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ORIGINAL ARTICLE

Anisotropic bulk viscous cosmological models in a modified gravity

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Abstract A spatially homogeneous Bianchi type-VI_0 space-time is considered in the frame work of $f(R,T)$ gravity proposed by Harko et al. (Phys. Rev. D 84:024020, 2011) when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings. Exact solutions of the field equations are obtained both in the absence and in the presence of cosmic strings under some specific plausible physical conditions. Some physical and kinematical properties of the model are, also, studied.

Keywords $f(R,T)$ gravity · Bulk viscous model cosmic strings · Bianchi type-VI_0 model

1 Introduction

Several astronomical observations indicate that the observable universe is undergoing a phase of accelerated expansion (Reiss et al. 1998; Perlmutter et al. 1998, 2003; de Bernardis et al. 2000) and this expansion of the universe is driven by an exotic energy with large negative pressure which is known as dark energy. In spite of the all observational evidence, dark energy is still a challenging problem in theoretical physics. The data indicates that the universe is spatially flat and is dominated by 76 % dark energy, 24 % by other matter (20 % dark matter and 4 % other cosmic matter). Thus dark energy has become important in modern cosmology and there has been a considerable interest in cosmological models with dark energy. A nice review of dark energy and dark energy models is presented by Mishra and Sahoo (2013). There are two major approaches to address this problem of cosmic acceleration either by introducing a dark energy component in the universe and study its dynamics or by interpreting as a failure of general relativity and consider modifying Einstein's theory of gravitation termed as 'modified gravity approach'. Among the various modifications of Einstein's theory $f(R)$ gravity (Caroll et al. 2004) is treated most suitable due to cosmologically important $f(R)$ models. In this theory, a more general action is chosen in which standard Einstein–Hilbert action is replaced by an arbitrary function of Ricci Scalar R, i.e. $f(R)$ so that this modified theory may explain the late time acceleration of the universe. It also describes the transition phase of the universe from deceleration to acceleration (Nojiri and Odintsov 2007). Several aspects of $f(R)$ gravity have been investigated by Capozziello et al. (2007, 2008), Multamaki and Vilja (2006, 2007), Sharif and Zubair (2010), Azadi et al. (2008), Nojiri and Odintsov (2003, 2004, 2007) and Chiba et al. (2007). A comprehensive review of $f(R)$ gravity has been given by Copeland et al. (2006).

Recently, a further generalization of $f(R)$ gravity theory has been proposed by Harko et al. (2011). In this, the gravitational Lagrangian is given by an arbitrary function of the Ricci Scalar R and of the trace T of the stress energy tensor T_{ij}. The field equations of $f(R,T)$ gravity are derived from Hilbert–Einstein type variational principle by taking the ac-

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tion

$$S=\frac{1}{16\pi}\int\left[f(R,T)+L_m\right]\sqrt{-g}d^4x \tag{1}$$

where L_m is the matter Lagrangian density. Now by the variation of the action S of the gravitational field with respect to the metric tensor components g^{ij}, the field equations of $f(R,T)$ gravity with the special choice of

$$f(R,T)=R+2f(T) \tag{2}$$

have been obtained as (for a detailed derivation of these field equations one can refer to Harko et al. 2011).

$$R_{ij}-\frac{1}{2}g_{ij}R=8\pi T_{ij}+2f'(T)T_{ij}+\left[2pf'(T)+f(T)\right]g_{ij} \tag{3}$$

where the overhead prime denotes derivative with respect to the argument and T_{ij} is given by

$$T_{ij}=(\rho+p)u_iu_j+pg_{ij} \tag{4}$$

ρ and p being energy density and isotropic pressure respectively.

It is well known that the present day universe is satisfactorily described by homogenous and isotropic models given by FRW line element. But, we cannot expect the universe in its early stages to have these properties. Hence the models with anisotropic background are suitable to describe the early stages of the universe. Therefore, Bianchi type models which are spatially homogenous and anisotropic have been widely studied in the frame work of general relativity and in alternative or modified theories of gravitation in the search for a realistic picture of the universe in its early stages. In particular, Barrow (1984) has pointed out that Bianchi type-VI_0 universe gives a better explanation of some of the cosmological problems like primordial helium abundance and they also isotropize in a special sense.

Strings are line like structures which arise due to spontaneous symmetry breaking during phase transition in the early universe. Massive strings serve as seeds for the large structures like galaxies and cluster of galaxies in the universe. Several important aspects of strings both in general relativity and in modified theories of gravitation have been investigated in Bianchi type space times by Stachel (1980), Letelier (1983), Vilenkin et al. (1987), Banerjee et al. (1990), Reddy (2003a, 2003b), Katore and Rane (2006), Sahoo (2008) and Tripathy et al. (2009)

Cosmological models with bulk viscosity have gained importance in recent years. Bulk viscosity contributes negative pressure term giving rise to an effective total negative pressure stimulating repulsive gravity which over comes attractive gravity of matter and gives an impetus for rapid expansion of the universe. It is well known that in several circumstances during cosmic evolution viscosity could arise and lead to an effective mechanism of entropy production (Misner 1968; Ellis and Sachs 1979; Hu 1983) Bianchi type bulk viscous cosmological models play a vital role in the discussion of early stages of evolution of the universe when galaxies were formed (Ellis and Sachs 1971). Also, Murphy (1973) and Heller and Klimek (1975) have shown that the big bang singularity can be avoided by the introduction of bulk viscosity. The importance of bulk viscosity in the cosmological models is also discussed clearly by Mohanty and Mishra (2001). Barrow (1986), Pavon et al. (1991), Martens (1995), Lima et al. (1993), Mohanty and Pradhan (1992) are some of the authors who have investigated bulk viscous cosmological models in general relativity. Also Johri (1994), Pimental (1994), Banerjee and Beesham (1996), Singh et al. (1997) have discussed bulk viscous models in Brans and Dicke (1961) theory of gravitation.

The study of spatially homogenous and anisotropic Bianchi models in the presence of bulk viscous fluid with one dimensional cosmic strings is attracting more and more attention in view of the fact that these models help in understanding the realistic picture of a universe immediately after the big bang. Naidu et al. (2012) studied LRS Bianchi type-II bulk viscous cosmic string model in Saez and Ballester (1986) scalar–tensor theory while Reddy et al. (2013a, 2013b) have investigated the same universe in $f(R,T)$ theory of gravity and in scale covariant theory of gravitation proposed by Canuto et al. (1977). Also, Reddy et al. (2013c) discussed Kaluza-Klein universe with bulk viscous cosmic strings in Saez-Ballester theory while the same has been studied by Naidu et al. (2013a) in Brans-Dicke theory and by Reddy et al. (2013d) in $f(R,T)$ gravity. Subsequently Kiran and Reddy (2013) established the non-existence of Bianchi type-III bulk viscous string cosmological model in $f(R,T)$ gravity. Very Recently, Naidu et al. (2013b) presented Bianchi type-V bulk viscous string model in $f(R,T)$ gravity while Reddy et al. (2013e) have obtained the same in Saez-Ballester theory. Also, Vidya Sagar et al. (2013) have studied Bianchi type-III bulk viscous String model in Saez-Ballester theory.

Inspired by the above discussion and investigations, we discuss the dynamics of anisotropic Bianchi type-VI_0 model in the presence of bulk viscous fluid with one dimensional cosmic strings. In Sect. 2, we present the anisotropic Bianchi type-VI_0 model and formulate the dynamical field equations, in $f(R,T)$ gravity. Section 3 deals with some exact solutions of the field equations and the models. In Sect. 4 we discuss the physical properties of the models and we summarize the results.

2 Metric and the field equations

Spatially homogenous and anisotropic Bianchi type-VI$_0$ metric is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2x}B^2(t)dy^2 - e^{-2x}C^2(t)dz^2 \quad (5)$$

where the scale factors A, B and C are functions of cosmic time t only

We assume the universe is filled with bulk viscous fluid with one dimensional cosmic strings. The combined energy momentum tensor for this matter source can be taken in the form

$$T_{ij} = (\rho + \overline{p})u_i u_j + \overline{p} g_{ij} - \lambda x_i x_j \quad (6)$$

and

$$\overline{p} = p - 3\zeta H \quad (7)$$

where ρ is the energy density of the fluid, $\zeta(t)$ is the coefficient of bulk viscosity, $3\zeta H$ is usually known as bulk viscous pressure, H is the Hubble's parameter and λ is the string tension density. Also $u^i = \delta^i_4$ is a four velocity which satisfies

$$g_{ij}u^i u_j = -x^i x_j = -1, \qquad u^i x_i = 0 \quad (8)$$

Here we consider $\rho, \overline{p}$ and λ as functions of cosmic time t only.

By adopting co-moving coordinates the field equations (3), for the metric (5) with the help of Eqs. (6)–(8) for the particular choice of the function (Harko et al. 2011) given by

$$f(T) = \mu(t) \quad (9)$$

can be written explicitly as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = \overline{p}(8\pi + 7\mu) - \lambda(8\pi + 3\mu) - \mu\rho \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \overline{p}(8\pi + 7\mu) - \mu\rho - \mu\lambda \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \overline{p}(8\pi + 7\mu) - \mu\rho - \mu\lambda \quad (12)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = -\rho(8\pi + 3\mu) + 5\mu\overline{p} - \mu\lambda \quad (13)$$

$$\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \quad (14)$$

where an overhead dot denotes differentiation with respect to time t.

Equation (14) yields

$$B = kC \quad (15)$$

where k is a constant of integration which can be chosen as unity without any loss of generality so that we have

$$B = C \quad (16)$$

Now, using Eq. (16), the field equations (10)–(13) reduce to the following independent equations:

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = \overline{p}(8\pi + 7\mu) - \lambda(8\pi + 3\mu) - \mu\rho \quad (17)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \overline{p}(8\pi + 7\mu) - \mu(\rho + \lambda) \quad (18)$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -\rho(8\pi + 3\mu) + 5\mu\overline{p} - \mu\lambda \quad (19)$$

Now we define some parameters for the Bianchi type-VI$_0$ model which are important in cosmological observations. The average scale factor and spatial volume are defined as

$$a = (AB^2)^{1/3}, \qquad V = a^3 = AB^2 \quad (20)$$

The anisotropic parameters of the expansion is expressed in terms of mean and directional Hubble parameter as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad (21)$$

where

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \quad (22)$$

is the mean Hubble parameter and H_i $(i = 1, 2, 3)$ represent the directional Hubble parameters in the directions of x, y and z axes respectively. For $\Delta = 0$, the space-time is isotropic. The physical parameters expansion scalar θ, shear scalar σ^2 are defined as follows:

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} = 3H \quad (23)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]^2 \quad (24)$$

The model approaches to isotropy continuously if $V \to \infty$, $\Delta \to 0$ and $\rho > 0$ (for comoving fluid to be realistic) as $t \to \infty$ (Collins and Hawking 1973; Akarsu and Kilinc 2010; Sharif and Zubair 2010).

3 Solutions of the field equations and the models

The field equations (17)–(19) are a system of three independent non-linear differential equations in five unknowns

A, B, p, ρ and λ. Hence we obtain the following physically important cosmological models.

Case(i) $\lambda = 0$, **bulk viscous model**

In this particular case the field equations (17)–(19) reduce to

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{2}{A^2} = 0 \tag{25}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -\rho(8\pi + 3\mu) + 5\mu\overline{p} \tag{26}$$

Now Eqs. (25) and (26) are two independent equations in four unknowns A, B, p and ρ Hence to find a determinate solution we use the following physically plausible conditions:

(i) The Shear Scalar σ^2 is proportional to scalar expansion θ so that we can take (Collins et al. 1980)

$$A = B^m \tag{27}$$

where $m \neq 0$ is a constant.

(ii) For a bratropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\overline{p} = p - 3\zeta H = \varepsilon\rho \tag{28}$$

where

$$\varepsilon = \varepsilon_0 - \beta \quad (0 \leq \varepsilon_0 \leq 1), \quad p = \varepsilon_0\rho \tag{29}$$

and ε_0 and β are constants

Now, using (27) in (25) we get the metric coefficients as

$$A = m(K_1 t + K_2), \qquad B = C = \left[m(K_1 t + K_2)\right]^{\frac{1}{m}} \tag{30}$$

where K_1 and K_2 are constants of integration and

$$K_1^2 = \frac{1}{m-1} \tag{31}$$

Now choosing $K_2 = 0$, the metric (5) with the help of (30) and (31) can, now, be written as

$$ds^2 = -dt^2 + \left(\frac{m^2}{m-1}\right)t^2 dx^2 + \left(\frac{m^2}{m-1}t^2\right)^{1/m}\left[e^{2x}dy^2 + e^{-2x}dz^2\right] \tag{32}$$

Equation (32) represents Bianchi type-VI$_0$ bulk viscous cosmological model in $f(R,T)$ gravity with the following physical and kinematical parameters which are important in the discussion of cosmological models.

Spatial volume in the model is

$$V = \frac{(mt)^{\frac{2+m}{m}}}{(m-1)^{\frac{m-2}{2m}}} \tag{33}$$

Hubble parameter is

$$H = \left(\frac{m+2}{m}\right)\frac{1}{t} \tag{34}$$

Scalar expansion is

$$\theta = 3H = \frac{3(m+2)}{m}\frac{1}{t} \tag{35}$$

The shear scalar is

$$\sigma^2 = \frac{(m-1)^2}{3m^2t^2} \tag{36}$$

The average anisotropic parameter is

$$\Delta = \frac{4}{3}\left(\frac{2m+1}{m+2}\right)^2 \tag{37}$$

The deceleration parameter in the model is

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -\frac{2}{2+m} \tag{38}$$

The energy density in the model is

$$\rho = \frac{\varepsilon_0}{m^2t^2}\left[\frac{m+2}{8\pi + \mu(3-5\varepsilon)}\right] \tag{39}$$

The isotropic pressure in the model is

$$p = \frac{\varepsilon_0}{m^2t^2}\left[\frac{m+2}{8\pi + \mu(3-5\varepsilon)}\right] \tag{40}$$

Bulk viscosity in the model is

$$\zeta = \left[\frac{\varepsilon_0 - \varepsilon}{3mt[8\pi + \mu(3-5\varepsilon)]}\right] \tag{41}$$

From the above results, we observe that the spatial volume of the universe (32) increases with the growth of cosmic time which shows that we have an expanding model. At the initial epoch of the universe i.e. at $t = 0$, the Hubble parameter, the scalar expansion, the shear scalar, the pressure, energy density and the bulk viscosity assume infinitely large values where as they all vanish as $t \to \infty$. Also, it can be seen that the deceleration parameter $q < 0$. If $q < 0$, the model accelerates and when $q > 0$, it decelerates in the standard way. Here the model accelerates which is in accordance with the present day scenario of accelerating universe. Also, since the bulk viscosity decreases with time we get, ultimately, inflationary model. It is worthwhile to mention, here, that the universe model does not approach to isotropy as the anisotropic parameter Δ is constant.

Case(ii) Bulk viscous string model ($\lambda \neq 0$)

In this particular case, the field equations (17)–(19) reduce to

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{2}{A^2} = -(8\pi + 2\mu)\lambda \tag{42}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -\rho(8\pi + 3\mu) + 5\mu\overline{\mu} - \mu\lambda \tag{43}$$

which are a system of two independent equations in five unknowns A, B, p, ρ and λ. Hence to get a determinate solution of the equations we use, in addition to the conditions (i) and (ii) of case (i), the special law of variation for Hubble's parameter proposed by Berman (1983) which yields constant deceleration parameter models of the universe defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = constant \tag{44}$$

which yields, on integration,

$$a = (ct + d)^{\frac{1}{1+q}} \tag{45}$$

where $c \neq 0$ and d are constants of integration. This equation implies that the condition for expansion of the universe is $1 + q > 0$. Using Eqs. (27) and (45) in Eq. (20), we obtain expression for the metric coefficients as

$$A = (ct + d)^{\frac{3m}{(m+2)(1+q)}}, \qquad B = C = (ct + d)^{\frac{3}{(1+q)(m+2)}} \tag{46}$$

where $c \neq 0$ and d are constants of integration. After a suitable choice of coordinates and constants (i.e. choosing $c = 1$, $d = o$) the metric (5) can, now, be written as

$$ds^2 = -dt^2 + t^{\frac{6m}{(1+q)(m+2)}} dx^2 + t^{\frac{6}{(1+q)(m+2)}} \left[e^{2x} dy^2 + e^{-2x} dz^2\right] \tag{47}$$

which represents a bulk viscous string cosmological model in $f(R, T)$ gravity with the following physical and kinematical parameters.

String tension density in the model is

$$(8\pi + 2\mu)\lambda = \frac{1}{t^2}\left[\frac{9(m-1)(m+2)}{(1+q)^2(m+2)^2} + \frac{3(1-m)}{(1+q)(m+2)} - 2t^{-\frac{6m}{(1+q)(m+2)}}\right] \tag{48}$$

Energy density is

$$\left[8\pi + \mu(3 - 5\varepsilon)\right]\rho = t^{-\frac{6m}{(1+q)(m+2)} - \frac{9(2m+1)}{(1+q)^2(m+2)^2 t^2}} \tag{49}$$

The isotropic pressure is

$$\left[8\pi + \mu(3 - 5\varepsilon)\right]p = \varepsilon_0\left[t^{-\frac{6m}{(1+q)(m+2)}} - \frac{9(2m+1)}{(1+q)^2(m+2)^2 t^2}\right] \tag{50}$$

The coefficient of bulk viscosity is

$$3\left[8\pi + \mu(3 - 5\varepsilon)\right]\zeta = (\varepsilon_0 - \varepsilon)(1+q)t\left[t^{-\frac{6m}{(1+q)(m+2)}} - \frac{9(2m+1)}{(1+q)^2(m+2)^2 t^2}\right] \tag{51}$$

Spatial volume is

$$V = t^{3/1+q} \tag{52}$$

The scalar expansion is

$$\theta = \frac{3}{(1+q)t} \tag{53}$$

The Hubble's parameter is

$$H = \frac{1}{(1+q)t} \tag{54}$$

The average anisotropy parameter is

$$\Delta = \frac{2(m-1)^2}{(m+2)^2} \tag{55}$$

Shear scalar is

$$\sigma^2 = \frac{3(m-1)^2}{[(1+q)(m+2)t]^2} \tag{56}$$

From the above results, we can see that at the initial epoch spatial volume vanishes and increases as $t \to \infty$. The expansion and shear scalars diverge at $t = 0$ and decrease with the increase in cosmic time. Thus the universe starts evolving with zero volume at the initial epoch with infinite rate of expansion and expansion rate slows down for later times of the universe. Also, the Hubble parameter, energy density and isotropic pressure decrease in later times of the universe and approach to zero as $t \to \infty$. The coefficient of bulk viscosity decreases with time and we, ultimately, get inflationary model. However, the average anisotropic parameter Δ remains constant throughout the evolution of the universe and it becomes zero when $m = 1$. Therefore when $m = 1$ and $\alpha = 0$, $\Delta = 0$ and $\sigma^2 = 0$, so that the universe becomes isotropic and shear free. It can be seen from Eq. (54) that Hubble parameter decreases with increase in time. However, the model will accelerate in finite time since the deceleration parameter $q < 0$ which is in accordance with the present day scenario of accelerated expansion of the universe. It is interesting to note that in both the models the bulk viscosity decreases with time so that we obtain, ultimately, inflationary models. Also the model does not admit initial singularity which supports the result of Murphy (1973) that the introduction of bulk viscosity avoids an initial singularity.

4 Summary and conclusions

In this paper, we have studied a spatially homogeneous and anisotropic Bianchi-VI_0 cosmological model in the presence of bulk viscous cosmic string source. We have presented cosmological models corresponding to bulk viscous fluid and bulk viscous cosmic strings. Exact solutions to field equations are obtained by using the conditions that expansion scalar is proportional to shear scalar and barotropic equation of state for the pressure and density. We have also used the special law of variation for Hubble`s parameter proposed by Berman (1983). It is observed that the bulk viscous model is expanding and accelerating in accordance with the present day observations.

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