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ORIGINAL ARTICLE

Bianchi type-III bulk viscous cosmic string model in a scalar-tensor theory of gravitation

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Abstract A spatially homogeneous and anisotropic Bianchi type-III space-time is considered in the presence of bulk viscous fluid containing one dimensional cosmic strings in the frame work of a scalar-tensor theory of gravity proposed by Saez and Ballester (in Phys. Lett. A 113:467, 1986). We have obtained a determinate solution of the field equations of this theory, using (i) a barotropic equation of state for the pressure and density and (ii) the bulk viscous pressure is proportional to the energy density. Some physical properties of the model are also discussed.

Keywords Scalar-tensor theory · Bulk viscosity · Cosmic string · Bianchi type-III model

1 Introduction

Scalar-tensor theories of gravitation are considered to be the most natural alternatives to Einstein's theory of gravitation. In these theories gravity is mediated by a long range scalar field in addition to the usual tensor fields present in Einstein's theory. Brans and Dicke (1961) formulated a scalar-tensor theory of gravitation which introduces an additional scalar field ϕ interacting equally with all forms of matter (with the exception of electromagnetism) besides the metric tensor g_{ij} and a dimensionless coupling constant ω. Subsequently, Saez and Ballester (1986) proposed a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. One particularly interesting result of this theory is appearance of antigravity regime, which suggests a possible connection to the missing matter problem in non-flat FRW cosmologies. In particular Amendariz-Picon et al. (2000, 2001) related this scenario to k-essence. The gravitational field equations of Saez-Ballester (SB) theory for the combined scalar and tensor fields are (with $8\pi G = C = 1$)

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij} \quad (1)$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \quad (2)$$

Also, we have

$$T^{ij}_{;j} = 0 \quad (3)$$

which is a consequence of the field equation (1) and (2). Here ω and n are constants. T_{ij} is the energy tensor of the matter, R_{ij} is the Ricci tensor, R is the Ricci scalar and comma and semicolon denote partial and covariant derivatives respectively.

Scalar-tensor theories have many interesting properties and have been extensively discussed in literature. The most fruitful area of their application is in cosmology where the scalar field is often applied as quintessence field to drive accelerating phase of the universe. The analysis of the large scale cosmic microwave background fluctuations confirm

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that our present day physical universe is isotropic, homogeneous, expanding and is well represented by FRW model. But other analyses reveal some inconsistency. Analysis of WMAP data sets shows us that the universe could have a preferred direction. Hence, the study of the anisotropic Bianchi models are important. The investigation of Bianchi type models in scalar-tensor Theories of gravity is also stimulating growing interest in anisotropic cosmological models of the universe. Singh and Agrawal (1991), Shri Ram and Singh (1995), Shri Ram et al. (1998), Reddy et al. (2001, 2006), Mohanty and Pattanaik (2001), Singh and Shri Ram (2003), Rao et al. (2007) and Suresh (2008) are some of the authors who have investigated Bianchi models and other aspects of SB theory of gravity.

During the past two decades, string cosmological models have received considerable attention of research workers because of their importance in structure formation in the early stages of evolution of the universe. During the phase transition in the early universe, spontaneous symmetry breaking gives rise to a random network of stable line like topological defects known as cosmic strings. It is well known that massive strings serve as seeds for the large structures like galaxies and cluster of galaxies in the universe. Letelier (1983), Stachel (1980), Vilenkin et al. (1987), Banerjee et al. (1990), Tripathy et al. (2009), Reddy (2003a, 2003b), Katore and Rane (2006) and Sahoo (2008) are some of the authors who have investigated several important aspects of string cosmological models either in the frame work of general relativity or in scalar-tensor theories of gravitation.

It is well known that viscosity plays an important role in cosmology (Singh and Devi 2011; Singh and Kale 2011; Setare and Sheyki 2010; Misner 1969). Also, bulk viscosity appears as the only dissipative phenomenon occurring in FRW models and has a significant role in getting accelerated expansion.of the universe popularly known as inflationary space. Bulk viscosity contributes negative pressure term giving rise to an effective total negative pressure stimulating repulsive gravity. The repulsive gravity overcomes attractive gravity of matter and gives an impetus for rapid expansion of the universe. Hence cosmological models with bulk viscosity have gained importance in recent years. Barrow (1986), Pavon et al. (1991), Maartens et al. (1995), Lima et al. (1993), and Mohanthy and Pradhan (1992) are some of the authors who have investigated bulk viscous cosmological models in general relativity. Johri and Sudharsan (1989), Pimental (1994), Banerjee and Beesham (1996), Singh et al. (1997), Rao et al. (2012), Naidu et al. (2012), Reddy et al. (2013) have studied bulk viscous string cosmological models in Brans-Dicke and Saez-Ballester theories of gravity respectively. Very recently, Kiran and Reddy (2013) have shown the non-existence of Bianchi type-III bulk viscous string cosmological model in $f(R,T)$ gravity proposed by Harko et al. (2011).

Motivated by the above discussion and investigations in scalar-tensor theories of gravity, we have taken up the study of Bianchi type-III bulk viscous cosmic string cosmological model in Saez-Ballester theory of gravity. This paper is organized as follows. Explicit field equations in SB theory of gravity are derived with the help of Bianchi type-III metric in the presence of bulk viscous fluid with one dimensional cosmic strings in Sect. 2. In Sect. 3 we have presented a determinate solution of the field equations using plausible physical conditions. Section 4 contains some conclusions.

2 Metric and field equations

We consider the spatially homogeneous and anisotropic Bianchi type-III metric given by

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{-ax} B^2(t)dy^2 + C^2(t)dz^2 \tag{4}$$

where A, B, C are cosmic scale factors and α is a positive constant.

The energy momentum tensor for a bulk viscous fluid containing one dimensional cosmic strings is given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} - \lambda x_i x_j \tag{5}$$

and

$$\overline{p} = p - 3\zeta H \tag{6}$$

ρ is the rest energy density of the system, $\zeta(t)$ is the coefficient of bulk viscosity, $3\zeta H$ is usually known as bulk viscous pressure, H is Hubble's parameter and λ is string tension density

Also, $u^i = \delta^i_4$ is a four-velocity vector which satisfies

$$g_{ij} u^i u_j = -x^i x_j = -1, \qquad u^i x_i = 0 \tag{7}$$

Here we also consider ρ, $\overline{p}$ and λ as functions of time t only.

By adopting co-moving coordinates, the field equations (1)–(3) for the metric (4) yield the following independent equations [Eq. (3) being consequence of Eqs. (1) and (2)].

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{\alpha^2}{A^2} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = -8\pi\rho \tag{8}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\omega}{2}\phi^n \dot{\phi}^2 = 8\pi\overline{p} \tag{9}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} + \frac{\omega}{2}\phi^n \dot{\phi}^2 = 8\pi\overline{p} \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\alpha^2}{A^2} + \frac{\omega}{2}\phi^n \dot{\phi}^2 = 8\pi(\overline{p} - \lambda) \tag{11}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{12}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{13}$$

where an over head dot indicates differentiation with respect to t.

The spatial volume is given by

$$V = ABC = a^3 \tag{14}$$

where $a(t)$ is the scale factor of the universe.

The expressions for scalar of expansion θ and shear scalar σ^2 are (kinematical parameters)

$$\theta = u^i_{;j} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{15}$$

$$\sigma^2 = \sigma^{ij}\sigma_{ij} = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{C}}{C}\frac{\dot{A}}{A}\right] \tag{16}$$

The Hubble parameter H and the mean anisotropy parameter are defined as

$$3H = \theta = 3\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{17}$$

$$3A_h = \sum_{I=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \quad \Delta H_i = H_i - H,\ i = 1, 2, 3 \tag{18}$$

3 Solutions and the model

Equation (12) gives on integration

$$A = kB \tag{19}$$

where k is a constant of integration which can be taken as unity without any loss of generality so that we have

$$A = B \tag{20}$$

Using Eq. (19), the field equations (8)–(11) and (13) reduce to

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\alpha^2}{B^2} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -8\pi\rho \tag{21}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = 8\pi\overline{p} \tag{22}$$

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{\alpha^2}{B^2} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = 8\pi(\overline{p} - \lambda) \tag{23}$$

$$\ddot{\phi} + \dot{\phi}\left(2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{24}$$

Now Eqs. (21)–(24) are a system of four independent equations in six unknowns $B, C, p, \rho, \lambda, \phi$. Also the above equations are highly non-linear differential equations. Hence to find a determinate solution we use the following physically plausible conditions.

(i) For a barotropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\overline{p} = p - 3\zeta H = \varepsilon\rho \tag{25}$$

where $\varepsilon = \varepsilon_0 - \beta$, $(0 \le \varepsilon_0 \le 1)$, $p = \varepsilon_0\rho$ and ε_0 and β are constants.

(ii) The shear scalar σ^2 is proportional to scalar expansion θ so that we can take (Collins et al. 1983)

$$B = C^m \tag{26}$$

where $m \neq 0$ is a constant

(iii) Variation of Hubble's parameter proposed by Berman (1983) that yields constant deceleration parameter models of the universe defined by

$$q = -a\frac{\ddot{a}}{\dot{a}^2} = \text{constant} \tag{27}$$

Which admits the solution.

$$a = (ct + d)^{\frac{1}{1+q}} \tag{28}$$

where $c \neq 0$ and d are constants of integration. This equation implies that the condition for accelerated expansion of the universe is $1 + q > 0$.

Now from Eqs. (14), (20), (26) and (28), we obtain

$$A = B = (ct + d)^{\frac{3m}{(2m+1)(1+q)}}, \qquad C = (ct + d)^{\frac{3}{(2m+1)(1+q)}} \tag{29}$$

Using Eq. (29) and by a suitable choice of coordinates and constants (i.e., taking $d = 0$ and $c = 1$), the metric (4) can be written as

$$ds^2 = -dt^2 + t^{\frac{6m}{(2m+1)(1+q)}}\left[dx^2 + e^{-\alpha x}dy^2\right] + t^{\frac{6}{(2m+1)(1+q)}}dz^2 \tag{30}$$

4 Some properties of the model

The model given by Eq. (30) represents Bianchi type-III anisotropic, bulk viscous and cosmic string model in the scalar-tensor theory of gravitation proposed by Saez and Ballester with the following physical and kinematical properties.

The physical and kinematical quantities of observational interest in cosmology are the energy density ρ, bulk viscous pressure $\overline{p}$, pressure p, Hubble's parameter H, coefficient of bulk viscosity ζ, scalar field ϕ, string tension density λ Spatial volume V, Scalar expansion θ, Shear scalar σ^2 and mean anisotropy parameter A_h, for the model Eq. (30) are given by

$$8\pi\rho = \frac{\omega}{2}\phi_0^2 t^{\frac{-6}{1+q}} + \alpha^2 t^{\frac{-6m}{(2m+1)(1+q)}} - \frac{9m(m+2)}{(2m+1)^2(1+q)^2t^2} \tag{31}$$

$$8\pi\overline{p} = \frac{\omega}{2}\phi_0^2 t^{\frac{-6}{1+q}} + \frac{1}{t^2}\left[\frac{9(m^2+m+1)}{(1+q)^2(2m+1)^2} - \frac{3(m+1)}{(2m+1)(1+q)}\right] \tag{32}$$

$$8\pi p = \varepsilon_0\left[\frac{\omega}{2}\phi_0^2 t^{\frac{-6}{1+q}} + \alpha^2 t^{\frac{-6m}{(2m+1)(1+q)}} - \frac{9m(m+2)}{(2m+1)^2(1+q)^2t^2}\right] \tag{33}$$

$$H = \frac{1}{(1+q)t} \tag{34}$$

$$8\pi\zeta = \frac{(\varepsilon_0-\varepsilon)}{3}(1+q)t\left[\frac{\omega}{2}\phi_0^2 t^{\frac{-6}{1+q}} + \alpha^2 t^{\frac{-6m}{(2m+1)(1+q)}} - \frac{9m(m+2)}{(2m+1)^2(1+q)^2t^2}\right] \tag{35}$$

$$\phi = \left[\frac{(n+2)}{2}\left(\phi_0 t^{\frac{(q-2)}{(1+q)}} + t_0\right)\right]^{\frac{2}{n+2}} \tag{36}$$

$$8\pi\lambda = \alpha^2 t^{\frac{-6m}{(2m+1)(1+q)}} - \frac{9(m-1)}{(2m+1)^2(1+q)^2t^2} \tag{37}$$

$$V = t^{\frac{3}{1+q}} \tag{38}$$

$$\theta = \frac{3}{(1+q)t} \tag{39}$$

$$\sigma^2 = \frac{3(m-1)^2}{[(2m+1)(1+q)t]^2} \tag{40}$$

$$A_h = \frac{4}{3} \tag{41}$$

It is well known that one can study the behavior of the above physical and kinematical parameters either by observing the analytical expressions or by graphical representation. Using the former procedure, from the above results, we can observe that the model (30) has no initial singularity i.e., at $t=0$. The spatial volume of the model increases as t increases, showing the expansion of the universe since $1+q>0$. We also observe that all the other physical and kinematical quantities except the scalar field ϕ diverge at the initial epoch, i.e., at $t=0$ and they all approach zero as t becomes infinitely large. It can also be seen that, since $A_h \neq 0$, the model remains anisotropic throughout the evolution of the universe. However, the model becomes isotropic for $m=1$ and $\alpha=0$. and the universe will be in a state of accelerated expansion. In this particular case, the string tension density vanishes and strings, in the isotropic accelerated expanding universe, do not survive. Also, since this model remains anisotropic throughout, it will be useful to discuss the structure formation at the early stages of evolution of the universe in scalar-tensor cosmology.

5 Conclusions

In this paper, we have studied the Bianchi type-III string cosmological model with bulk viscosity in Saez and Ballester (1986) scalar-tensor theory of gravitation. Scalar field cosmology is very important in the study of early universe and particularly in the investigation of inflation. It is observed that the model obtained is non-singular and expanding which is in accordance with recent observations. It is also observed that all the physical and kinematical parameters of cosmology diverge at the initial epoch and they approach zero for large t. The bulk viscosity decreases with time so that we get, ultimately, inflationary model. This will help us to study the role of scalar fields and the influence of bulk viscosity on the evolution of homogeneous and anisotropic Bianchi type-III models.

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