ORIGINAL ARTICLE

A family of exact solutions of Einstein-Maxwell field equations in isotropic coordinates: an application to optimization of quark star mass

Neeraj Pant · N. Pradhan · Mohammad Hassan Murad

Abstract In the present paper, we have obtained a class of charged super dense star models, starting with a static spherically symmetric metric in isotropic coordinates for perfect fluid by considering Hajj-Boutros (in J. Math. Phys. 27:1363, 1986) type metric potential and a specific choice of electrical intensity which involves a parameter K. The resulting solutions represent charged fluid spheres joining smoothly with the Reissner-Nordstrom metric at the pressure free interface. The solutions so obtained are utilized to construct the models for super-dense star like neutron stars ($\rho_b = 2$ and 2.7×10^{14} g/cm^3) and Quark stars ($\rho_b = 4.6888 \times 10^{14}$ g/cm^3). Our solution is well behaved for all values of n satisfying the inequalities $4 < n \leq 4(4 + \sqrt{2})$ and K satisfying the inequalities $0 \leq K \leq 0.24988$, depending upon the value of n. Corresponding to $n = 4.001$ and $K = 0.24988$, we observe that the maximum mass of quark star $M = 2.335 M_\odot$ and radius $R = 10.04$ km. Further, this maximum mass limit of quark star is in the order of maximum mass of stable Strange Quark Star established by Dong et al. (in arXiv:1207.0429v3, 2013). The robustness of our results is that the models are alike with the recent discoveries.

Keywords Isotropic coordinates · General relativity · Reissner-Nordström · Fluid ball

1 Introduction

Ever since the formulation of Einstein-Maxwell field equations, the relativists have been proposing different models of immensely gravitating astrophysical objects by considering the distinct nature of matter or radiation (energy-momentum tensor) present in them. Einstein-Maxwell field equations have more importance over Einstein field equations due to following rationale justifications:

- The presence of some charge may avert the catastrophic gravitational collapse by counter balancing the gravitational attraction by the electric repulsion in addition to the pressure gradient.
- The inclusion of charge inhibits the growth of space time curvature which has a great role to avoid singularities (Ivanov 2002; de Felice et al. 1995).
- Bonnor (1965), pointed out that a dust distribution of arbitrarily large mass and small radius can remain in equilibrium against the pull of gravity by a repulsive force produced by a small amount of charge.
- The solutions of Einstein-Maxwell equations are useful to study the cosmic matter.
- The charge dust models and electromagnetic mass models are providing some clue about the structure of electron (Bijalwan 2011) and Lepton model (Kiess 2013).
- Several solutions which do not satisfy some or all the conditions for well behaved nature can be renewed into well behaved nature by charging them.

Thus it is desirable to study the insinuations of Einstein-Maxwell field equations with reference to the general rela-

N. Pant (✉)
Mathematics Department, National Defence Academy,
Khadakwasla, Pune 411023, India
e-mail: neeraj.pant@yahoo.com

N. Pradhan
Physics Department, National Defence Academy, Khadakwasla,
Pune 411023, India
e-mail: npradhan20569@gmail.com

M.H. Murad
Department of Natural Sciences, Daffodil International
University, 102, Sukrabad, Mirpur Road, Dhaka 1207,
Bangladesh
e-mail: murad@daffodilvarsity.edu.bd

tivistic prediction of gravitational collapse. For this purpose charged fluid ball models are required. The external field of such ball is to be matched with *Reissener-Nordström* solution. The solutions of Einstein-Maxwell field equations successfully explain the characteristics of massive objects like neutron star, quark star or other super-dense objects. Further, these stars are specified in terms of their masses and densities:

(a) A Neutron Star has surface density $\rho_b = 2 \times 10^{14}$ g/cm^3 (Brecher and Caporaso 1976) or 2.7×10^{14} g/cm^3 (Astashenok et al. 2013) and mass $1.4M_\odot$–$2.9M_\odot$. However, Astashenok et al. (2013) established that $f(R)$ models with realistic equation of state of neutron star has upper limit mass $2.0M_\odot$ and minimal radius close to 9 km.

(b) A Strange Quark Star has surface density $\rho_b = 4.6888 \times 10^{14}$ g/cm^3 (Fatema and Murad 2013; Zdunik 2000) and possible maximum mass $2M_\odot$. However, Dong et al. (2013) established that due to presence of half skyrmions in the dense baryonic matter the stable Strange Quark Star can have upper mass limit $2.4M_\odot$.

2 Conditions for well behaved solution

For well behaved nature of the solution in isotropic coordinates, the following conditions should be satisfied:

(i) The solution should be free from physical and geometrical singularities i.e. finite and positive values of central pressure, central density and non zero positive values of e^ω and e^υ.

(ii) The solution should have positive and monotonically decreasing expressions for pressure and density (p and ρ) with the increase of r. The solution should have positive value of ratio of pressure-density and less than 1 (weak energy condition) and less than $1/3$ (strong energy condition) throughout within the star, monotonically decreasing as well.

(iii) The causality condition should be obeyed i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e.

$$\frac{d}{dr}\left(\frac{dp}{d\rho}\right) < 0 \quad \text{or} \quad \left(\frac{d^2p}{d\rho^2}\right) > 0$$

for $0 \leq r \leq r_b$ i.e. the velocity of sound is increasing with the increase of density. In this context it is worth mentioning that the equation of state at ultra-high distribution, has the property that the sound speed is decreasing outwards (Canuto and Lodenquai 1975).

(iv) $\frac{p}{\rho} \leq \frac{dp}{d\rho}$, everywhere within the ball.

$$\gamma = \frac{d \log_e p}{d \log_e \rho} = \frac{\rho}{p}\frac{dp}{d\rho} \quad \Rightarrow \quad \frac{dp}{d\rho} = \gamma \frac{p}{\rho},$$

for realistic matter $\gamma \geq 1$.

(v) The red shift Z should be positive, finite and monotonically decreasing in nature with the increase of r.

(vi) Electric intensity E, such that $E_{(r=0)} = 0$, is taken to be monotonically increasing.

Under these conditions, we have to assume the one of the gravitational potential components in such a way that the field equation (8) can be integrated and solution should be well behaved. Further, the mass of the such modeled super dense object can be maximized by assuming surface density, for Neutron star (Brecher and Caporaso 1976) $\rho_b = 2 \times 10^{14}$ g/cm^3 and for SQM, Strange Quark star (Fatema and Murad 2013; Zdunik 2000) $\rho_b = 4.6888 \times 10^{14}$ g/cm^3.

Keeping in view this aspect, several authors obtained the parametric class of exact solutions; Das et al. (2011), Pant et al. (2011a, 2011b), Pant and Negi (2012), Kiess (2012), Gupta and Maurya (2011), Maurya and Gupta (2011), Pant (2011a, 2011b), Murad and Fatema (2013), Takisa and Maharaj (2013) etc. These coupled solutions are well behaved with some positive values of charge parameter K and completely describe the interior of the super-dense astrophysical object with charged matter. However, the works of Pant (2011b), Kiess (2012), Gupta and Maurya (2011) and Fatema and Murad (2013) are well behaved with their neutral counterparts. Most of the findings are in curvature coordinates. By motivation of Ivanov (2012) and in continuation our previous work (Pradhan and Pant 2014), in this paper we present a new family of solutions of Einstein-Maxwell field equations in isotropic coordinates for perfect fluid assuming a particular form of one of the metric potentials and suitable choice of electric intensity.

3 Field equations in isotropic coordinates

We consider the static and spherically symmetric metric in isotropic co-ordinates

$$ds^2 = -e^\omega\big[dr^2 + r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)\big] + c^2 e^\upsilon dt^2 \qquad (1)$$

where, ω and υ are functions of r.

Einstein-Maxwell field equations of gravitation for a non empty space-time are

$$R^i_j - \frac{1}{2}R\delta^i_j = -\frac{8\pi G}{c^4}T^i_j$$

$$= -\frac{8\pi G}{c^4}\Big[(p + \rho c^2)v^i v_j - p\delta^i_j$$

$$+ \frac{1}{4\pi}\Big(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F_{mn}F^{mn}\Big)\Big] \qquad (2)$$

where R_{ij} is Ricci tensor, T_{ij} is energy-momentum tensor, R is the scalar curvature and F_{jm} is the electromagnetic field

tensor, p denotes the pressure distribution, ρ the density distribution and v_i the velocity vector, satisfying the relation

$$g_{ij}v^iv^j = 1 \tag{3}$$

Since the field is static, therefore

$$v^1 = v^2 = v^3 = 0 \quad \text{and} \quad v^4 = \frac{1}{\sqrt{g_{44}}} \tag{4}$$

Thus we find that for the metric (1) under these conditions and for matter distributions with isotropic pressure the field equation (2) reduces to the following Das et al. (2011):

$$\frac{8\pi G}{c^4}p = e^{-\omega}\left[\frac{(\omega')^2}{4} + \frac{\omega'}{r} + \frac{\omega'v'}{2} + \frac{v'}{r}\right] + \frac{q^2}{r^4} \tag{5}$$

$$\frac{8\pi G}{c^4}p = e^{-\omega}\left[\frac{\omega''}{2} + \frac{v''}{2} + \frac{(v')^2}{4} + \frac{\omega'}{2r} + \frac{v'}{2r}\right] - \frac{q^2}{r^4} \tag{6}$$

$$\frac{8\pi G}{c^2}\rho = -e^{-\omega}\left[\omega'' + \frac{(\omega')^2}{4} + \frac{2\omega'}{r}\right] - \frac{q^2}{r^4} \tag{7}$$

where, prime (′) denotes differentiation with respect to r. From Eqs. (5) and (6) we obtain following differential equation in ω and v.

$$e^{-\omega}\left(\omega'' + v'' + \frac{(v')^2}{2} - \frac{(\omega')^2}{2} - \omega'v' - \frac{\omega'}{r} - \frac{v'}{r}\right) - 4\frac{q^2}{r^4} = 0 \tag{8}$$

Our task is to explore the solutions of Eq. (8) and obtain the fluid parameters p and ρ from Eq. (5) and Eq. (7). To solve the above equation we consider a seed solution as Pant et al. (2013), Murad and Pant (2014) and the electric intensity E of the following form:

$$\frac{E^2}{C} = \frac{q^2}{Cr^4} = \frac{KCr^2(1+Cr^2)^{\frac{2-2n}{n}}}{B^2} \tag{9}$$

where K is a positive constant. The electric intensity is so assumed that the model is physically significant and well behaved i.e. E remains regular and positive throughout the sphere. In addition, E vanishes at the center of the star and increases towards the boundary.

4 Boundary conditions in isotropic coordinates

For exploring the boundary conditions, we use the principle that the metric coefficients g_{ij} and their first derivatives $g_{ij,k}$ in interior solution (I) as well as in exterior solution (E) are continuous upto and on the boundary B. The continuity of metric coefficients g_{ij} of I and B on the boundary is known first fundamental form. The continuity of derivatives of metric coefficients g_{ij} of I and B on the boundary is known second fundamental form.

The exterior field of a spherically symmetric static charged fluid distribution is described by Reissner-Nordström metric given by

$$ds^2 = \left(1 - \frac{2GM}{c^2R} + \frac{q^2}{R^2}\right)c^2dt^2$$
$$- \left(1 - \frac{2GM}{c^2R} + \frac{q^2}{R^2}\right)^{-1}dR^2 - R^2d\theta^2$$
$$- R^2\sin^2\theta d\phi^2 \tag{10}$$

where M is the mass of the ball as determined by the external observer and R is the radial coordinate of the exterior region.

Since Reissner-Nordström (10) is considered as the exterior solution, thus we shall arrive at the following conclusions by matching first and second fundamental forms:

$$e^{v_b} = \left[1 - 2\frac{GM}{c^2R_b} + \frac{q_b^2}{R_b^2}\right] \tag{11}$$

$$R_b = r_b.e^{\frac{\omega_b}{2}} \quad \text{and} \quad q_{(\text{at } r=r_b)} = q_b \tag{12}$$

$$\frac{1}{2}\left(\omega' + \frac{2}{r}\right)_b r_b = \left(1 - 2\frac{GM}{c^2R_b} + \frac{q_b^2}{R_b^2}\right)^{1/2} \tag{13}$$

$$\frac{1}{2}(v')_b r_b = \left(\frac{GM}{c^2R_b} - \frac{q_b^2}{R_b^2}\right)\left(1 - 2\frac{GM}{c^2R_b} + \frac{q_b^2}{R_b^2}\right)^{-1/2} \tag{14}$$

Equations (11) to (14) are four conditions, known as boundary conditions in isotropic coordinates. Moreover, (12) and (14) are equivalent to zero pressure of the interior solution on the boundary.

5 A new class of solution

Equation (8) is solved by assuming the seed solution as Hajj-Boutros (1986), Murad and Pant (2014) and the electric intensity E in such a manner that the solution can be obtained and physically viable. Thus we have,

$$e^{\omega/2} = B(1+Cr^2)^{-\frac{1}{n}}, \quad x = Cr^2, \quad y = \frac{dv}{dx} \quad \text{and}$$
$$q^2 = \frac{K}{B^2C}x^3(1+x)^{\frac{2-2n}{n}} \tag{15}$$

On substituting the above in Eq. (8), we get the following Riccati differential equation in y,

$$\frac{dy}{dx} + \frac{2}{n}\frac{1}{(1+x)}y + \frac{1}{2}y^2 = -\frac{2n-2}{n^2}\frac{1}{(1+x)^2} + \frac{K}{(1+x)^2} \tag{16}$$

Which yields the following solution,

$$e^{\frac{\nu}{2}} = \frac{\{1 + A(1 + Cr^2)^{\frac{2S}{2n}}\}(1 + Cr^2)^{\frac{n-2-S}{2n}}}{B^2} \quad (17)$$

where A, B, C and K are arbitrary constants and

$$S = \sqrt{2n^2 K + n^2 - 8n + 8} \quad (18)$$

S is real for $K > \dfrac{8n - n^2 - 8}{2n^2}$ (18a)

K is non-zero for $4 - 2\sqrt{2} < n < 4 + 2\sqrt{2}$

The expressions for density and pressure are given by

$$\frac{8\pi G\rho}{c^2} = \frac{C}{n^2 B^2 f^{4n-4}}\left(12n + (4n-4)Cr^2 - n^2 K Cr^2\right) \quad (19)$$

$$\frac{8\pi Gp}{c^4}$$
$$= \frac{C}{n^2 B^2 f^{4n-4}\{1 + Af^{2S}\}}$$
$$\times \left[Cr^2\left[Af^{2S}(n^2 K + 2n^2 - 12n + 12 + 2nS - 4S) \right.\right.$$
$$\left. + (n^2 K + 2n^2 - 12n + 12 - 2nS + 4S)\right]$$
$$\left. + 2n\left[Af^{2S}(n-4+S) + (n-4-S)\right]\right] \quad (20)$$

where, $f = \left(1 + Cr^2\right)^{\frac{1}{2n}}$ (21)

6 Properties of the new solution

The central values of pressure and density are given by

$$\left(\frac{8\pi Gp}{c^4}\right)_{r=0} = \frac{2C}{nB^2(1+A)}\left[(n-4)(1+A) + S(A-1)\right] \quad (22)$$

$$\left(\frac{8\pi G\rho}{c^2}\right)_{r=0} = \frac{12C}{nB^2} \quad (23)$$

The central values of pressure and density will be non zero positive definite, if the following conditions will be satisfied.

$$A > (S - n + 4)/(S + n - 4), \qquad C > 0. \quad (24)$$

In view of (19) and (20) the ratio of pressure-density is given by

$$\frac{p}{\rho c^2} = \left(Cr^2\left[Af^{2S}(n^2 K + 2n^2 - 12n + 12 + 2nS - 4S)\right.\right.$$
$$\left. + (n^2 K + 2n^2 - 12n + 12 - 2nS + 4S)\right]$$
$$\left. + 2n\left[Af^{2S}(n-4+S) + (n-4-S)\right]\right)$$

$$\times \left((1 + Af^{2S})\left[12n + (4n-4)Cr^2 - n^2 K Cr^2\right]\right)^{-1} \quad (25)$$

Subjecting the condition that positive value of ratio of pressure-density and less than 1 at the centre i.e. $\frac{p_0}{\rho_0 c^2} \le 1$ which leads to the following inequality,

$$\left\{\frac{p}{\rho c^2}\right\}_{r=0} = \frac{(n-4)(1+A) + SA - S}{6(1+A)}$$
$$= \frac{n-4}{6} - \frac{S(1-A)}{6(1+A)} \quad (26)$$

All the values of A which satisfy Eq. (24), will also lead to the condition $\frac{p_0}{\rho_0 c^2} \le 1$.

Differentiating (20) with respect to r, we get

$$\frac{8\pi G}{c^4}\frac{dp}{dr}$$
$$= \frac{C^2 r}{n^3 B^2 \{1 + Af^{2S}\}^2 f^{6n-4}}\left[-Cr^2(2n-4)\right.$$
$$\times \left[A^2 f^{4S}\{n^2 K + 2n^2 - 12n + 12 + (2n-4)S\}\right.$$
$$+ \{n^2 K + 2n^2 - 12n + 12 - (2n-4)S\}$$
$$\left. + 2Af^{2S}(n^2 K + 2n^2 - 12n + 12 - 2S^2)\right]$$
$$+ 2n\left[A^2 f^{4S}\{n^2 K - 2n^2 + 8n - 4 - 2nS\}\right.$$
$$+ (n^2 K - 2n^2 + 8n - 4 + 2nS)$$
$$\left.\left. + 2Af^{2S}(n^2 K - 2n^2 + 8n - 4 + 2S^2)\right]\right] \quad (27)$$

Thus it is found that extrema of p occur at the centre i.e.

$$p' = 0 \quad \Rightarrow \quad r = 0 \quad \text{and} \quad \frac{8\pi G}{c^4}(p'')_{r=0} = -ve \quad (28)$$

Thus the expression of right hand side of Eq. (28) is negative for all values of A satisfying condition (24), showing thereby that the pressure (p) is maximum at the centre and monotonically decreasing.

Now differentiating Eq. (19) with respect to r we get

$$\frac{8\pi G}{c^2}\frac{d\rho}{dr} = \frac{C^2 r}{n^3 B^2 f^{6n-4}}\left[-40n^2 + 40n - 2n^3 K\right.$$
$$\left. - 8(n^2 - 3n + 2)Cr^2 + (2n^3 - 4n^2)K Cr^2\right] \quad (29)$$

Thus the extrema of ρ occur at the centre if

$$\rho' = 0 \quad \Rightarrow \quad r = 0$$

$$\frac{8\pi G}{c^2}(\rho'')_{r=0} = \frac{(-40n^2 + 40n - 2n^3 K)C^2}{n^3 B^2} \quad (30)$$

Thus, the expression of right hand side of (30) is negative showing thereby that the density ρ is maximum at the centre and monotonically decreasing.

The square of adiabatic sound speed at the centre, $\frac{1}{c^2}(\frac{dp}{d\rho})_{r=0}$, is given by

$$\frac{1}{c^2}\left(\frac{dp}{d\rho}\right)_{r=0} = \frac{(n^2K - 2n^2 + 8n - 4)(1+A)^2 + 2nS + 4AS^2 - 2nSA^2}{(20 - 20n - n^2K)(1+A)^2} < 1 \quad \text{and positive} \tag{31}$$

The causality condition is obeyed at the centre for all values of constants satisfying condition (24).

Due to cumbersome expressions of (25) and (31), the trend of pressure-density ratio and adiabatic sound speed is studied analytically after applying the boundary conditions.

Applying the boundary conditions from (11) to (14), we get the values of the arbitrary constants in terms of Schwarzschild parameters $u = \frac{GM}{c^2 R_b}$ and radius of the star R_b.

$$C = \frac{n(1-d)}{(nd - n + 2)r_b^2} > 0 \quad \text{for} \quad u \le \frac{2n-2}{n^2} + \frac{q_b^2}{2R_b^2} \tag{32}$$

$$A = \frac{n.(u - \frac{q_b^2}{R_b^2}).(1 + Cr_b^2)^{\frac{n-2-S}{2n}} - (n-2-S).Cr_b^2.d.(1+Cr_b^2)^{\frac{-n-2-S}{2n}}}{(n-2+S).d.Cr_b^2.(1+Cr_b^2)^{\frac{-n-2+S}{2n}} - n(u - \frac{q_b^2}{R_b^2}).(1+Cr_b^2)^{\frac{n-2+S}{2n}}} \tag{33}$$

$$B = \sqrt{\frac{(1+Cr_b^2)^{\frac{n-2-S}{2n}} + A(1+Cr_b^2)^{\frac{n-2+S}{2n}}}{d}} \tag{34}$$

where we define a new parameter called as Reissner-Nordström parameter 'd' given by

$$d = \sqrt{1 - 2u + \frac{q_b^2}{R_b^2}} \tag{35}$$

Whose value lies between $(n-2)/n < d < 1$ for $Cr_b^2 > 0$.

Surface density is given by

$$\frac{8\pi G}{c^2}\rho_b R_b^2 = \frac{Cr_b^2[12n + 4(n-1)Cr_b^2 - n^2KCr_b^2]}{n^2(1+Cr_b^2)^2} \tag{36}$$

Central red-shift is given by

$$Z_0 = \left[\frac{B^2}{1+A} - 1\right] \tag{37}$$

The surface red shift is given by

$$Z_b = \left[e^{-\frac{v_b}{2}} - 1\right] \tag{38}$$

state and the conditions for well behaved nature are satisfied. Our solution is well behaved for the all values of n satisfying the inequalities $4 < n \le 4(4+\sqrt{2})$ and K satisfying the inequalities $0 \le K \le 0.24988$. However, for $n > 4(4+\sqrt{2})$, there exist no value of K for which the nature of adiabatic sound speed is monotonically decreasing from centre to pressure free interface. In view of Fig. 2: With the increase of n the maximum mass decreases.

In Table 2, we present a neutron star modeling based on the particular solution; corresponding to $n = 4.001$ for which $K_{max} = 0.24988$ and $u_{max} = 0.3448$ the maximum mass of neutron star for surface density 2×10^{14} gm/cc is found to be $M = 3.58M_\odot$ and radius $R = 15.37$ km and that for surface density 2.7×10^{14} gm/cc is found to be $M = 3.08M_\odot$ and radius $R = 13.23$ km.

In Table 3, we present a quark star modeling based on the particular solution; corresponding to $n = 4.001$ for which $K_{max} = 0.24988$ and $u_{max} = 0.3448$ and by assuming the surface density the maximum mass of quark star $M = 2.335M_\odot$ and radius $R = 10.04$ km.

7 Discussions

In view of Table 1 and Fig. 1 it has been observed that all the physical parameters (p, ρ, $\frac{p}{\rho c^2}$, $\frac{dp}{d\rho}$, z, γ and E) are positive at the centre and within the limit of realistic equation of

8 Conclusion

We find that all the physical parameters (p, ρ, $\frac{p}{\rho c^2}$, $\frac{dp}{d\rho}$, Z, γ and E) are showing the desired trend in our range for K ($0 \le K \le 0.24988$) and for the prescribed value for n;

Table 1 The march of pressure, density, pressure-density ratio, square of adiabatic speed of sound, γ, red shift and electric field intensity within the ball corresponding to $n = 4.001$ for which $K_{max} = K_{min} = 0.24988$, $d_{min} = 0.6552$ and $u_{max} = 0.3448$

$\frac{r}{r_b}$	$\frac{8\pi G}{c^4} p r_b^2$ (10^{-6})	$\frac{8\pi G}{c^2} \rho r_b^2$ (10^{-7})	$\frac{p}{\rho c^2}$ (10^{-7})	$\frac{1}{c^2}(\frac{dp}{d\rho})$ (10^{-7})	$\gamma = \frac{dp}{d\rho}/\frac{p}{\rho}$	Z	$E.r_b$
0	0.1626	5.5598	0.2925	0.4133	1.4129	1.0448	0.0000000
0.1	0.1560	5.3993	0.2889	0.4118	1.4252	1.0336	0.0998146
0.2	0.1382	4.9653	0.2784	0.4075	1.4639	1.0017	0.1903867
0.3	0.1142	4.3700	0.2612	0.4010	1.5349	0.9537	0.2655015
0.4	0.0888	3.7310	0.2380	0.3927	1.6503	0.8950	0.3230645
0.5	0.0655	3.1304	0.2092	0.3834	1.8329	0.8309	0.3642153
0.6	0.0457	2.6086	0.1753	0.3732	2.1290	0.7653	0.3917283
0.7	0.0298	2.1752	0.1369	0.3624	2.6475	0.7009	0.4087300
0.8	0.0172	1.8238	0.0945	0.3510	3.7152	0.6391	0.4180159
0.9	0.0075	1.5421	0.0487	0.3391	6.9673	0.5807	0.4218196
1	0.0000	1.3168	0.0000	0.3264	infinity	0.5262	0.4218197

Table 2 The variation of maximum mass of neutron star, corresponding radius R_b, surface red shift and electric field with u for $n = 4.001$ and $K = 0.24988$

u	$\frac{8\pi G \rho r_b^2}{c^2}$	R_b (km) $(\rho_b = 2 \times 10^{14}$ g/cc$)$	$\frac{M}{M_\oplus}$	R_b (km) $(\rho_b = 2.7 \times 10^{14}$ g/cc$)$	$\frac{M}{M_\oplus}$	Z_b	$E_b.r_b$
0.010	0.10011	3.98069	0.02686	3.42604	0.02311	0.01010	0.0130
0.020	0.19585	5.58163	0.07531	4.80390	0.06482	0.02041	0.0260
0.040	0.37442	7.75630	0.20932	6.67556	0.18015	0.04167	0.0517
0.080	0.68170	10.56996	0.57049	9.09718	0.49100	0.08696	0.1024
0.100	0.81136	11.58804	0.78180	9.97339	0.67287	0.11111	0.1274
0.140	1.02521	13.15114	1.24216	11.31869	1.06908	0.16279	0.1766
0.180	1.18178	14.24872	1.73035	12.26334	1.48925	0.21951	0.2251
0.200	1.23988	14.65740	1.97776	12.61508	1.70218	0.25000	0.2490
0.240	1.31840	15.23216	2.46638	13.10975	2.12272	0.31579	0.2965
0.280	1.35045	15.51135	2.93018	13.35004	2.52190	0.38889	0.3438
0.300	1.35056	15.54574	3.14644	13.37964	2.70802	0.42857	0.3675
0.320	1.34096	15.51093	3.34869	13.34968	2.88209	0.47059	0.3915
0.3448	1.31677	15.37104	3.57546	13.22928	3.07727	0.52620	0.4218

Fig. 1 The variation of p, ρ, $\frac{p}{\rho c^2}$, Z, $\frac{1}{c^2}(\frac{dp}{d\rho})$, γ and E from centre to surface for $n = 4.001$ and $K = 0.24988$ is shown in the following graphs
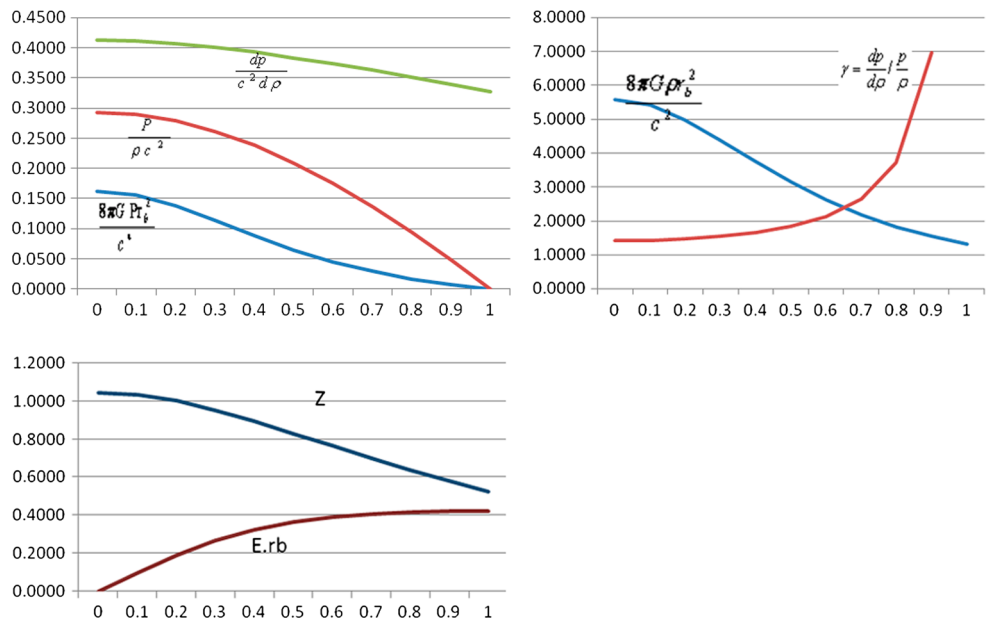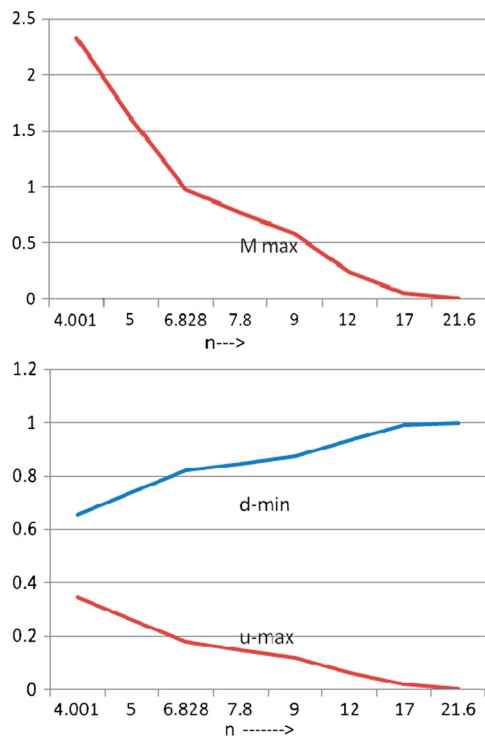
Fig. 2 Variation of maximum mass M of quark star with n

Table 3 Variation of maximum mass of quark star (surface density $\rho_b = 4.6888 \times 10^{14}$ g/cc) and its radius with u for $n = 4.001$ and $K = 0.24988$

u	R_b (km)	$\frac{M}{M_\oplus}$
0.010	2.59982	0.01754
0.020	3.64540	0.04919
0.040	5.06569	0.13671
0.080	6.90331	0.37259
0.100	7.56822	0.51060
0.140	8.58909	0.81126
0.180	9.30593	1.13011
0.200	9.57284	1.29169
0.240	9.94822	1.61081
0.280	10.13057	1.91372
0.300	10.15303	2.05496
0.320	10.13029	2.18705
0.3448	10.03893	2.33516

$4(4 + \sqrt{2}) \geq n > 4$. However, for $4 + 2\sqrt{2} \geq n > 4$, we observe that with the increase of K, i.e. increasing the charge, the mass of the star increases. This is consistent with the observed phenomena. The ratio of mass/radius is increasing as we increase the charge i.e. with increasing charge; the proportion of mass to the radius of the star is increasing. Hence the denser star can be stabilized with the increase of charge. Second important aspect of the solutions is that the range of well behaved nature of solutions of Murad and Pant (2014) (Neutral solutions) is augmented substantially from $4(4 + \sqrt{2}) \geq n > 4 + 2\sqrt{2}$ to $4(4 + \sqrt{2}) \geq n > 4$. The maximum mass of the quark star is very close to the value suggested by Dong et al. (2013). Further, for some suitable choices of input parameters n and u, we have generated in Table 3 compact fluid spheres similar to the mass and radius of some possible strange star candidates like Her X-1, 4U 1538–52, LMCX-4, SAX J1808.4–3658 (Gangopadhyay et al. 2013).

References

Astashenok, A.V., et al.: (2013). arXiv:1309.1978v2
Bijalwan, N.: Astrophys. Space Sci. **336**, 485 (2011)
Bonnor, W.B.: Mon. Not. R. Astron. Soc. **137**, 239 (1965)
Brecher, K., Caporaso, G.: Nature **259**, 377–378 (1976)
Canuto, V., Lodenquai, J.: Phys. Rev. C **12**, 2033 (1975)
Das, B., et al.: Int. J. Mod. Phys. D **20**, 1675 (2011)
de Felice, F., Yu, Y., Fang, J.: Mon. Not. R. Astron. Soc. **277**, L17–L19 (1995)
Dong, H., et al.: (2013). arXiv:1207.0429v3
Fatema, S., Murad, M.H.: Int. J. Theor. Phys. (2013). doi:10.1007/s10773-013-1538-y
Gangopadhyay, T., et al.: Mon. Not. R. Astron. Soc. (2013). doi:10.1093/mnras/stt401
Gupta, Y.K., Maurya, S.K.: Astrophys. Space Sci. **334**(1), 155 (2011)
Hajj-Boutros, J.: J. Math. Phys. **27**, 1363 (1986)
Ivanov, B.V.: Phys. Rev. D **65**, 104001 (2002)
Ivanov, B.V.: Gen. Relativ. Gravit. **44**, 1835 (2012)
Kiess, T.E.: Astrophys. Space Sci. (2012). doi:10.1007/s10509-012-1013-x
Kiess, T.E.: Int. J. Mod. Phys. D **22**, 1350088 (2013)
Maurya, S.K., Gupta, Y.K.: Astrophys. Space Sci. **332**, 481 (2011)
Murad, H.M., Fatema, S.: Astrophys. Space Sci. **343**, 587 (2013)
Murad, H.M., Pant, N.: Astrophys. Space Sci. **350**, 349 (2014)
Pant, N.: Astrophys. Space Sci. **334**, 267 (2011a)
Pant, N.: Astrophys. Space Sci. **332**, 403 (2011b)
Pant, N., Negi, P.S.: Astrophys. Space Sci. **338**, 163 (2012)
Pant, N., et al.: Astrophys. Space Sci. **332**, 473 (2011a)
Pant, N., et al.: Astrophys. Space Sci. **333**, 161 (2011b)
Pant, N., et al.: Int. J. Theor. Phys. (2013). doi:10.1007/s10773-013-1892-9
Pradhan, N., Pant, N.: Astrophys. Space Sci. (2014). doi:10.1007/s10509-014-1905-z
Takisa, P.M., Maharaj, S.D.: Gen. Relativ. Gravit. (2013). doi:10.1007/s10714-013-1570-5
Zdunik, J.L.: Astron. Astrophys. **359**, 311–315 (2000)