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ORIGINAL ARTICLE

On the radiative and thermodynamic properties of the cosmic radiations using *COBE* FIRAS instrument data: I. Cosmic microwave background radiation

Anatoliy I. Fisenko · Vladimir Lemberg

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Abstract Using the explicit form of the functions to describe the monopole and dipole spectra of the Cosmic Microwave Background (CMB) radiation, the exact expressions for the temperature dependences of the radiative and thermodynamic functions, such as the total radiation power per unit area, total energy density, number density of photons, Helmholtz free energy density, entropy density, heat capacity at constant volume, and pressure in the finite range of frequencies $v_1 \leq v \leq v_2$ are obtained. Since the dependence of temperature upon the redshift z is known, the obtained expressions can be simply presented in z representation. Utilizing experimental data for the monopole and dipole spectra measured by the *COBE* FIRAS instrument in the 60–600 GHz frequency interval at the temperature $T = 2.72548$ K, the values of the radiative and thermodynamic functions, as well as the radiation density constant a and the Stefan-Boltzmann constant σ are calculated. In the case of the dipole spectrum, the constants a and σ, and the radiative and thermodynamic properties of the CMB radiation are obtained using the mean amplitude $T_{\mathrm{amp}} = 3.358$ mK. It is shown that the Doppler shift leads to a renormalization of the radiation density constant a, the Stefan-Boltzmann constant σ, and the corresponding constants for the thermodynamic functions. The expressions for new astrophysical parameters, such as the entropy density/Boltzmann constant, and number density of CMB photons are obtained. The radiative and thermodynamic properties of the Cosmic Microwave Background radiation for the monopole and dipole spectra at redshift $z \approx 1089$ are calculated.

A.I. Fisenko (✉) · V. Lemberg
ONCFEC Inc., 250 Lake Street, Suite 909, St. Catharines, Ontario L2R 5Z4, Canada
e-mail: afisenko@oncfec.com

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1 Introduction

In many areas of astrophysics and physics associated with the Far InfraRed Absolute Spectrophotometry (FIRAS), Fourier Transform InfraRed (FTIR) spectroscopy, radiation pyrometry, or any other area of study of electromagnetic radiation, the spectrum of a real body in the finite frequency interval needs to be measured.

In FIRAS, for example, only a small region of the electromagnetic spectrum is of present concern; 2–20 cm^{-1} frequency interval for the measurement of the cosmic microwave background (CMB) radiation (Fixsen et al. 1994) or 5–80 cm^{-1} frequency interval to observe the spectrum of the extragalactic far infrared background radiation (Fixsen et al. 1998). This instrument has been developed to determine the spectral radiation intensity seen through these short radiation windows. The theoretical experiments utilizing a computer require the calculation of the total radiation, number density of photons, as well as the thermodynamic functions, as seen through these short windows. A numerical solution for the computer calculation of these functions should be obtained over a specified range of the spectrum. The importance of the use of a finite frequency range in astrophysics and physics was noted by Michels (1968).

The present work is devoted to the construction of the exact expressions for the radiative and thermodynamic functions of the CMB radiation in the finite range of frequencies, which can be used for the computer calculation.

It is well-known that the discovery of the cosmic microwave background (CMB) radiation by Penzias and Wilson (1965) provides a strong observational foundation for confirming the Gamow's primordial-fireball hypothesis (Sunyaev and Zel'dovich 1980). The theoretical prediction that the CMB spectrum is so close to a blackbody was confirmed by the *COBE* FIRAS observation (Mather et al. 1990). However, the perfect fitting of the measured spectrum with the spectrum of a blackbody was achieved only at the peak of the blackbody in the 2–20 cm^{-1} frequency interval. This range belongs to the Planck part of the total spectrum. In the Rayleigh-Jeans approximation at low frequency, most experiments are consistent with $T = 2.72548$ K, but some are not (Kogut et al. 1990, 1996). With respect to the Wien part of the total spectrum, the spectral distortions of the spectrum are difficult to measure due to the foreground signal from interstellar dust at high frequencies (Gawiser and Silk 2000). Above a few hundred GHz, the detected an isotropic far infrared background dominates the cosmic microwave background (Fixen et al. 1998; Puget et al. 1996; Dwek et al. 1998; Schlegel et al. 1998; Burigana and Popa 1998).

As a result, the theoretical prediction that the CMB spectrum has a blackbody one in the semi-infinite range of frequencies should be confirmed by new experiments, especially in Wien's part of the spectrum.

In order to construct the thermodynamics of the CMB radiation, knowledge of integral characteristics of a system, such as the total energy density and the number density of photons, is necessary. In this case, the experimental data of the CMB radiation spectrum in a wide range of frequencies should be used. Currently, only the data measured by the *COBE* FIRAS instrument within a finite range of frequencies, ranging from 2 cm^{-1} to 20 cm^{-1}, which cover the Planck region, is reliable for the calculation of the thermodynamic functions of the CMB radiation.

In previous studies (Fisenko and Ivashov 2009; Fisenko and Lemberg 2012, 2013), the thermodynamics of the thermal radiation of real bodies, such as molybdenum, luminous flames, stoichiometric carbides of hafnium, titanium and zirconium, and ZrB_2-SiC-based ultra-high temperature ceramics in the finite range of frequencies at high temperatures were constructed. The calculated values of radiative and thermodynamic functions were in good agreement with experimental data.

In the present work, the exact expressions for the temperature dependences of the radiative and thermodynamic functions, such as the total radiation power per unit area, total energy density, radiation density constant a, Stefan-Boltzmann's constant σ, number density of photons, Helmholtz free energy density, entropy density, heat capacity at constant volume, and pressure for the monopole and dipole spectra in the finite range of frequencies are constructed. These results can be presented in the redshift z representation. For the monopole spectrum, the values of the radiative and thermodynamic functions at the mean temperature $T = 2.72548$ K are calculated. In the case of the dipole spectrum, the calculated values of the radiative and thermodynamic functions were obtained using the mean amplitude $T_{\mathrm{amp}} = 3.358$ mK. It is shown that the Doppler shift leads to a renormalization of the radiation density constant, the Stefan-Boltzmann constant, and the corresponding constants for the thermodynamic functions. The radiative and thermodynamic functions of the CMB radiation for the monopole and dipole spectra at the redshift $z \approx 1089$ are presented.

2 General relationships for monopole and dipole spectra

According to Fixsen et al. (1994), Mather et al. (1994), the cosmological anisotropy is predicted to have a Planckian spectrum of the following form

$$I_0(\tilde{\upsilon}) \approx B_{\tilde{\upsilon}}(T_0) + \left.\frac{\partial B_{\tilde{\upsilon}}(T)}{\partial T}\right|_{T=T_0} \Delta T. \tag{1}$$

Here two terms are the monopole and dipole spectra with the temperature T_0. $T_0 = 2.72548$ K is the mean temperature of the CMB radiation (Mather et al. 2013; Fixsen 2009). The temperature fluctuation $\Delta T = T - T_0$ is the temperature anisotropy in a given direction in the sky and can be presented in the form $\Delta T \approx (\frac{\upsilon}{c}) T_{\mathrm{amp}} \cos(\theta)$. Here υ is a velocity of moving observer with respect to the rest frame of the blackbody radiation, θ is the angle between the direction of observation and the dipole direction $(l, b) = (264.°26, +48.°22)$ (Bennett et al. 1996, 2003), and the dipole mean amplitude $T_{\mathrm{amp}} = 3.358$ mK (Hinshaw et al. 2007; Robitaille 2007). $B_{\tilde{\upsilon}}(T)$ at the temperature T is given by the Planck law

$$B_{\tilde{\upsilon}}(T) = 2hc^2 \frac{\tilde{\upsilon}^3}{e^{\frac{hc\tilde{\upsilon}}{k_B T}} - 1}, \tag{2}$$

were h is the Planck constant and c is the speed of light.

Let us note that in Mather et al. (1994), the variable $\tilde{\upsilon} = \upsilon c$ calls as the frequency with the unit [cm^{-1}], where υ is the frequency [Hz]. However, according to (http://en.wikipedia.org/wiki/Wavenumber), the variable $\tilde{\upsilon}$ is called as wavenumber. Further, we will call $\tilde{\upsilon}$ as the wavenumber and υ as the frequency.

According to Eq. (1), the total energy density of the CMB radiation in the wavenumber domain received in the frequency interval from $\tilde{\upsilon}_1$ to $\tilde{\upsilon}_2$ has the following structure:

$$B_0(\tilde{\upsilon}_1, \tilde{\upsilon}_2, T) = \frac{4\pi}{c}\left\{\int_{\tilde{\upsilon}_1}^{\tilde{\upsilon}_2} B_{\tilde{\upsilon}}(T_0) d\tilde{\upsilon} + \Delta T \int_{\tilde{\upsilon}_1}^{\tilde{\upsilon}_2} \left.\frac{\partial B_{\tilde{\upsilon}}(T)}{\partial T}\right|_{T=T_0} d\tilde{\upsilon}\right\} \tag{3}$$

For constructing the thermodynamics of the CMB radiation, using the *COBE* FIRAS instrument data in the finite range of wavenumbers, hereinafter, it is convenient to present the Planck function Eq. (2) in the frequency domain. Using the relationship

$$B_{\tilde{\upsilon}}(T)d\tilde{\upsilon} = B_{\upsilon(\tilde{\upsilon})}(T)d\upsilon, \tag{4}$$

where $\tilde{\upsilon}$ stands for wavenumber, that can be related to the frequency υ via the transformation $\upsilon(\tilde{\upsilon})$, then

$$B_\upsilon(T) = B_{\tilde{\upsilon}}(T)\frac{d\tilde{\upsilon}}{d\upsilon} = \frac{2h}{c^2}\frac{\upsilon^3}{e^{\frac{h\upsilon}{k_B T}} - 1}. \tag{5}$$

Equation (5) is called the Planck function in the frequency domain.

Using the following relationship between the spectral energy density $I_\upsilon(T)$ and Eq. (5)

$$I_\upsilon(T) = \frac{4\pi}{c}B_\upsilon(T), \tag{6}$$

we obtain

$$I_\upsilon(T) = \frac{8\pi h}{c^3}\frac{\upsilon^3}{e^{\frac{h\upsilon}{k_B T}} - 1}. \tag{7}$$

Then, according to Eq. (3), the total energy density of the CMB radiation in the frequency domain has the form

$$I_0(\upsilon_1, \upsilon_2, T) = \int_{\upsilon_1}^{\upsilon_2} I_\upsilon(T)\mathrm{d}\nu + \Delta T \int_{\upsilon_1}^{\upsilon_2} \frac{\partial I_\upsilon(T)}{\partial T}\bigg|_{T=T_0} \mathrm{d}\nu. \tag{8}$$

The first term is the total energy density for the monopole spectrum $I_0^{\mathrm{M}}(T)$ and the second one for the dipole spectrum $I_0^{\mathrm{D}}(T)$.

The thermodynamic functions of the CMB radiation is determined as follows (Landau and Lifshitz 1980):

(1) Helmholtz free energy density $f = \frac{F}{V}$:
   (a) Monopole:

$$f = \frac{8\pi k_B T}{c^3}\int_{\upsilon_1}^{\upsilon_2} \upsilon^2 \ln\left(1 - e^{\frac{h\upsilon}{k_b T}}\right)d\upsilon \tag{9}$$

   (b) Dipole

$$f' = \frac{3}{2}T_{\mathrm{amp}}\frac{\partial f}{\partial T} \tag{10}$$

(2) Entropy density $s = \frac{S}{V}$:
   (a) Monopole

$$s = -\frac{\partial f}{\partial T} \tag{11}$$

   (b) Dipole

$$s = -\frac{\partial f'}{\partial T} \tag{12}$$

(3) Heat capacity at constant volume per unit volume $c_V = \frac{C_V}{V}$:
   (a) Monopole

$$c_V = \left(\frac{\partial I_0^{\mathrm{M}}(\upsilon_1, \upsilon_2, T)}{\partial(T)}\right)_V \tag{13}$$

   (b) Dipole

$$c_V = \left(\frac{\partial I_0^{\mathrm{D}}(\upsilon_1, \upsilon_2, T)}{\partial(T)}\right)_V \tag{14}$$

(4) Pressure of photons P:
   (a) Monopole

$$P = -f \tag{15}$$

   (b) Dipole

$$P = -f' \tag{16}$$

(5) The number density of photons $n = \frac{N}{V}$:
   (a) Monopole:

$$n = \frac{8\pi}{c^3}\int_{\upsilon_1}^{\upsilon_2} \frac{\upsilon^2}{e^x - 1}d\upsilon \tag{17}$$

   (b) Dipole

$$n = \frac{8\pi}{c^3}T_{\mathrm{amp}}\frac{\partial}{\partial T}\int_{\upsilon_1}^{\upsilon_2} \frac{\upsilon^2}{e^x - 1}d\upsilon \tag{18}$$

   Here T is the temperature for the monopole spectrum, T_{amp} is the dipole amplitude and V is the volume of emitted object.

Equations (8)–(18) describe the temperature dependences of the radiative and thermodynamic properties of the CMB radiation. To convert to the redshift dependences, the following relationships $T(z) = T_0(1+z)$, $T_{\mathrm{amp}}(z) = T_{\mathrm{amp}}(1+z)$ and $\upsilon(z) = \upsilon(1+z)$ should be used.

3 Monopole spectrum

Now let us construct the thermodynamics of the CMB radiation for the monopole spectrum. In this case, the total energy density of the CMB radiation received in the finite range of frequencies from υ_1 to υ_2 is described by the first term in Eq. (8). Substituting Eq. (7) in Eq. (8) and after computing the integral, we obtain

$$\begin{aligned} I_0^{\mathrm{M}}(x_1, x_2, T) &= \int_{\upsilon_1}^{\upsilon_2} I_\upsilon(T)\mathrm{d}\nu = \frac{8\pi h}{c^3}\int_{\upsilon_1}^{\upsilon_2} \frac{\upsilon^3}{e^{\frac{h\upsilon}{k_B T}} - 1}d\upsilon \\ &= \frac{48\pi(k_B T)^4}{c^3 h^3}\left[P_3(x_1) - P_3(x_2)\right]. \end{aligned} \tag{19}$$

Here $x = \frac{h\upsilon}{k_B T}$. $P_3(x)$ is defined as

$$P_3(x) = \sum_{s=0}^{3}\frac{(x)^s}{s!}\mathrm{Li}_{4-s}\left(e^{-x}\right), \tag{20}$$

where

$$\mathrm{Li}_{4-s}\left(e^{-x}\right) = \sum_{k=1}^{\infty}\frac{e^{-kx}}{k^{4-s}} \tag{21}$$

is the polylogarithm function (Abramowitz and Stegun 1972).

In the semi-infinite range ($0 \le \upsilon \le \infty$), Eq. (16) can be re-written as

$$I_0^{\mathrm{M}}(0,\infty,T) = \frac{48\pi k_{\mathrm{B}}^4}{c^3h^3}T^4\big(P_3(0) - P_3(\infty)\big). \tag{22}$$

Since $P_3(0) = \mathrm{Li}_4(1) = \xi(4) = \frac{\pi^4}{90}$ and $P_3(\infty) = 0$, Eq. (19) transforms to the well-known expression for the total energy density of the blackbody radiation (Landau and Lifshitz 1980)

$$I_0^{\mathrm{M}}(0,\infty,T) = aT^4. \tag{23}$$

Here a is the radiation density constant

$$a = \frac{8\pi^5 k_{\mathrm{B}}^4}{15c^3h^3}. \tag{24}$$

The numerical value of a is $a = 7.5657 \times 10^{-16}\ \frac{\mathrm{J}}{\mathrm{m}^3\,\mathrm{K}^4}$. The Stefan-Boltzmann constant σ can be determined using the following relationship $\sigma = \frac{ac}{4}$ and, in accordance with Eq. (24), takes the value $\sigma = 5.67037 \times 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}^4}$ (Landau and Lifshitz 1980). Then, the Stefan-Boltzmann law or the total radiation power per unit area received in the semi-infinite frequency range has the well-known form

$$I_0^{\prime\mathrm{M}}(0,\infty,T) = \sigma T^4. \tag{25}$$

Let us note that according to Eq. (25), the total radiation power $I_{\mathrm{total}}^{\mathrm{M}}(T)$ emitted from the area A of the early universe, which is presented in the form of a spherical shell of finite thickness at distance of almost 15 billion light years, is defined as follows

$$I_{\mathrm{total}}^{\mathrm{M}}(T) = AI_0^{\prime\mathrm{M}}(0,\infty,T). \tag{26}$$

Here it is important to note that the area A may be obtained from experiment data by measuring the total radiation power $I_{\mathrm{Measured}}^{\mathrm{M}}(T) = I_{\mathrm{total}}^{\mathrm{M}}(T)$, and then using the Eq. (26). In this case, we have

$$A = \frac{I_{\mathrm{Measured}}^{\mathrm{M}}(T)}{I_0^{\prime\mathrm{M}}(0,\infty,T)}. \tag{27}$$

For this purpose, an optical device for measuring the total radiation power emitted from the area A of a surface at low temperature will be useful.

Let us Eq. (19) present in the form

$$I_0^{\mathrm{M}}(x_1,x_2,T) = a'(x_1,x_2)T^4, \tag{28}$$

where

$$a'(x_1,x_2) = \frac{48\pi k_{\mathrm{B}}^4}{c^3h^3}\big[P_3(x_1) - P_3(x_2)\big]. \tag{29}$$

$a'(x_1,x_2)$ can be called as the radiation density constant in the finite range of frequencies $\upsilon_1 \le \upsilon \le \upsilon_2$. Then, the Stefan-Boltzmann law has the following form

$$I_0^{\prime\mathrm{M}}(x_1,x_2,T) = \sigma'(x_1,x_2)T^4. \tag{30}$$

Here

$$\sigma'(x_1,x_2) = \frac{a'(x_1,x_2)c}{4} = \frac{12\pi k_{\mathrm{B}}^4}{c^2h^3}\big[P_3(x_1) - P_3(x_2)\big]. \tag{31}$$

$\sigma'(x_1,x_2)$, as in the case of the radiation density constant, can be named as the Stefan-Boltzmann constant in the finite range of frequencies.

Importantly, the Eq. (27) has the same structure, within a finite range of frequencies in which values $I_{\mathrm{Measured}}^{\mathrm{M}}(T)$ and $I_0^{\prime\mathrm{M}}(0,\infty,T)$ should be replaced by $I_{\mathrm{Measured}}^{\mathrm{M}}(x_1,x_2,T)$ and $I_0^{\prime\mathrm{M}}(x_1,x_2,T)$.

In accordance with Eq. (17), the number density of photons of the CMB radiation with a photon energy from $h\upsilon_1$ to $h\upsilon_2$, defined as

$$n = \frac{16\pi k_{\mathrm{B}}^3}{c^3h^3}T^3\big[P_2(x_1) - P_2(x_2)\big]. \tag{32}$$

In the case of the semi-infinite range of frequencies, Eq. (29) simplifies to

$$n = \frac{16\pi k_{\mathrm{B}}^3}{c^3h^3}T^3\big[P_2(0) - P_2(\infty)\big]. \tag{33}$$

Since $P_2(0) = L_{i_3}(1) = \xi(3)$ and $P_2(\infty) = 0$, Eq. (30) transforms to the well-known expression (Landau and Lifshitz 1980)

$$n \approx 0.244\left(\frac{2\pi k_{\mathrm{B}}T}{hc}\right)^3. \tag{34}$$

According to Eqs. (9), (11), (13), (15), the thermodynamic functions of the CMB radiation for the monopole spectrum in the finite frequency range have the following structure:

(1) Helmholtz free energy density f:

$$f = -\frac{16\pi k_{\mathrm{B}}^4}{c^3h^3}T^4\bigg[\big(P_3(x_1) - P_3(x_2)\big) - \frac{1}{6}\big(x_1^3\mathrm{Li}_1\big(e^{-x_1}\big) - x_2^3\mathrm{Li}_1\big(e^{-x_2}\big)\big)\bigg] \tag{35}$$

(2) Entropy density s:

$$s = \frac{64\pi k_{\mathrm{B}}^4}{c^3h^3}T^3\bigg[\big(P_3(x_1) - P_3(x_2)\big) - \frac{1}{24}\big(x_1^3\mathrm{Li}_1\big(e^{-x_1}\big) - x_2^3\mathrm{Li}_1\big(e^{-x_2}\big)\big)\bigg] \tag{36}$$

(3) Heat capacity at constant volume per unit volume, c_V

$$c_V = \frac{192\pi k_{\mathrm{B}}^4}{c^3h^3}T^3\bigg[\big(P_3(x_1) - P_3(x_2)\big) + \frac{1}{24}\big(x_1^4\mathrm{Li}_0\big(e^{-x_1}\big) - x_2^4\mathrm{Li}_0\big(e^{-x_2}\big)\big)\bigg] \tag{37}$$

(4) Pressure P:

$$P = \frac{16\pi k_{\mathrm{B}}^4}{c^3h^3}T^4\bigg[\big(P_3(x_1) - P_3(x_2)\big) - \frac{1}{6}\big(x_1^3\mathrm{Li}_1\big(e^{-x_1}\big) - x_2^3\mathrm{Li}_1\big(e^{-x_2}\big)\big)\bigg] \tag{38}$$

Table 1 Calculated values of the radiative and thermodynamic state functions for the monopole and dipole spectra in the 60–600 GHz frequency interval at $T = 2.72548$ K and $T_{\text{amp}} = 3.358$ mK

Quantity	Monopole $0 \leq \tilde{v} \leq \infty$	Monopole $v_1 \leq v \leq v_2$	Dipole $v_1 \leq v \leq v_2$	Dipole $0 \leq \tilde{v} \leq \infty$
a', a'' [$\mathrm{J\,m^{-3}\,K^{-4}}$]	7.5657×10^{-16}	7.2167×10^{-16}	2.9263×10^{-15}	3.0263×10^{-15}
σ', σ'' [$\mathrm{W\,m^{-3}\,K^{-4}}$]	5.6704×10^{-8}	5.4088×10^{-8}	2.1932×10^{-7}	2.2681×10^{-7}
$I_0^{\mathrm{M}}(v_1, v_2, T)$	4.1902×10^{-14}	3.9845×10^{-14}	–	–
$I_0^{\mathrm{D}}(v_1, v_2, T)$ [$\mathrm{J\,m^{-3}}$]	–	–	1.9894×10^{-16}	2.0571×10^{-16}
$I_0'^{\mathrm{M}}(v_1, v_2, T)$	3.1404×10^{-6}	2.9845×10^{-6}	–	–
$I_0'^{\mathrm{D}}(v_1, v_2, T)$ [$\mathrm{W\,m^{-2}}$]	–	–	1.4910×10^{-8}	1.5420×10^{-8}
f [$\mathrm{J\,m^{-3}}$]	-1.3967×10^{-14}	-1.2260×10^{-14}	-9.6252×10^{-17}	-1.0287×10^{-16}
s [$\mathrm{J\,m^{-3}\,K^{-1}}$]	2.0478×10^{-14}	1.9109×10^{-14}	1.0652×10^{-16}	1.1323×10^{-16}
P [$\mathrm{J\,m^{-3}}$]	1.3967×10^{-14}	1.2260×10^{-14}	9.6252×10^{-17}	1.0287×10^{-16}
c_V [$\mathrm{J\,m^{-3}\,K^{-1}}$]	6.1439×10^{-14}	5.9244×10^{-14}	2.1219×10^{-16}	2.2542×10^{-16}
n [$\mathrm{m^{-3}}$]	4.1186×10^{8}	3.4404×10^{8}	1.3787×10^{6}	1.5180×10^{6}

It is not difficult to show that Eqs. (35)–(38) in the semi-infinite range of frequencies are converted to the well-known expressions for the thermodynamic functions (Landau and Lifshitz 1980).

By definition (Landau and Lifshitz 1980), the chemical potential density $\mu = (\frac{\partial f}{\partial n})_{T,V}$, as clearly seen from Eq. (35), is equal to zero.

Now let us apply obtained expressions for calculating the radiative and thermodynamic functions of the CMB radiation for the monopole spectrum at the temperature $T = 2.72548$ K (Mather et al. 2013). Using the data obtained by the *COBE* FIRAS instrument in the 60–600 GHz frequency interval, the radiative and thermodynamic properties of the CMB radiation are calculated and presented in Table 1. As seen in Table 1, the calculated values in the finite range slightly differ from the corresponding values for the semi-infinite range. For example, the radiation density constant in the range from $v_1 = 60$ GHz to $v_2 = 600$ GHz is 95 % from the corresponding value for the semi-infinite interval. As for entropy density, we have 97 %. It means that observed part of spectrum from $v_1 = 60$ GHz to $v_2 = 600$ GHz covers a significant portion of the total spectrum.

One of the interesting questions is the following. What contribution to the radiative and thermodynamic properties of the CMB radiation gives the Wien part to the total spectrum? The obtained above expressions allow us to answer this question. Indeed, if we assume that the Rayleigh-Jeans region is described by the Planck formula, we have to calculate the radiative and thermodynamic functions of the CMB radiation in the range $0 \leq v \leq 60$ GHz. Then, the radiative and thermodynamic properties in Wien's region 600 GHz $\leq v \leq \infty$ can be calculated by subtracting the range $0 \leq v \leq 60$ GHz of the Rayleigh-Jeans part and the Planck part of the spectrum 60 GHz $\leq v_1 \leq$ 600 GHz from the total spectrum in the semi-infinite frequency range. Performing the calculation for the Wien part of the spectrum 600 GHz $\leq v \leq \infty$, we obtain $a = 4.8203 \times 10^{-18}\ \frac{\mathrm{J}}{\mathrm{m^3\,K^4}}$ and $s = 3.8661 \times 10^{-16}\ \frac{\mathrm{J}}{\mathrm{m^3\,K}}$. Then, the contribution of Wien's part of the spectrum to the radiation density constant a is 0.64 %. As for the contribution to entropy density, we have 1.79 %. As seen, the Wien part gives a small contribution to the radiative and thermodynamic properties of the CMB radiation.

4 Dipole spectrum

The first anisotropy discovered was the dipole anisotropy. The dipole spectrum of the CMB radiation is generally interpreted as a Doppler shift due to the Earth's motion relative to the CMB radiation field. The *Cobe* Firas instrument was used for the measurement of the dipole spectrum in the wavenumber range between 2 and 20 $\mathrm{cm^{-1}}$ (Fixsen et al. 1994, 1996). The observed dipole spectrum, a second term in Eq. (1), was fitted by the following expression

$$I^D(\tilde{v}, T) = \frac{I_0(\tilde{v})}{(\frac{v}{c})\cos(\theta)} = T_{\text{amp}} \frac{\partial B_{\tilde{v}}(T)}{\partial T}\bigg|_{T=T_0}. \tag{39}$$

Here $T_{\text{amp}} = 3.358$ mK is the mean dipole amplitude (Robitaille 2007; Hinshaw et al. 2007).

According to Eq. (3), the total energy density is defined as

$$\begin{aligned} I^D(\tilde{v}_1, \tilde{v}_2, T) &= \frac{4\pi}{c} T_{\text{amp}} \int_{\tilde{v}_1}^{\tilde{v}_2} \frac{\partial B_{\tilde{v}}(T)}{\partial T} d\tilde{v} \\ &= \frac{4\pi}{c} T_{\text{amp}} \frac{\partial}{\partial T} \int_{\tilde{v}_1}^{\tilde{v}_2} B_{\tilde{v}}(T) d\tilde{v}. \end{aligned} \tag{40}$$

To convert to the redshift dependences, the following relationships: $T(z) = T_0(1+z)$, $T_{\mathrm{amp}}(z) = T_{\mathrm{amp}}(1+z)$, and $\tilde{v}(z) = \tilde{v}(1+z)$ should be used.

Using a similar procedure to switch to the frequency domain, as in the case of the monopole spectrum, the expression for the total energy density of the CMB radiation for the dipole spectrum in the finite frequency range has the following form:

$$\begin{aligned} I_0^D(x_1, x_2, T) &= T_{\mathrm{amp}} \frac{\partial}{\partial T} \int_{\nu_1}^{\nu_2} I_\nu(T) \mathrm{d}\nu \\ &= T_{\mathrm{amp}} \frac{48\pi k_{\mathrm{B}}}{c^3 h^3} \frac{\partial}{\partial T} \big[T^4 \big(P_3(x_1) - P_3(x_2) \big) \big]. \end{aligned} \tag{41}$$

After differentiating Eq. (41) is presented as follows

$$\begin{aligned} I_0^D(x_1, x_2, T) &= T_{\mathrm{amp}} \frac{192\pi k_{\mathrm{B}}^4}{c^3 h^3} T^3 \bigg[\big(P_3(x_1) - P_3(x_2) \big) \\ &\quad + \frac{1}{24} \big(x_1^4 \mathrm{Li}_0 \big(e^{-x_1} \big) - x_2^4 \mathrm{Li}_0 \big(e^{-x_2} \big) \big) \bigg]. \end{aligned} \tag{42}$$

Since $P_3(0) = \mathrm{Li}_4(1) = \xi(4) = \frac{\pi^4}{90}$, $P_3(\infty) = 0$, and $x_1^4 \mathrm{Li}_0(e^{-x_1})|_{x_1=0} = x_2^4 \mathrm{Li}_0(e^{-x_2})|_{x_2=\infty} = 0$ in the semi-infinite range of frequencies, Eq. (42) takes the form

$$I_0^D(0, \infty, T) = \frac{32\pi^5 k_{\mathrm{B}}^4}{15 c^3 h^3} T_{\mathrm{amp}} T^3. \tag{43}$$

Here

$$a'' = 4a' \tag{44}$$

can be named as the radiation density constant for the dipole spectrum in the semi-infinite range of frequencies. The numerical value is $a'' = 3.0263 \times 10^{-15} \ \frac{\mathrm{J}}{\mathrm{m^3 K^4}}$. The Stefan-Boltzmann constant for the dipole spectrum is determined by the following relationship $\sigma'' = \frac{a'' c}{4}$. Then, σ'' has the following structure:

$$\sigma'' = 4\sigma'. \tag{45}$$

The numerical value is $\sigma'' = 2.2681 \times 10^{-7}\ \mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$.

The Stefan-Boltzmann law or the total radiation power per unit area $I_0'^{\mathrm{D}}(T)$ for the dipole spectrum is defined as

$$I_0'^{\mathrm{D}}(0, \infty, T) = \frac{8\pi^5 k_{\mathrm{B}}^4}{15 c^2 h^3} T_{\mathrm{amp}} T^3. \tag{46}$$

According to Eq. (46), the total radiation power emitted from the area A' of the universe for the dipole spectrum can be determined as follows

$$I_{\mathrm{total}}'^{\mathrm{D}}(T) = A' I_0'^{\mathrm{D}}(0, \infty, T). \tag{47}$$

Therefore, as in the case of the monopole spectrum, the area A' may be obtained from the experiment data by the measurement of the total radiation power $I_{\mathrm{Measured}}^{\mathrm{D}}(\nu_1, \nu_2, T)$. Then, in accordance with Eq. (47), we have

$$A' = \frac{I_{\mathrm{Measured}}^{\mathrm{D}}(T)}{I_0'^{\mathrm{D}}(0, \infty, T)}. \tag{48}$$

Here it is important to note that the radiation density constant and the Stefan-Boltzmann constant for the dipole spectrum, Eq. (44) and Eq. (45), in the semi-infinite range of frequencies differ from the corresponding constants for the monopole spectrum. This means that Doppler shift leads to a renormalization of the corresponding constants. This situation is similar for the radiative and thermodynamic functions of the CMB radiation for the dipole spectrum.

Let us Eq. (42) present in the form

$$I_0^D(x_1, x_2, T) = a''(x_1, x_2) T_{\mathrm{amp}} T^3, \tag{49}$$

where

$$\begin{aligned} a''(x_1, x_2) &= \frac{192\pi k_{\mathrm{B}}^4}{c^3 h^3} \bigg[\big(P_3(x_1) - P_3(x_2) \big) \\ &\quad + \frac{1}{24} \big(x_1^4 \mathrm{Li}_0 \big(e^{-x_1} \big) - x_2^4 \mathrm{Li}_0 \big(e^{-x_2} \big) \big) \bigg] \end{aligned} \tag{50}$$

can be called as the radiation density constant for the dipole spectrum in the finite range of frequencies. The Stefan-Boltzmann constant $\sigma''(\nu_1, \nu_2, T)$ is defined as

$$\begin{aligned} \sigma''(x_1, x_2) &= \frac{4\pi a''(x_1, x_2)}{c} \\ &= \frac{48\pi k_{\mathrm{B}}^4}{c^2 h^3} \bigg[\big(P_3(x_1) - P_3(x_2) \big) \\ &\quad + \frac{1}{24} \big(x_1^4 \mathrm{Li}_0 \big(e^{-x_1} \big) - x_2^4 \mathrm{Li}_0 \big(e^{-x_2} \big) \big) \bigg] \end{aligned} \tag{51}$$

Then, the Stefan-Boltzmann law or the total radiation power per unit area in the finite range of frequencies for the dipole spectrum has the structure

$$I_0'^{\mathrm{D}}(x_1, x_2, T) = \sigma''(x_1, x_2) T_{\mathrm{amp}} T^3. \tag{52}$$

According to Eq. (18), the number density of photons with photon energy from $h\nu_1$ to $h\nu_2$ is defined as follows

$$n = \frac{16\pi k_{\mathrm{B}}^3}{c^3 h^3} T_{\mathrm{amp}} \frac{\partial}{\partial T} \big[T^3 \big(P_2(x_1) - P_2(x_2) \big) \big]. \tag{53}$$

After differentiating Eq. (53) takes the form

$$\begin{aligned} n &= \frac{48\pi k_{\mathrm{B}}^3}{c^3 h^3} T_{\mathrm{amp}} T^2 \bigg\{ \big[P_2(x_1) - P_2(x_2) \big] + \frac{1}{6} \big(x_1^2 \mathrm{Li}_1 \big(e^{-x_1} \big) \\ &\quad - x_2^2 \mathrm{Li}_1 \big(e^{-x_{21}} \big) \big) \bigg\}. \end{aligned} \tag{54}$$

In the case of the semi-infinite range $0 \le \upsilon \le \infty$, Eq. (54) is simplified as

$$n = \frac{48\pi k_{\mathrm{B}}^3}{c^3 h^3} T_{\mathrm{amp}} T^2 \big[P_2(0) - P_2(\infty) \big]. \tag{55}$$

Since $P_2(0) = L_{i_3}(1) = \varsigma(3)$ and $P_2(\infty) = 0$, Eq. (55) is defined as

$$n = \frac{57.696\pi k_{\mathrm{B}}^3}{c^3 h^3} T_{\mathrm{amp}} T^2. \tag{56}$$

Using Eqs. (10), (12), (14), (16) and Eq. (42), the thermodynamic functions of the CMB radiation for the dipole spectrum in the finite $\upsilon_1 \le \upsilon \le \upsilon_2$ and the semi-infinite $0 \le \upsilon \le \infty$ ranges of frequencies have the following structure:

(1) Helmholtz free energy density f':
(a) $\upsilon_1 \le \upsilon \le \upsilon_2$

$$f' = -\frac{96\pi k_B^4}{c^3h^3} T_{\text{amp}} T^3 \Big[\big(P_3(x_1) - P_3(x_2)\big) - \frac{1}{24}\big(x_1^3 \text{Li}_1\big(e^{-x_1}\big) - x_2^3 \text{Li}_1\big(e^{-x_2}\big)\big) \Big] \tag{57}$$

(b) $0 \le \upsilon \le \infty$

$$f' = -\frac{16\pi^5 k_B^4}{15c^3h^3} T_{\text{amp}} T^3 \tag{58}$$

(2) Entropy density s:
(a) $\upsilon_1 \le \upsilon \le \upsilon_2$

$$s = \frac{288\pi k_B^4}{c^3h^3} T_{\text{amp}} T^2 \Big[\big(P_3(x_1) - P_3(x_2)\big) + \frac{1}{24}\big(x_1^4 \text{Li}_0\big(e^{-x_1}\big) - x_2^4 \text{Li}_0\big(e^{-x_2}\big)\big) \Big] \tag{59}$$

(b) $0 \le \upsilon \le \infty$

$$s = \frac{16\pi^5 k_B^4}{5c^3h^3} T_{\text{amp}} T^2 \tag{60}$$

(3) Heat capacity at constant volume per unit volume c_V:
(a) $\upsilon_1 \le \upsilon \le \upsilon_2$

$$c_V = \frac{576\pi k_B^4}{c^3h^3} T_{\text{amp}} T^2 \Big[\big(P_3(x_1) - P_3(x_2)\big) + \frac{1}{24}\big(x_1^4 \text{Li}_0\big(e^{-x_1}\big) - x_2^4 \text{Li}_0\big(e^{-x_2}\big)\big) + \frac{1}{72}\big(x_1^5 \text{Li}_{-1}\big(e^{-x_1}\big) - x_2^5 \text{Li}_{-1}\big(e^{-x_2}\big)\big) \Big] \tag{61}$$

(b) $0 \le \upsilon \le \infty$

$$c_V = \frac{32\pi^5 k_B^4}{5c^3h^3} T_{\text{amp}} T^2 \tag{62}$$

(4) Pressure P:
(a) $\upsilon_1 \le \upsilon \le \upsilon_2$

$$P = \frac{96\pi k_B^4}{c^3h^3} T_{\text{amp}} T^3 \Big[\big(P_3(x_1) - P_3(x_2)\big) - \frac{1}{24}\big(x_1^4 \text{Li}_0\big(e^{-x_1}\big) - x_2^4 \text{Li}_0\big(e^{-x_2}\big)\big) \Big] \tag{63}$$

(b) $0 \le \upsilon \le \infty$

$$P = \frac{16\pi^5 k_B^4}{15c^3h^3} T_{\text{amp}} T^3 \tag{64}$$

Now let us calculate the thermodynamic and radiative properties of the CMB radiation for the dipole spectrum using *Cobe* FIRAS instrument data (Fixsen et al. 1994, 1996). In Table 1, the values for the thermodynamic and radiative functions for the dipole spectrum in the finite and semi-infinite ranges of frequencies at $T = 2.72548$ K and $T_{\text{amp}} = 3.358$ mK are presented. As seen in Table 1, the radiation density constant a'' and the Stefan-Boltzmann constant σ'' for the dipole spectrum differ from those of the constants a' and σ' for the monopole spectrum. This means that Doppler shift leads to a renormalization of these constants, as well as the corresponding constants for the thermodynamic and radiative functions.

The obtained expressions for radiative and thermodynamic functions are important for obtaining new astrophysical parameters. Indeed, in accordance with the table of astrophysical constants and parameters (Groom 2013), three fundamental constants and parameters for the monopole spectrum, such as the present day CMB temperature, entropy density/Boltzmann constant, and number density of CMB photons are presented. The entropy density/Boltzmann constant parameter, for example, has the form

$$\frac{s}{k_B} = 2.8912 \left(\frac{T}{T_0}\right)^3 \text{cm}^{-3}, \tag{65}$$

where $T_0 = 2.72548$ K is present day CMB temperature.

As for the dipole spectrum, only the present day CMB dipole amplitude is the fundamental constant. However, as in the case of monopole spectrum, the expressions for the entropy density/Boltzmann constant parameter and number density of CMB photons for the dipole spectrum can be considered as additional parameters to the astrophysical parameters. In accordance with Eq. (56) and Eq. (60), their analytical expressions are presented in the form:

(1) Entropy density/Boltzmann constant

$$\frac{s}{k_B} = 8.2012 \left(\frac{T}{T_0}\right)^2 \left(\frac{T_{\text{amp}}}{T_{\text{amp0}}}\right) \text{cm}^{-3} \tag{66}$$

(2) Number density of CMB photons for the dipole spectrum

$$n = 1.5180 \left(\frac{T}{T_0}\right)^2 \left(\frac{T_{\text{amp}}}{T_{\text{amp0}}}\right) \text{cm}^{-3}. \tag{67}$$

Here $T_{\text{amp0}} = 3.358 \times 10^{-3}$ K is the present day CMB dipole amplitude.

Let us note that when $T_0 = 2970.77$ K and $T_{\text{amp}} = 3.66$ K, the Eq. (66) has a value that is shown in Table 2 for the entropy density at redshift $z = 1089$.

It is important to note that the exact expressions for the radiative and thermodynamic functions obtained above can

Table 2 Calculated values of the radiative and thermodynamic state functions for the monopole and dipole spectra in the 65.4–654 THz frequency interval at the redshift $z \approx 1089$ and at $T = 2970.77$ K and $T_{\mathrm{amp}} = 3.660$ K

Quantity	Monopole $0 \le \tilde{v} \le \infty$	Monopole $v_1 \le v \le v_2$	Dipole $v_1 \le v \le v_2$	Dipole $0 \le \tilde{v} \le \infty$
a', a'' $[\mathrm{J\,m^{-3}\,K^{-4}}]$	7.5657×10^{-16}	7.2169×10^{-16}	2.9261×10^{-15}	3.0263×10^{-15}
σ', σ'' $[\mathrm{W\,m^{-3}\,K^{-4}}]$	5.6704×10^{-8}	5.4089×10^{-8}	2.1930×10^{-7}	2.2681×10^{-7}
$I_0^{\mathrm{M}}(v_1, v_2, T)$	5.9149×10^{-2}	5.6212×10^{-2}	–	–
$I_0^{\mathrm{D}}(v_1, v_2, T)$ $[\mathrm{J\,m^{-3}}]$	–	–	2.8079×10^{-4}	2.9040×10^{-4}
$I_0'^{\mathrm{M}}(v_1, v_2, T)$	4.4331×10^{6}	4.2129×10^{6}	–	–
$I_0'^{\mathrm{D}}(v_1, v_2, T)$ $[\mathrm{W\,m^{-2}}]$	–	–	2.1044×10^{4}	2.1765×10^{4}
f $[\mathrm{J\,m^{-3}}]$	-1.9716×10^{-5}	-1.7309×10^{-5}	-1.3587×10^{-7}	-1.4520×10^{-7}
s $[\mathrm{J\,m^{-3}\,K^{-1}}]$	2.6522×10^{-5}	2.4748×10^{-5}	1.3796×10^{-7}	1.4663×10^{-7}
P $[\mathrm{J\,m^{-3}}]$	1.9716×10^{-5}	1.7309×10^{-5}	1.3587×10^{-7}	1.4520×10^{-7}
c_V $[\mathrm{J\,m^{-3}\,K^{-1}}]$	7.9567×10^{-5}	7.6718×10^{-5}	2.7471×10^{-7}	2.9191×10^{-7}
n $[\mathrm{m^{-3}}]$	5.3338×10^{17}	4.4563×10^{17}	1.7763×10^{15}	1.9658×10^{15}

be used to construct the thermodynamics of the CMB radiation for the quadrupole and higher-order contributions. Indeed, in the case of the quadrupole spectrum, for example, we need to include in Eq. (1) the quadrupole term

$$I^Q(\tilde{v}, T) = \frac{1}{2}(\Delta T)^2 \frac{\partial^2 B_{\tilde{v}}(T)}{\partial T^2}\bigg|_{T=T_0}.$$

Using Eq. (42), for

$$\frac{\partial^2 B_{\tilde{v}}(T)}{\partial T^2}\bigg|_{T=T_0},$$

we get

$$\frac{\partial^2 B_{\tilde{v}}(T)}{\partial T^2}\bigg|_{T=T_0} = \frac{\partial I_0^D(v_1, v_2, T)}{\partial T}\bigg|_{T-T_0}.$$

After differentiating, the thermodynamics of the CMB radiation for the quadrupole spectrum will be constructed and the radiative properties will be determined. New renormalized values of the radiation density a and the Stefan-Boltzmann σ will be obtained. As a result, the contributions of the quadrupole and higher-order contributions to the total spectrum of the CMB radiation will be obtained.

In conclusion, it should be noted that the cosmic microwave background radiation is the largest observed redshift, which corresponds to the greatest distance. According to the Cosmic Detectives (2013), this value of the redshift is around $z = 1089$ and it shows the state of the Universe about 13.8 billion years ago. Now we calculate the radiative and thermodynamic functions for the monopole and dipole spectra in the finite and semi-infinite ranges of frequencies at the redshift $z = 1089$. It is well known that in an expanding universe the temperature T and the frequency v depends on the redshift z (Sunyaev and Zel'dovich 1980), in accordance with the formulas $v = v_0(1 + z)$ and $T = T_0(1 + z)$. In this case, the obtained expressions for the radiative and thermodynamic functions of the CMB radiation for the monopole and dipole spectra have the same structure, in which the temperatures $T = 2.72548$ K and $T_{\mathrm{amp}} = 3.358$ mK should be replaced by $T = 2970.77$ K and $T_{\mathrm{amp}} = 3.660$ K, and the frequencies $v_1 = 60$ GHz and $v_2 = 600$ GHz by $v_1 = 65.4$ THz and $v_2 = 654$ THz. In Table 2, the radiative and the thermodynamic functions for the monopole and dipole spectra in the finite and semi-infinite frequency ranges at $z = 1089$ are presented.

5 Conclusions

In this paper, the exact expressions for the calculation of the temperature dependences of the radiative and thermodynamic functions of the cosmic microwave background (CMB) radiation, such as the total radiation power per unit area, total energy density, number density of photons, Helmholtz free energy density, entropy density, heat capacity at constant volume, and pressure in the finite range of frequencies are obtained.

Utilizing the experimental data for the monopole spectrum measured by the *COBE* FIRAS instrument in the finite range of frequencies 60 GHz $\le v \le$ 600 GHz at the temperature $T = 2.72548$ K, the values of the radiative and thermodynamic functions, as well as the radiation density constant a' and the Stefan-Boltzmann constant σ' are calculated. For the dipole spectrum, the constants a'' and σ'', and the radiative and thermodynamic properties of CMB radiation are obtained using the mean amplitude $T_{\mathrm{amp}} = 3.358$ mK. The results are presented in Table 1. It is shown that the Doppler shift leads to a renormalization of the radiation density constant, the Stefan-Boltzmann constant, and the corresponding constants for the thermodynamic functions.

Knowing the dependence of the temperature T on the redshift z allows us to study the thermodynamic and radiative state of the Universe many years ago. As an example, the thermodynamic and radiative functions, which belong to the state of the Universe at redshift $z = 1089$, corresponding to the state of Universe about 13.8 billion years ago, for the monopole and dipole spectra in the semi-infinite $0 \le v \le \infty$ and finite 64.5 THz $\le v \le$ 654 THz ranges of frequencies are calculated. The calculated values are presented in Table 2. These values differ significantly from corresponding values at present day $z = 0$.

The analytical expressions for the radiative and thermodynamic functions for the dipole spectrum allow us to construct new astrophysical parameters, such as the entropy density/Boltzmann constant, and number density of CMB photons.

It well-know that in cosmological models of heat transfer is typically used Stephan-Boltzmann law in the form σT^4. Nevertheless, we must also take into account the contribution of the dipole component. As a result, in accordance with Eq. (25) and Eq. (46), the total radiation power per unit area should have the following structure $\sigma' T^4 + \sigma'' T^3 T_{\mathrm{amp}}$. This fact should be taken into account in the construction of cosmological models.

The results of the present paper allow estimating the contribution of the radiative and thermodynamic properties of the thermal continuum radiation (photon gas) to the total radiation of other particles (protons, alpha and beta particles etc.).

In conclusion, it is important to note the following directions for the future research:

(a) It is desirable to investigate the contributions of the quadrupole and higher-order contributions to the total spectrum of the CMB radiation (Eq. (1)). In this case, we obtain new temperature dependencies of the radiative and thermodynamic properties of the CMB radiation.
(b) One important issue is the construction of the thermodynamic and radiative functions of galactic radiation using the far infrared spectrum. The latter has the form $v^n B_v(T_{dust})$. The index n changes from 1.65 to 2. Fixen et al. (1996) studied the FIRAS Galaxy spectrum and found that it was fitted using $n = 2$. As a result, the exact expressions for the thermodynamic and radiative properties of the far infrared galactic radiation can be obtained.
(c) Particular attention should be paid to the investigation of the radiative and thermal properties of the extragalactic far infrared background (FIRB) radiation (Fixen et al. 1998). In this case, the radiative and thermodynamic functions of the FIRB radiation in the frequency interval $\tilde{v} = 5$–$80\ \mathrm{cm}^{-1}$ at $T = 18.5 \pm 1.2$ K can be defined.
(d) In the future, it is of interest in the study of the temperature T and redshift z dependences of the Wien displacement law for the dipole spectrum of the CMB radiation. As a result, different law of the relationships between the position of the maximum of the spectral energy density and temperature will be established (Fisenko and Ivashov 1999).
(e) Developed in this work approach can be useful to consider the Hawking radiation. This radiation is predicted to be released by black holes that emit exactly blackbody radiation.

These and other topics will be points of discussion in subsequent publication.

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