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ORIGINAL ARTICLE

Exact solutions in (2 + 1)-dimensional anti-de Sitter space-time admitting a linear or non-linear equation of state

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Abstract Gravitational analyzes in lower dimensions has become a field of active research interest ever since Bañados, Teitelboim and Zanelli (BTZ) (Phys. Rev. Lett. 69:1849, 1992) proved the existence of a black hole solution in (2+1) dimensions. The BTZ metric has inspired many investigators to develop and analyze circularly symmetric stellar models which can be matched to the exterior BTZ metric. We have obtained two new classes of solutions for a (2+1)-dimensional anisotropic star in anti-de Sitter background space-time which have been obtained by assuming that the equation of state (EOS) describing the material composition of the star could either be linear or non-linear in nature. By matching the interior solution to the BTZ exterior metric with zero spin, we have demonstrated that the solutions provided here are regular and well-behaved at the stellar interior.

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1 Introduction

Lower dimensional gravity, due to its comparatively simpler setting, plays a crucial role towards our understanding of many conceptual issues relating to Einstein's gravity. Gravitational analyzes in three dimensions got a tremendous impetus when Bañados et al. (1992) (henceforth BTZ) proposed a model for a circularly symmetric charged body in an anti-de Sitter back ground space-time which was found to admit a black hole solution in the presence of a negative cosmological constant. The BTZ metric is characterized by its mass, angular momentum and charge.

The existence of a black hole in (2 + 1) dimensions has inspired many investigators to construct and study circularly symmetric star models. Different techniques have so far been adopted to generate static interiors solutions corresponding to the BTZ exterior metric. For example, Cruz and Zanelli (1995) have obtained an exact solution for an incompressible fluid in (2 + 1) dimensions. For a given density profile, Cruz et al. (2005) have obtained new class of solutions corresponding to exterior BTZ metric. Cataldo and Salgado (1996) have analyzed an Einstein-Maxwell system in (2 + 1) dimensions. For a polytropic equation of state (EOS), Sá (1999) has proposed a formalism to obtain interior solutions corresponding to the BTZ exterior metric. Sharma et al. (2011) have assumed a particular mass function to obtain new class of solutions in (2 + 1) dimensions. García and Campuzano (2003) have proposed a formalism to obtain circularly symmetric solutions from known density profile or EOS of the fluid source. Making use of Finch and Skea (1989) ansatz in (2 + 1) dimensions, Banerjee et al. (2013) have generated new class of physically acceptable interior solutions corresponding to the BTZ exterior. Rahaman et al. (2013) have studied the properties of BTZ black hole by proposing new exact solutions of Einstein's field equations in $(2+1)$ dimensional anti-de Sitter back ground space-time in the context of non-commutative geometry.

Motivated by such developments, we propose here two new classes of exact solutions describing the interior of a circularly symmetric star with zero angular momentum in an anti-de Sitter back ground space-time. In our construction, we assume that the material composition of the star is anisotropic in nature and generate solutions for linear as well as non-linear EOS. We analyze physical behaviour of the model by matching the interior solution to the BTZ exterior metric with zero spin and show that the solutions generated here are regular and well-behaved at the stellar interior.

2 Interior space-time

We write the line element for a static circularly symmetric star with zero angular momentum in the form

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\mu(r)}dr^2 + r^2 d\theta^2, \tag{1}$$

where $\nu(r)$ and $\mu(r)$ are yet to be determined. The Einstein's field equations for an anisotropic fluid in the presence of a negative cosmological constant ($\Lambda < 0$) are then obtained as (we set $G = c = 1$)

$$2\pi\rho + \Lambda = \frac{\mu' e^{-2\mu(r)}}{r}, \tag{2}$$

$$2\pi p_r - \Lambda = \frac{\nu' e^{-2\mu(r)}}{r}, \tag{3}$$

$$2\pi p_t - \Lambda = e^{-2\mu}\left(\nu'' + \nu'^2 - \nu'\mu'\right), \tag{4}$$

where, ρ is the energy density, p_r is the radial pressure and p_t is the tangential pressure. Equations (2)–(4) may be combined to yield

$$(\rho + p_r) + p_r' + \frac{1}{r}(p_r - p_t) = 0, \tag{5}$$

which is analogous to the generalized Tolman-Oppenheimer-Volkoff (TOV) equation in $(3+1)$ dimensions. Defining the mass within a radius r as

$$m(r) = \int_0^r 2\pi\rho\, \tilde{r}\, d\tilde{r}, \tag{6}$$

Eq. (2) yields

$$2m(r) = C - e^{2\mu(r)} - \Lambda r^2, \tag{7}$$

where C is integrating constant. Following an earlier treatment (Sharma et al. 2011), we set $C = 1$ and assume $2\mu(r) = Ar^2$ so as to ensure regular behaviour of the mass function $m(r)$ at the centre. The energy density is then obtained as

$$\rho = \frac{1}{2\pi}\left[Ae^{-Ar^2} - \Lambda\right]. \tag{8}$$

The constant A can be determined from the central density

$$\rho_c = \rho(r=0) = \frac{1}{2\pi}[A - \Lambda]. \tag{9}$$

To determine the unknown metric potential $\nu(r)$, we prescribe an EOS corresponding to the material composition of the star in the form

$$p_r = p_r(\rho, \alpha_1, \alpha_2), \tag{10}$$

where α_1 and α_2 are two positive arbitrary constants constraining the EOS. The physical radius R of the star can be obtained by ensuring that

$$p_r\left(\rho(R), \alpha_1, \alpha_2\right) = 0. \tag{11}$$

The EOS (10) can be either linear or non-linear in nature and accordingly we consider the two possibilities separately.

2.1 Case I: Solution admitting a linear EOS

Let us first assume a linear EOS of the form

$$p_r = \alpha_1\rho + \alpha_2. \tag{12}$$

For the choice (12), the system (2)–(4) can be solved analytically and we get

$$\nu(r) = \frac{\alpha_1 A}{2}r^2 - \left(\frac{\alpha_1\Lambda + \Lambda - 2\pi\alpha_2}{2A}\right)e^{Ar^2} + C_1, \tag{13}$$

$$p_r = \frac{\alpha_1}{2\pi}\left[Ae^{-Ar^2} - \Lambda\right] + \alpha_2, \tag{14}$$

$$\begin{aligned} p_t = \frac{1}{2\pi}\Bigg[& r^2 e^{-Ar^2}\bigg(\alpha_1 A - (\alpha_1\Lambda + \Lambda - 2\pi\alpha_2)^2 e^{2Ar^2} \\ & + \frac{\alpha_1 A}{r^2} - \alpha_1 A^2\bigg) \\ & - (\alpha_1\Lambda + \Lambda - 2\pi\alpha_2)\left(1 + Ar^2\right) + \Lambda\Bigg], \end{aligned} \tag{15}$$

where C_1 is integrating constant. We also have

$$\begin{aligned} \Delta &= p_t - p_r \\ &= \frac{1}{2\pi}\Big[r^2 e^{-Ar^2}\Big(\big(\alpha_1 A - (\alpha_1\Lambda + \Lambda - 2\pi\alpha_2)e^{Ar^2}\big)^2 \\ &\quad - \alpha_1 A^2\Big) - (\alpha_1\Lambda + \Lambda - 2\pi\alpha_2)Ar^2\Big], \end{aligned} \tag{16}$$

which is the measure of anisotropy. Note that the anisotropy vanishes at the centre which is a desirable feature of a realistic star.

2.2 Case II: Solution admitting a non-linear EOS

Assuming a non-linear EOS of the form

$$p_r = \gamma_1 \rho + \frac{\gamma_2}{\rho}, \tag{17}$$

where γ_1 and γ_2 are positive arbitrary constants, we solve the system (2)–(4) and obtain

$$\nu(r) = \frac{\gamma_1 A}{2} r^2 - \left(\frac{(\gamma_1+1)\Lambda}{2A}\right) e^{Ar^2} - \frac{2\pi^2\gamma_2}{A\Lambda}\left(e^{Ar^2} + \frac{A}{\Lambda}\ln\left(A - \Lambda e^{Ar^2}\right)\right) + C_2, \tag{18}$$

$$p_r = \frac{\gamma_1}{2\pi}\left(Ae^{-Ar^2} - \Lambda\right) + \frac{2\pi\gamma_2}{(Ae^{-Ar^2} - \Lambda)}, \tag{19}$$

$$\begin{aligned} p_t(r) = \frac{1}{2\pi}\Bigg[&\left((\gamma_1 - 1)Ar^2 + 1\right)\gamma_1 Ae^{-Ar^2} \\ &- \Lambda(\gamma_1+1)\left(1+2Ar^2\right) \\ &+ A(1-2\gamma_1)\left(\Lambda\gamma_1 + \Lambda - \frac{4\pi^2\gamma_2}{Ae^{-Ar^2} - \Lambda}\right) r^2 \\ &+ \left(\Lambda\gamma_1 + \Lambda - \frac{4\pi^2\gamma_2}{Ae^{-Ar^2} - \Lambda}\right)^2 r^2 e^{Ar^2} \\ &+ 4\pi^2\gamma_2\left(\frac{1+2Ar^2}{Ae^{-Ar^2} - \Lambda} + \frac{2A^2r^2e^{-Ar^2}}{(Ae^{-Ar^2} - \Lambda)^2}\right) r^2 \\ &+ \Lambda\Bigg]. \end{aligned} \tag{20}$$

In Eq. (18), C_2 is an integration constant.

The measure of anisotropy in this case turns out to be

$$\begin{aligned} \Delta &= p_t - p_r \\ &= \frac{1}{2\pi}\Bigg[(\gamma_1 - 1)\gamma_1 A^2 r^2 e^{-Ar^2} - 2\Lambda(\gamma_1+1)Ar^2 \\ &\quad + A(1-2\gamma_1)\left(\Lambda\gamma_1 + \Lambda - \frac{4\pi^2\gamma_2}{Ae^{-Ar^2} - \Lambda}\right) r^2 \\ &\quad + \left(\Lambda\gamma_1 + \Lambda - \frac{4\pi^2\gamma_2}{Ae^{-Ar^2} - \Lambda}\right)^2 r^2 e^{Ar^2} \\ &\quad + 4\pi^2\gamma_2\left(\frac{2Ar^2}{Ae^{-Ar^2} - \Lambda} + \frac{2A^2r^2e^{-Ar^2}}{(Ae^{-Ar^2} - \Lambda)^2}\right)\Bigg], \end{aligned} \tag{21}$$

which vanishes at the centre.

3 Exterior space-time and boundary conditions

We assume that the exterior space-time of our circularly symmetric star is described by the BTZ metric (with zero angular momentum)

$$ds^2 = -\left(-M_0 - \Lambda r^2\right)dt^2 + \left(-M_0 - \Lambda r^2\right)^{-1} dr^2 + r^2 d\theta^2, \tag{22}$$

where M_0 is the conserved charge associated with asymptotic invariance under time displacements. At the boundary $r = R$, continuity of the metric potentials yield the following junction conditions:

$$e^{2\nu(R)} = -M_0 - \Lambda R^2, \tag{23}$$

$$e^{-2\mu(R)} = -M_0 - \Lambda R^2. \tag{24}$$

Moreover, the radial pressure must vanish at the boundary, i.e., $p_r(r = R) = 0$. These three conditions can be utilized to fix the values of the constants A, C and R for two different cases, namely solution admitting a linear EOS (Case I) and solution admitting a non-linear EOS (Case II).

3.1 Case I

$$A = -\frac{1}{R^2}\ln\left(-M_0 - \Lambda R^2\right), \tag{25}$$

$$C_1 = \frac{\alpha_1\Lambda + \Lambda - 2\pi\alpha_2}{2A(-M_0 - \Lambda R^2)} - \left(\frac{1+\alpha_1}{2}\right)\Lambda R^2, \tag{26}$$

$$R = \frac{1}{\sqrt{A}}\left[\ln\left(\frac{A\alpha_1}{\alpha_1\Lambda - 2\pi\alpha_2}\right)\right]^{\frac{1}{2}}. \tag{27}$$

3.2 Case II

$$A = -\frac{1}{R^2}\ln\left(-M_0 - \Lambda R^2\right), \tag{28}$$

$$\begin{aligned} C_2 = &\frac{\Lambda(\gamma_1+1)}{4A(-M_0 - \Lambda R^2)} - \frac{\Lambda R^2(\gamma_1+1)}{2} \\ &+ \frac{2\pi^2\gamma_2}{A\Lambda}\Bigg[\frac{1}{(-M_0 - \Lambda R^2)} \\ &+ \frac{A}{\Lambda}\ln\left(A - \frac{\Lambda}{(-M_0 - \Lambda R^2)}\right)\Bigg], \end{aligned} \tag{29}$$

$$R = \frac{1}{\sqrt{A}}\left[\ln\left(\frac{A\sqrt{\gamma_1}}{\Lambda\sqrt{\gamma_1} + \sqrt{-4\pi^2\gamma_2}}\right)\right]^{\frac{1}{2}}. \tag{30}$$

4 Physical acceptability and regularity of the model

For a physically acceptable model, we impose the following restrictions:

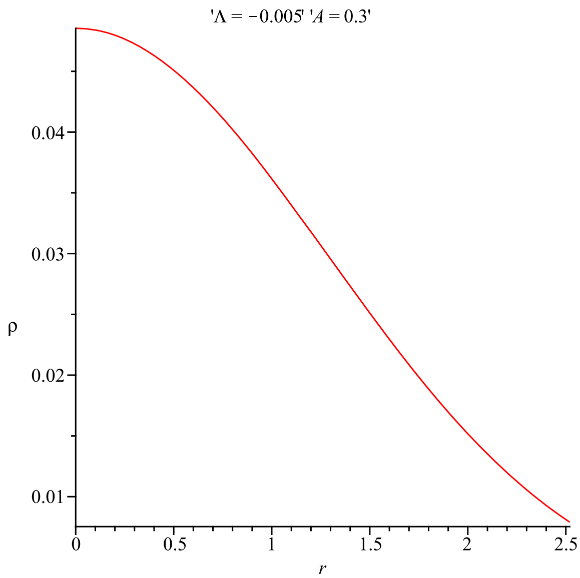

Fig. 1 Energy density plotted against the radial parameter r

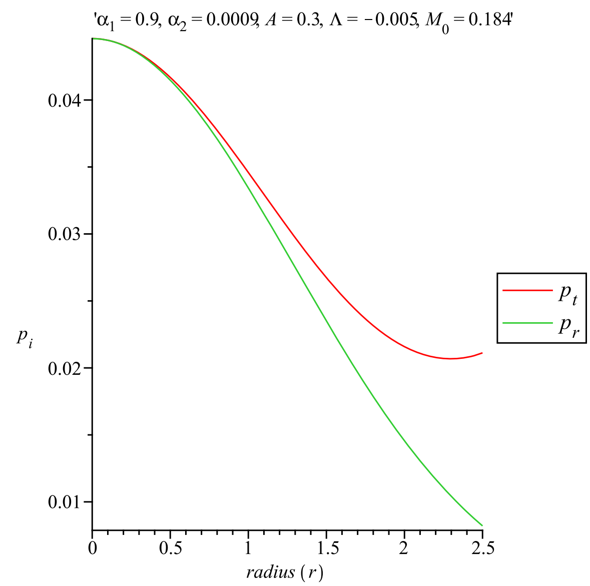

Fig. 2 Radial and transverse pressures are plotted against the radial parameter r for linear EOS

- Regularity of the curvature invariants.
- Energy-density and pressure should be monotonically decreasing functions of r.
- Radial sound speed and transverse sound speed should be less than unity i.e.,

$$0 < v_{sr}^2\left(=\frac{dp_r}{d\rho}\right) < 1, \qquad 0 < v_{st}^2\left(=\frac{dp_t}{d\rho}\right) < 1.$$

4.1 Case I

From Eqs. (5) and (6), we obtain

$$\frac{d\rho}{dr} = -\frac{A^2}{\pi}re^{-Ar^2}, \tag{31}$$

$$\frac{dp_r}{dr} = -\frac{A^2\alpha_1}{\pi}re^{-Ar^2}, \tag{32}$$

$$\left.\frac{d^2\rho}{dr^2}\right|_{r=0} = -\frac{A^2}{\pi} < 0, \tag{33}$$

$$\left.\frac{d^2p_r}{dr^2}\right|_{r=0} = -\frac{A^2\alpha_1}{\pi} < 0. \tag{34}$$

Eqs. (31)–(34) show that, for $\alpha_1 > 0$, both energy-density and radial pressure decrease from their maximum values at the centre. Variations of energy-density and the two pressures have been shown in Figs. 1 and 2, respectively. The Ricci scalar assumed the following form

$$R = 2e^{-Ar^2}\big[(1-2\alpha_1)A + (1-\alpha_1)\alpha_1A^2r^2 - (\alpha_1\Lambda + \Lambda - 2\pi\alpha_2)^2r^2e^{2Ar^2} + (\alpha_1\Lambda + \Lambda - 2\pi\alpha_2)\big(2+(1+2\alpha_1)Ar^2\big)e^{Ar^2}\big]. \tag{35}$$

One can note that R is regular at the origin and well behaved in the stellar interior.

4.2 Case II

Here,

$$\frac{d\rho}{dr} = -\frac{A^2}{\pi}re^{-Ar^2}, \tag{36}$$

$$\frac{dp_r}{dr} = -\frac{A^2\gamma_1}{\pi}re^{-Ar^2} + \frac{4\pi A^2\gamma_2}{(Ae^{-Ar^2}-\Lambda)^2}re^{-Ar^2}, \tag{37}$$

$$\left.\frac{d^2\rho}{dr^2}\right|_{r=0} = -\frac{A^2}{\pi} < 0, \tag{38}$$

$$\left.\frac{d^2p_r}{dr^2}\right|_{r=0} = -\frac{A^2\gamma_1}{\pi} + \frac{4\pi A^2\gamma_2}{(A-\Lambda)^2} < 0. \tag{39}$$

Though it is not straight forward, we note that Eq. (39) holds for appropriate choices of the values of γ_1 and γ_2. Equations (36)–(39) show that both energy-density and radial pressure decrease from their maximum values at the centre. Behaviour of two pressures have been shown in Fig. 3 and the Ricci scalar is given by

$$R = 2e^{-Ar^2}\Bigg[(1-2\gamma_1)A + \gamma_1A^2r^2 + \left((\gamma_1+1)\Lambda + \frac{4\pi^2\gamma_2}{\Lambda}\right)(1+Ar^2)e^{Ar^2} - \frac{4\pi^2\gamma_2A}{\Lambda(A-\Lambda e^{Ar^2})}$$

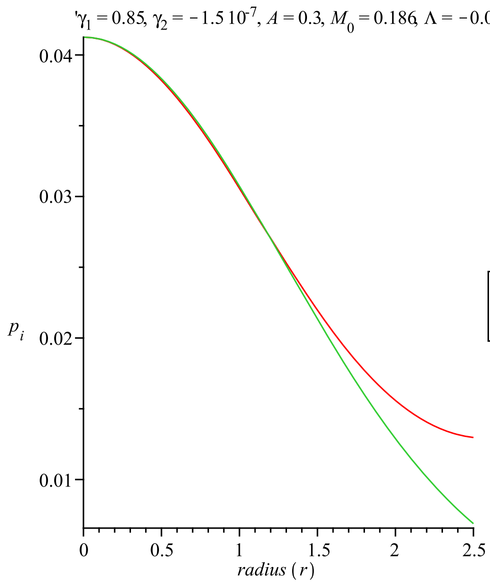

Fig. 3 Radial and transverse pressures are plotted against the radial parameter r for non linear EOS

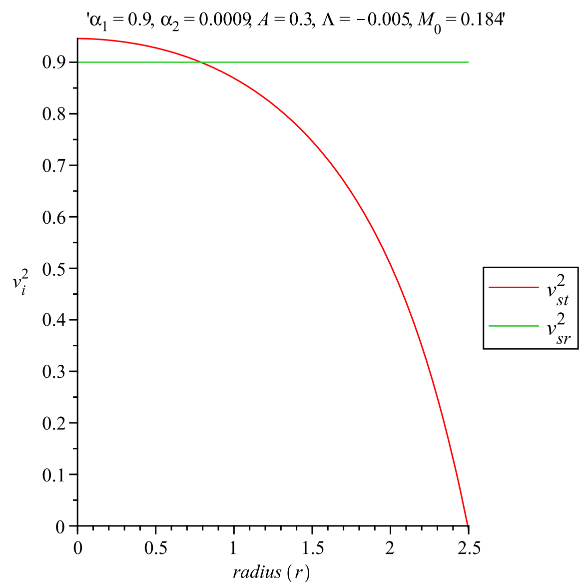

Fig. 4 Radial and tangential sound speeds plotted against r (Case I)

$$\times\left(1-Ar^2+\frac{A(1+2Ar^2)-\Lambda e^{Ar^2}}{(A-\Lambda e^{-Ar^2})}\right)e^{Ar^2} - \left(\gamma_1 Ar-\left((\gamma_1+1)\Lambda+\frac{4\pi^2\gamma_2}{\Lambda}\right)re^{Ar^2} + \frac{4\pi^2\gamma_2 A}{\Lambda(A-\Lambda e^{Ar^2})}re^{Ar^2}\right)^2\Bigg] \tag{40}$$

which is also regular at the stellar interior. Another important 'physical acceptability condition' is the causal property of the radial and tangential sound speeds which have been addressed in the following sub-sections.

4.3 Case I

Combining Eqs. (31)–(34), we obtain

$$v_{sr}^2=\frac{dp_r}{d\rho}=\alpha_1, \tag{41}$$

$$v_{st}^2=\frac{dp_t}{d\rho}=1-(\alpha_1^2-\alpha_1)(1-Ar^2) + (2\alpha_1+1)\left(\frac{\alpha_1\Lambda+\Lambda-2\pi\alpha_2}{A}\right)e^{Ar^2} - \left(\frac{\alpha_1\Lambda+\Lambda-2\pi\alpha_2}{A}\right)^2(1+Ar^2). \tag{42}$$

In Fig. 4, we have shown the nature of two sound speeds for specific choices of the model parameters which clearly shows regular behaviour of both the radial and transverse sound speeds.

4.4 Case II

From Eqs. (36)–(39), we determine

$$v_{sr}^2=\frac{dp_r}{d\rho}=\gamma_1-\frac{4\pi^2\gamma_2}{(Ae^{-Ar^2}-\Lambda)^2}, \tag{43}$$

$$\begin{aligned} v_{st}^2=\frac{dp_t}{d\rho}=\Bigg[&-\gamma_1\big((\gamma_1-1)(1-Ar^2)-1\big) + \frac{2\Lambda(\gamma_1+1)}{A}e^{Ar^2}-\big(3-A(1-2\gamma_1)r^2\big) \\ &\times\left(\frac{4\pi^2\gamma_2}{(Ae^{-Ar^2}-\Lambda)^2}\right) - \frac{e^{Ar^2}}{A}\left(\Lambda\gamma_1+\Lambda-\frac{4\pi^2\gamma_2}{Ae^{-Ar^2}-\Lambda}\right) \\ &\times\left((1-2\gamma_1)-\frac{8\pi^2Ar^2}{(Ae^{-Ar^2-\Lambda})^2}\right) - \frac{(1+Ar^2)e^{2Ar^2}}{A^2}\left(\Lambda\gamma_1+\Lambda - \frac{4\pi^2\gamma_2}{Ae^{-Ar^2}-\Lambda}\right)^2 - \frac{2\pi^2\gamma_2 e^{Ar^2}}{A^2 r} \\ &\times\left(\frac{4Ar}{Ae^{-Ar^2}-\Lambda}+\frac{8A^4r^3e^{-2Ar^2}}{(Ae^{-Ar^2}-\Lambda)^3}\right)\Bigg]. \end{aligned} \tag{44}$$

Figure 5 shows regular behaviour of radial and transverse sound speeds for the non-linear case as well for specific choices of the model parameters.

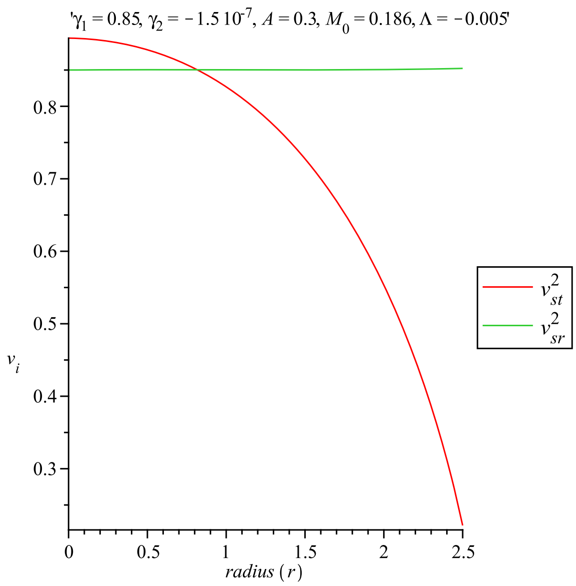

Fig. 5 Radial and tangential sound speeds plotted against r (Case II)

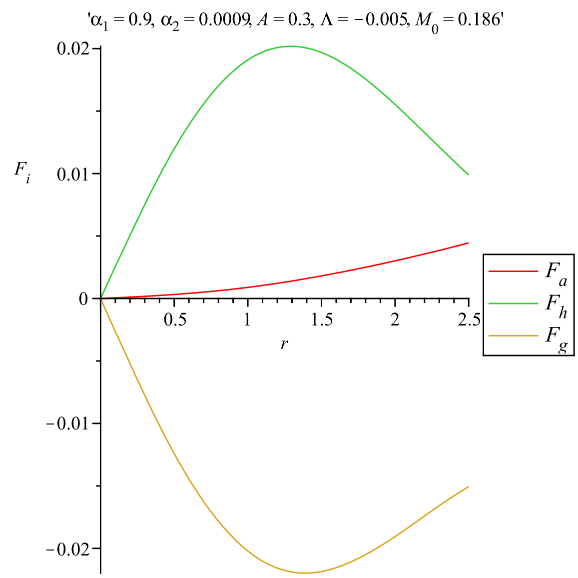

Fig. 6 Behaviour of three different forces acting on the fluid in static equilibrium at the stellar interior (Case I)

5 Some features of the model

We rewrite Eq. (5) in the form

$$-\frac{M_G(\rho+p_r)}{r}e^{\frac{\mu-\nu}{2}}-\frac{d}{dr}\left(p_r-\frac{\Lambda}{2\pi}\right)+\frac{1}{r}(p_t-p_r)=0, \tag{45}$$

where $M_G(r)$ is the Tolman-Whittaker mass (Ponce de León 1993) and is given by

$$M_G(r)=re^{\frac{\nu-\mu}{2}}\nu'. \tag{46}$$

Equation (45) provides the equilibrium condition which implies that the stellar configuration will be in equilibrium under the combined impacts of gravitational force (F_g), hydrostatic force (F_h) and another force term due to pressure anisotropy (F_a) given respectively by

$$F_g=-\frac{M_G(\rho+p_r)}{r}e^{\frac{\mu-\nu}{2}}, \tag{47}$$

$$F_h=-\frac{d}{dr}\left(p_r-\frac{\Lambda}{2\pi}\right), \tag{48}$$

$$F_a=\frac{1}{r}(p_t-p_r). \tag{49}$$

In Figs. 6 and 7, the nature of the three forces have been shown for Case I and Case II, respectively.

The total gravitational mass $M(r=R)$ in our model can be obtained from Eq. (7). Plugging in $C=1$ and $2\mu(r)=Ar^2$ in Eq. (7), we obtain

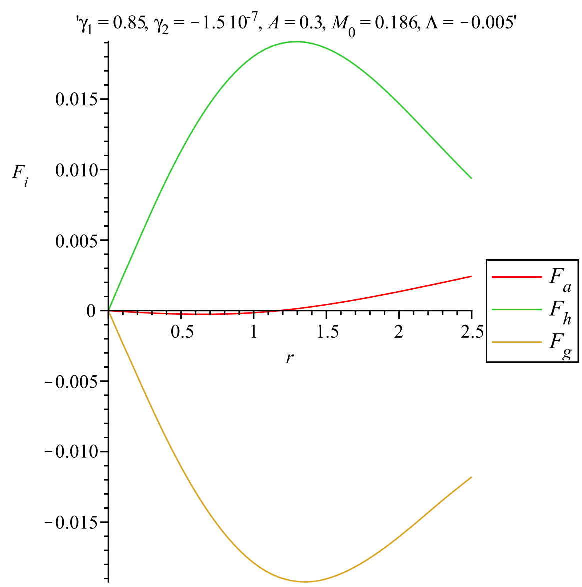

Fig. 7 Behaviour of three different forces acting on the fluid in static equilibrium at the stellar interior (Case II)

$$m(r)=\frac{1}{2}-\frac{e^{-Ar^2}}{2}-\frac{\Lambda r^2}{2}, \tag{50}$$

which can be utilized to determine the compactness of the star as

$$\frac{M}{R}=\frac{1-\Lambda R^2-e^{-AR^2}}{2R}. \tag{51}$$

For $R=2.5$, the compactness $\frac{M}{R}$ is found to be 0.172 for $\Lambda=-0.002$ and 0.194 for $\Lambda=-0.02$.

The corresponding surface red-shift

$$Z_s = \left(1 - \frac{2M}{R}\right)^{-\frac{1}{2}} - 1, \qquad (52)$$

turns out be 0.234 and 0.279, respectively.

6 Discussions

In this work, we have generated new analytic solutions for a circularly symmetric star which admits a linear or non-linear equation of state. The matter composition of the star has been assumed to be anisotropic in nature. The values of the constants in our solution have been fixed by matching the interior solution to the BTZ exterior metric. The cosmological constant Λ remains a free parameter in our construction. In the absence of any definite value of Λ, we have made some specific choices of the cosmological constant and shown that the solutions provided here are well behaved and can be utilized to develop physically acceptable model of a static circularly symmetric star in AdS space-time.

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