac{cm_0}{z^4}+\frac{cQ^2}{2z^5}\right)'$$
$$\times\left(\frac{8\pi z^3P_{z0}+2m_0-32\pi^2E_0^2z^3}{2z(16\pi^2z^3E_0^2-2m_0)}\right.$$
$$\left.\left.+\frac{4\pi z^3\mu_0-m_0+16\pi^2E_0^2z^3}{2z(8\pi^2z^3E_0^2-m_0)}\right)\right]+\kappa^2eE_0$$
$$+\frac{4\kappa c}{z^2}\left(P_{\perp0}+2\pi E_0^2\right)\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)$$
$$+ze'+2e+2\kappa z(P_{z0}-P_{\perp0})\left(\frac{c}{z^3}\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right)'$$
$$-4\pi\kappa\left\{\frac{cE_0'}{z}\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right.$$
$$+2\left(\frac{c}{z^2}\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right)'\left(zE_0'+2E_0\right)$$
$$\left.\times\left(\frac{e}{E_0}-\frac{c}{z^3}\left(1-\frac{m_0}{z}+\frac{Q^2}{2z^2}\right)\right)\right\}=0. \tag{54}$$

To obtain the instability conditions at N regime, we choose $\mu_0 \gg P_{z0}$, $\mu_0 \gg P_{\perp0}$ and neglect the ppN order terms as

$$\kappa z\mu_0\left(\frac{c}{z^3}\right)'-2\pi\mu_0\kappa^2ez^2E_0+\frac{2\alpha c^2}{z^3}\left(1+\frac{2m_0}{z}\right)$$
$$-32\pi\left(P_{\perp0}+2\pi E_0^2\right)+16\pi(a_0+a_1z)\left(P_{\perp0}+2\pi E_0^2\right)$$
$$-2c\kappa\left(P_{z0}+2\pi E_0^2\right)+\frac{4\kappa c}{z^2}\left(P_{\perp0}+2\pi E_0^2\right)+4\pi\kappa eE_0$$
$$-\frac{2c\alpha\kappa}{z^2}+\kappa^2eE_0+ze'+2e+2z\kappa(P_{z0}-P_{\perp0})\left(\frac{c}{z^3}\right)'$$
$$-4\pi\kappa\left(\frac{cE_0'}{z}\right)=0. \tag{55}$$

We know that pressure along z-direction decreases with the evolution of expansion-free collapse, i.e., $P_{z0}'<0$. Thus, the above equation leads to

$$\kappa z\mu_0\left(\frac{c}{z^3}\right)'-2\pi\mu_0\kappa^2ez^2E_0-32\pi\left(P_{\perp0}+2\pi E_0^2\right)$$
$$+16\pi(a_0+a_1z)\left(P_{\perp0}+2\pi E_0^2\right)-2c\kappa\left(P_{z0}+2\pi E_0^2\right)$$
$$+\frac{4\kappa c}{z^2}\left(P_{\perp0}+2\pi E_0^2\right)+4\pi\kappa eE_0-\frac{2c\alpha\kappa}{z^2}+\kappa^2eE_0$$
$$+ze'+2e+2z\kappa(P_{z0}-P_{\perp0})\left(\frac{c}{z^3}\right)'-4\pi\kappa\left(\frac{cE_0'}{z}\right)$$

$$+\frac{2\alpha c^2}{z^3}\left[1+\frac{2}{z}\left\{4\pi\int_{z_{\Sigma^{(i)}}}^{z}\mu_0 z^2 dz\right.\right.$$
$$\left.\left.+8\pi\int_{z_{\Sigma^{(i)}}}^{z}\left(3z^2E_0dz+2E_0E_0'\right)dz\right\}\right]=0. \tag{56}$$

The instability analysis at N limit requires all the terms in the above equation to be positive independently. Since the quantities a_0, a_1 are positive constants, so the instability is based on the positivity of the remaining terms, hence we can have $P_{z0} > P_{\perp 0}$ and $E_0 > 0$. For the positivity of the last terms, we require power law solution for charge distribution and energy density, i.e., $E_0 = \eta z^m$ and $\mu_0 = \xi z^w$, where $\eta, \xi > 0$ are constants and n, w lies in the interval $(-\infty, \infty)$. Consequently, the last term turns out to be

$$\frac{2\alpha c^2}{z^3}\left[1+\frac{2}{z}\left\{4\pi\times\int_{z_{\Sigma^{(i)}}}^{z}\mu_0 z^2 dz\right.\right.$$
$$\left.\left.+8\pi\int_{z_{\Sigma^{(i)}}}^{z}\left(3z^2E_0dz+2E_0E_0'\right)dz\right\}\right]$$
$$=\frac{2\alpha c^2}{z^3}\left[1+\frac{8\xi\pi z^{m+2}}{3}+8\pi^2z^3E_0^2\right.$$
$$+\frac{4\pi\xi m z}{3(m+3)}\left(z^{w+3}-z^{w+3}_{\Sigma^{(i)}}\right)$$
$$\left.+\frac{16\pi^2\eta^2 m z}{2m+3}\left(z^{2m+3}-z^{2m+3}_{\Sigma^{(i)}}\right)\right]. \tag{57}$$

The above equation reduces for the positivity of each term as follows

$$z^{w+3} > z^{w+3}_{\Sigma^{(i)}}, \qquad z^{2m+3} > z^{2m+3}_{\Sigma^{(i)}}. \tag{58}$$

The above two inequalities yield the instability regions for plane symmetric collapsing matter in the expansion-free scenario. It is interesting to mention here that it depends on the specific length of the planar geometry.

5 Concluding remarks

In this paper, we have discussed the issue of instability constraints of planar collapse in the presence of electromagnetic field. For this purpose, we have considered charged plane symmetric spacetime with pressure anisotropic source and explored its evolution by means of expansion-free scenario. The field equations, dynamical equations as well as junction conditions are formulated using perturbation scheme. The static and non-static forms of the set of governing equations are found and written separately. The terms of $O(c)$, $O(c^2)$ and $O(c^4)$ are separated after expansion of static configuration of the second dynamical equation. This equation is then used to explore the stability conditions of the charged planar expansion-free fluid upto pN regime.

We have also examined the role of adiabatic index in the stability analysis of the system. Since the expansion-free constraint makes fluid incompressible in the collapse, so the adiabatic index is not involved in the instability conditions. These conditions at both N and pN regimes depend on static form of energy density, electric charge and anisotropic pressure. We have seen that the system would be unstable as long as it obeys (58) at N and pN approximations. We also note that matter instability relies on the values of n, w and length of the plane symmetric surface. It is found that the instability of anisotropic plane diminishes in the presence of charge and the system becomes more stable as the evolution proceeds. This result is well-consistent with Sharif and Yousaf (2013d). Finally, one concludes that our results reduce to charge-free case with $s = 0$ (Sharif and Azam 2014).

References

Anguige, K.: Class. Quantum Gravity **17**, 2117 (2000)
Bayin, S.S.: Phys. Rev. D **26**, 6 (1982)
Chan, R.: Mon. Not. R. Astron. **316**, 588 (2000)
Chan, R., Herrera, L., Santos, N.O.: Mon. Not. R. Astron. Soc. **256**, 533 (1993)
Chan, R., Herrera, L., Santos, N.O.: Mon. Not. R. Astron. Soc. **267**, 637 (1994)
Chandrasekhar, S.: Astrophys. J. **140**, 417 (1964)
Chao-Guang, H.: Acta Phys. Sin. **4**, 617 (1995)
Consenza, M., Herrera, L., Esculpi, M., Witten, M.: J. Math. Phys. **22**, 118 (1981)
Darmois, G.: Memorial des Sciences Mathematiques. Gauthier-Villars, Paris (1927)
Govinder, K.S., Govender, M., Maartens, R.: Mon. Not. R. Astron. Soc. **299**, 809 (1998)
Harada, T.: Prog. Theor. Phys. **98**, 359 (1997)
Harrison, B.K., Thorne, K.S., Wakano, M., Wheeler, J.A.: Gravitation Theory and Gravitational Collapse. University of Chicago Press, Chicago (1965)
Harrison, E.R.: Phys. Rev. Lett. **30**, 188 (1973)
Herrera, L., Santos, N.O.: Astrophys. J. **438**, 308 (1995)
Herrera, L., Santos, N.O.: Phys. Rep. **286**, 53 (1997)
Herrera, L., Santos, N.O., Wang, A.: Phys. Rev. D **78**, 084026 (2008)
Herrera, L., Le Denmat, G., Santos, N.O.: Phys. Rev. D **79**, 087505 (2009)
Herrera, L., Le Denmat, G., Santos, N.O.: Class. Quantum Gravity **27**, 135017 (2010)
Herrera, L., Le Denmat, G., Santos, N.O.: Gen. Relativ. Gravit. **44**, 1143 (2012)
Krori, K.D., Bargohain, P., Devi, R.: Can. J. Phys. **62**, 239 (1984)
Letelier, P.S.: Phys. Rev. D **22**, 807 (1980)
Maharaj, S.D., Maartens, R.: Gen. Relativ. Gravit. **21**, 899 (1989)
Nouri-Zonoz, M., Tavanfar, A.R.: Class. Quantum Gravity **18**, 4293 (2001)
Pinheiro, G., Chan, R.: Gen. Relativ. Gravit. **40**, 2149 (2008)
Pradhan, A., Panday, P.: Czechoslov. J. Phys. **55**, 749 (2005)
Pradhan, A., Pandey, P., Singh, S.K.: Int. J. Theor. Phys. **46**, 1584 (2007)
Saha, B., Shikin, G.N.: Int. J. Theor. Phys. **44**, 1459 (2005)
Sawyer, R.F.: Phys. Rev. Lett. **29**, 382 (1972)
Seifert, M.D.: Phys. Rev. D **76**, 064002 (2007)
Sharif, M., Azam, M.: Gen. Relativ. Gravit. **44**, 1181 (2012)
Sharif, M., Azam, M.: Gen. Relativ. Gravit. **46**, 1647 (2014)

Sharif, M., Bhatti, M.Z.: Mod. Phys. Lett. A **27**, 1250141 (2012)
Sharif, M., Bhatti, M.Z.: J. Cosmol. Astropart. Phys. **10**, 056 (2013a)
Sharif, M., Bhatti, M.Z.: J. Cosmol. Astropart. Phys. **11**, 014 (2013b)
Sharif, M., Bhatti, M.Z.: Phys. Lett. A **378**, 469 (2014a)
Sharif, M., Bhatti, M.Z.: Astropart. Phys. **56**, 35 (2014b)
Sharif, M., Bhatti, M.Z.: Astrophys. Space Sci. **352**, 883 (2014c)
Sharif, M., Yousaf, Z.: Int. J. Mod. Phys. D **21**, 1250095 (2012)
Sharif, M., Yousaf, Z.: Mon. Not. R. Astron. Soc. **434**, 2529 (2013a)
Sharif, M., Yousaf, Z.: Mon. Not. R. Astron. Soc. **440**, 3479 (2013b)
Sharif, M., Yousaf, Z.: Eur. Phys. J. C **73**, 2633 (2013c)
Sharif, M., Yousaf, Z.: Phys. Rev. D **88**, 024020 (2013d)
Sharif, M., Yousaf, Z.: Mon. Not. R. Astron. Soc. **440**, 3479 (2014)
Sharma, R., Tikekar, R.: Gen. Relativ. Gravit. **44**, 2503 (2012)
Skripkin, V.A.: Sov. Phys. Dokl. **135**, 1183 (1960)
Tayler, R.J.: Mon. Not. R. Astron. Soc. **161**, 365 (1973)
Varela, V., Rahaman, F., Ray, S., Chakraborty, K.: Phys. Rev. D **82**, 044052 (2010)
Yazadjiev, S.S.: Class. Quantum Gravity **20**, 3365 (2003)
Yoshida, S., Kiuchi, K., Shibata, M.: Phys. Rev. D **86**, 044012 (2012)
Zannias, T.: Phys. Rev. D **41**, 3252 (1990)
Zeldovich, Ya.B., Ruzmainkin, A.A., Sokoloff, D.D.: Magnetic Fields in Astrophysics. Gordon & Breach, New York (1993)