

# Some Kaluza-Klein cosmological models in $f(R, T)$ gravity theory

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**Abstract** Algorithms are derived for constructing five dimensional Kaluza-Klein cosmological space-times in the presence of a perfect fluid source in the framework of  $f(R, T)$  gravity theory proposed by Harko et al. (Phys. Rev. D 84:024020, 2011). Starting from the solution of Reddy et al. (Int. J. Theor. Phys 51:3222-3227, 2012b) some classes of new solutions are generated which correspond to accelerating models of the Universe. The physical and kinematical behaviors of the models are studied.

**Keywords** Kaluza-Klein theory · Cosmological models ·  $f(R, T)$  gravity

## 1 Introduction

It is widely believed that a consistent unification of all fundamental forces in nature would be possible within the space-time with an extra dimension beyond those four observed so far. Higher dimensional theories of Kaluza-Klein (KK)-type have been considered to study some aspects of early Universe (Chodos and Detweiler 1980; Freund 1982; Sahdev 1984; Shafi and Wetterich 1984). In such KK theory it has been assumed that the extra dimension form a compact manifold of very small size undetectable at present day energies. Thus, in such higher dimensional theories one would expect that at the grand unification scale the word manifold has more than one dimension. The Kaluza-Klein theory is

attractive because it has an elegant presentation in terms of geometry. In certain sense, it looks just like ordinary gravity in free space, except that it is phrased in five dimensions instead of four. Kaluza et al. (1921) and Klein (1926a, 1926b) attempted to unify gravitation and electromagnetism. An interesting possibility known as the “cosmological reduction process” is based on the idea that at very early stage all dimensions in the universe are comparable. Later, the scale of the extra dimension becomes so small as to be unobservable by experiencing contraction. Such cosmological models were investigated by Forgacs and Horvath (1979), Guth (1981), Alvarez and Gavela (1983) observed that during the contraction process extra dimensions produce large amount of entropy, which provides an alternative resolution to the flatness and horizon problem, as compared to usual inflationary scenario. Gross and Perry (1983) have shown that the five-dimensional Kaluza-Klein theory of unified gravity and electromagnetism admits soliton solutions. Further, they explained the inequality of the gravitational and inertial masses due to the violation of Birkoff’s theorem in Kaluza-Klein theories, which is consistent with the principle of equivalence. Appelquist and Chodos (1983), Randjbar-Daemi et al. (1984) claimed through solution of the field equations that there is an expansion of four-dimensional space-time while fifth dimension contracts to the unobservable Planckian length scale or remains constant as needed for the real universe.

Recent observations of type Ia Supernovae (SNe Ia) at red shift  $z < 1$  provide startling and puzzling evidence that the expansion of the universe at the present time appears to be accelerating, behavior attributed to “Dark Energy” with negative pressure. These observations (Chatterjee 1992; Friedman and Waga 1998; Carlberg et al. 1996; Ozer and Thha 1987; Freese et al. 1987; Carvalho et al. 1992; Silveira and Waga 1988; Ratra and Peebles 1988), strongly favor a sig-

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nificant and positive value of  $\Lambda$ . A number of models for dark energy to explain the late-time cosmic acceleration without the cosmological constant has been proposed. For example, a canonical scalar field, so-called quintessence, a non-canonical scalar field such as phantom, tachyon scalar field motivated by string theories, and a fluid with a special equation of state (EoS) called as Chaplygin gas. Nojiri and Odintsov (2003a, 2003b) have presented a review a various modified gravities which have considered as gravitational alternative for dark energy. Nojiri and Odintsov (2004) proposed that dark energy may become over standard matter due to universe expansion. Carroll et al. (2004) explained the presence of late time cosmic acceleration of the universe in  $f(R)$  gravity and proposed that dark energy model for specific  $\frac{1}{R}$  modified gravity. Allemandi et al. (2005) discussed the dark energy dominance cosmic acceleration in first order Palatini formalism. There also exists a proposal of holographic dark energy. One of the most important quantity to describe the features of dark energy models is the equation of state parameter (EoS)  $\omega$ , which is the ratio of the pressure  $p$  to the energy density  $\rho$  of dark energy, defined as  $\omega = \frac{p}{\rho}$ . There are two ways to describe dark energy models. One is a fluid description and the other is to describe the action of a scalar field theory. In both description, we can write the gravitational field equations, so that we can describe various cosmologies, e.g., the  $\Lambda$  CDM model, in which  $\omega$  is a constant and exactly equal to  $-1$ , quintessence model, where  $\omega$  is a dynamical quantity and  $-1 < \omega < -\frac{1}{3}$ , and phantom model, where  $\omega$  also varies in time and  $\omega < -1$ . This means that one cosmology may be described equivalently by different model descriptions discussed by Bamba et al. (2012). In view of the late time acceleration of the universe and the existence of dark energy and dark matter, several modified theories of gravitation have been proposed as alternative to Einstein's theory. The most important among them are scalar-tensor theories of gravitation and Rosen's bi-metric theory of gravitation. Noteworthy amongst them is the  $f(R)$  gravity theory. Nojiri and Odintsov (2006a) developed the general scheme for modified  $f(R)$  gravity reconstruction from any realistic FRW cosmology. They have shown that the modified  $f(R)$  gravity indeed represents the realistic alternative to general relativity, being more consistent in dark epoch. Nojiri and Odintsov (2006b), developed a general program for unification of matter-dominated era with acceleration epoch for scalar-tensor theory or dark fluid. Nojiri and Odintsov (2007) have reviewed various modified gravities considered as gravitational alternative for dark energy. They have considered the version of  $f(R)$ ,  $f(G)$  or  $f(R, G)$  gravity, model with non-linear gravitational coupling or string inspired model with Gauss-Bonnet-dilaton coupling in the late universe. Nojiri and Odintsov (2011), have studied  $f(R)$  gravity in different context. Bertolami et al. (2007) proposed a generalization of  $f(R)$  theory of

gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar  $R$  with the matter Lagrangian density  $L_m$ . Shamir (2010), proposed a physically viable  $f(R)$  gravity model, which show the unification of early time inflation and the late time acceleration.

Harko et al. (2011) developed a generalized  $f(R, T)$  gravity where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar  $R$  and of the trace  $T$  of the energy-momentum tensor. It is to be noted that the dependence from  $T$  may be induced by exotic imperfect fluid or quantum effects. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion of test particles, which follow from the covariant divergence of the stress-energy tensor. They have derived some particular models corresponding to specific choices of the function  $f(R, T)$  and have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of the function  $f(R, T)$ . Subsequently Adhav (2012), Reddy et al. (2012a), Chaubey and Shukla (2013), Ram et al. (2013, to appear), presented Bianchi types cosmological models in the presence of perfect fluid in  $f(R, T)$  gravity theory. Recently, Reddy et al. (2012b), investigated a five dimensional Kaluza-Klein space-time in the presence of a perfect fluid source in  $f(R, T)$  theory of gravitation with negative constant deceleration parameter.

In this paper, we present some new classes of five dimensional Kaluza-Klein cosmological models in the presence of a perfect fluid source in  $f(R, T)$  gravity theory. The paper is organized as follows: In Sect. 2, we revisit the field equations presented by Reddy et al. (2012b). We then derive algorithms for generating new solutions of the field equations in Sect. 3. In Sect. 4, starting with solution of Reddy et al. (2012b), we obtain some solutions of the field equations which represent accelerating cosmological models. The physical and kinematical properties of the models are also discussed. Conclusions are given in Sect. 4.

## 2 Metric and field equations

We consider a five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\Psi^2 \quad (1)$$

where  $A(t)$  and  $B(t)$  are the scale factors. The fifth coordinate  $\Psi$  is taken to be space-like.

The field equations in  $f(R, T)$  theory of gravity for the function  $f(R, T) = R + 2f(T)$  when the matter source is perfect fluid are given by (Harko et al. 2011).

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \quad (2)$$

where

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad (3)$$

and the prime denotes differentiation with respect to the argument.

Choosing

$$f(T) = \lambda T \quad (4)$$

where  $\lambda$  is a constant, the field equations (2) for the metric (1) in comoving coordinates lead to the following equations

$$3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}\dot{B}}{AB} = (8\pi + 3\lambda)\rho - p\lambda, \quad (5)$$

$$\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -(8\pi + 3\lambda)p + \rho\lambda, \quad (6)$$

$$3\frac{\ddot{A}}{A} + 3\left(\frac{\dot{A}}{A}\right)^2 = -(8\pi + 3\lambda)p + \rho\lambda. \quad (7)$$

Here an overhead over dot denotes ordinary differentiation with respect to time  $t$ .

For the metric (1), the spatial volume  $V$  and the average scale factor  $a$  are given by

$$V = a^4 = A^3 B \quad (8)$$

where  $a$  is the scale factor.

The mean Hubble parameter  $H$  has the expression

$$H = \frac{\dot{a}}{a} = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \quad (9)$$

where  $H_x = H_y = H_z = \frac{\dot{A}}{A}$  and  $H_\psi = \frac{\dot{B}}{B}$  are directional Hubble parameters. The scalar expansion  $\theta$  and shear scalar  $\sigma$  are given by

$$\theta = \frac{3}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \quad (10)$$

$$\sigma^2 = \frac{3}{4}\theta^2. \quad (11)$$

In next sect. we follow (Hajj-Boutros 1986) to derive algorithms for generating new solutions of the field equations of KK-type perfect fluid cosmological models within the framework of  $f(R, T)$  gravity theory.

### 3 Generating technique

From Eqs. (6) and (7) we obtain

$$\frac{2\ddot{A}}{A} + 2\frac{\dot{A}^2}{A^2} - 2\frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} = 0. \quad (12)$$

To treat Eq. (12), we introduce new functions  $R$  and  $S$  given by

$$R = \frac{\dot{A}}{A}, \quad S = \frac{\dot{B}}{B}, \quad (13)$$

By use of (13), Eq. (12), becomes

$$2\dot{R}_0 + 4R_0^2 - 2R_0S - \dot{S} - S^2 - 2\frac{\dot{X}}{X} + 8R_0\frac{1}{X} + \frac{4}{X^2} - 2\frac{S}{X} = 0. \quad (14)$$

The nonlinear equation (14) can be treated as a Riccati equation in  $R$  or  $S$ .

If we treat Eq. (14) as a Riccati equation in  $R$ , it can be linearized by means of change of function

$$R = R_0 + \frac{1}{X}. \quad (15)$$

Using (15) in Eq. (14), we obtain

$$\dot{X} + (\dot{S} - 4R_0)X = 2 \quad (16)$$

where  $R_0$  is a particular solution of Eq. (14). Equation (16) is linear first-order differential equation which has the general solution given by

$$X = \frac{A_0^4}{B} \left( \int 2\frac{B}{A_0^4} dt + k_1 \right), \quad (17)$$

$k_1$  being an integration constant. From Eqs. (15) and (17), we obtain after integration

$$A = A_0 k_2 \exp \left[ \frac{dt}{\frac{A_0^4}{B} \left( \int 2\frac{B}{A_0^4} dt + k_1 \right)} \right] \quad (18)$$

where  $k_2$  being another constant. Hence, from metric function  $[A_0, B]$  we can generate new function  $[A, B]$  where  $(A)$  is given by Eq. (18) and  $B$  remains invariable.

If (14) is regarded as a Riccati equation in  $S$ , we can be linearized it by the change of function

$$S = S_0 + \frac{1}{Y}. \quad (19)$$

Introducing (19) into Eq. (14), we obtain

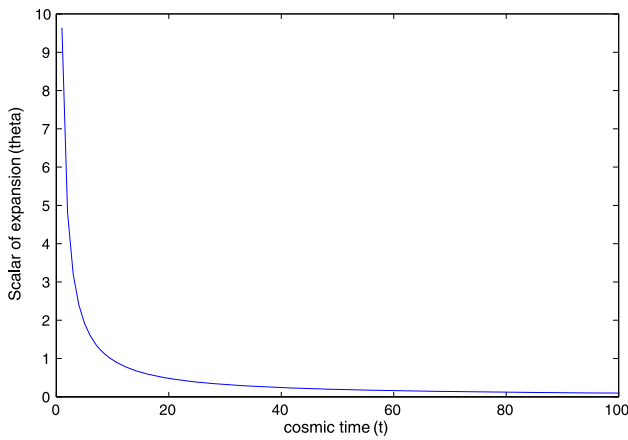
$$\dot{Y} - (2R + 2S_0)Y = 1 \quad (20)$$

where  $S_0$  is a particular solution of (14). Equation (20), on integration, gives

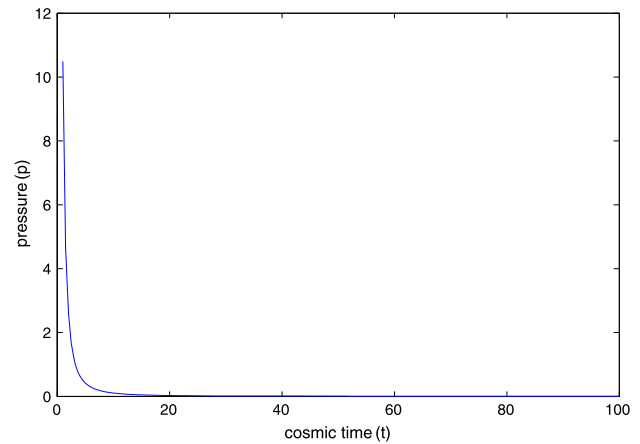
$$Y = A^2 B_0^2 \left( \int \frac{dt}{A^2 B_0^2} + k_3 \right) \quad (21)$$

where  $k_3$  being a constant. From Eqs. (19) and (21), we obtain

$$B = B_0 k_4 \exp \left[ \int \frac{dt}{A^2 B_0^2 \left( \int \frac{dt}{A^2 B_0^2} + k_3 \right)} \right] \quad (22)$$



**Fig. 1** The plot of scalar expansion  $\theta$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$



**Fig. 2** The plot of pressure  $p$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$

where  $k_4$  is another constant of integration. Thus, from the couple  $[A, B_0]$  we can generate  $[A, B]$  where  $B$  is given by Eq. (22) and  $A$  remains invariable.

Reddy et al. (2009) have presented the solutions of the field equations (5)–(7) in  $f(R, T)$  gravity theories has given by the metric

$$ds^2 = dt^2 - [kt]^{\frac{2}{k}}(dx^2 + dy^2 + dz^2) - [kt]^{\frac{2m}{k}}d\Psi^2 \quad (23)$$

where  $k = \frac{m^2+2m-3}{m-1}$ ,  $m \neq 1$ . Starting with this metric, we now generate new solutions of the field equations (5)–(7) by applying the generating techniques (18) and (22).

**Model I**

To apply our generation technique (22) to the metric (23), we take

$$A = (kt)^{\frac{1}{k}}, B_0 = (kt)^{\frac{m}{k}}$$

Then, performing the integration in (22), the new metric function  $B$  is obtained as

$$B = k_4(k)^{\frac{m}{k}} [t]^{\frac{k^2-2m^2-2m+km}{k(k-2m-2)}} \quad (24)$$

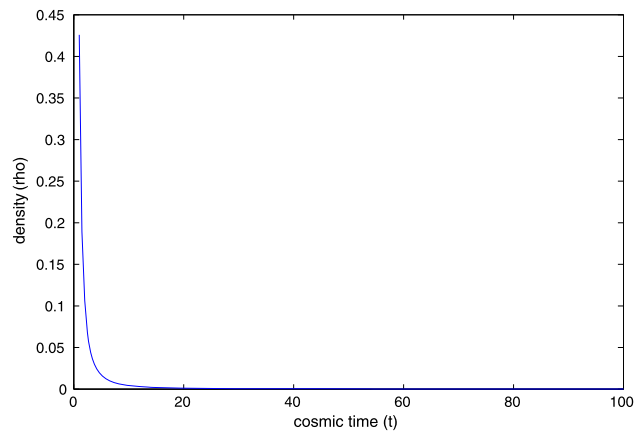
by putting  $k_3 = 0$ . Hence the metric of our new solution can be written in the form (Figs. 1, 2 and 3).

$$ds^2 = dt^2 - [kt]^{\frac{2}{k}}(dx^2 + dy^2 + dz^2) - \left\{ k^{\frac{m}{k}} t^{\frac{k^2-2m^2-2m+km}{k(k-2m-2)}} \right\}^2 d\Psi^2 \quad (25)$$

For the model (25) the physical and kinematical parameters are given by

$$H = \frac{1}{4mkt(k-2m-2)} \times [k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k], \quad (26)$$

$$\begin{aligned} \theta &= 3H \\ &= \frac{3}{4mkt(k-2m-2)} \\ &\times [k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k], \quad (27) \end{aligned}$$



**Fig. 3** The plot of density  $\rho$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$

$$\begin{aligned} \sigma^2 &= \frac{3}{4}\theta^2 \\ &= \frac{3}{4} \left( \frac{3}{4mkt(k-2m-2)} \right. \\ &\quad \left. \times [k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k] \right)^2. \quad (28) \end{aligned}$$

$$V = k^{\frac{m+3}{k}} t^{\frac{k^2-2m-2m+mk}{k(k-2m-2)} + \frac{3}{k}}. \quad (29)$$

The deceleration parameter  $q$  defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \quad (30)$$

which has the value given by

$$q = \frac{4k}{3(k-2m-2) + k^2 - 2m^2 - 2m + mk} - 1. \quad (31)$$

The pressure and energy density are obtained as

$$p = \left[ \frac{\lambda[k(k-2m-2)E_1 - (8\pi + 3\lambda)E_2m^2]}{(8\pi + 4\lambda)(8\pi + 2\lambda)m^2k^2(k-2m-2)t^2} \right], \tag{32}$$

$$\rho = \left[ \frac{3(1 + \frac{k^2-2m^2-2m+mk}{k(k-2m-2)})}{m^2t^2(8\pi + 3\lambda)} + \frac{\lambda^2(k(k-2m-2) - \frac{(8\pi+3\lambda)}{\lambda}E_2m^2)}{(8\pi + 2\lambda)(8\pi + 3\lambda)(8\pi + 4\lambda)m^2k^2t^2} \right] \tag{33}$$

where

$$E_1 = k(k-2m-2) + 3m(k^2 - 2m^2 - 2m + mk),$$

$$E_2 = [2 - m + (2m - 1)(k^2 - 2m^2 - 2m + mk) \\ \times (k(k-2m-2))] + (k^2 - 2m^2 - 2m + mk)$$

From the above results we observed that the model has initial singularity at  $t = 0$  if  $k > 2(m + 1)$  which leads to  $m < 1$ . We see that  $\theta, \sigma, H, p$  and  $\rho$  have infinite value at the initial singularity  $t = 0$ . These parameters are decreasing function of time which tend to zero for large time. Since  $\frac{\sigma^2}{\theta^2} \neq 0$ , the model is anisotropic throughout the evolution of the universe. We also find that the deceleration parameter  $q$  is negative, which corresponds to an accelerating model of the universe in five-dimensional Kaluza-Klein theory.

**Model II**

We apply formula (18) for the metric (23) to generate the new function  $A$  by taking

$$A_0 = (kt)^{\frac{1}{k}}, B = (k)^{\frac{m}{k}} [t]^{\frac{k^2-2m^2-2m+km}{k(k-2m-2)}}$$

Then, after integration, we obtain

$$A = k_2(k)^{\frac{1}{k}} [t]^{\frac{mk(k-2m-2)}{2m(k^2-2m^2-2m+mk)+(mk-4k)(k-2m-2)} + \frac{1}{k}} \tag{34}$$

assuming  $k_1 = 0$ . The metric of the solution can be written in the form

$$ds^2 = dt^2 - \left\{ (k)^{\frac{1}{k}} [t]^{\frac{mk(k-2m-2)}{2m(k^2-2m^2-2m+mk)+(mk-4k)(k-2m-2)} + \frac{1}{k}} \right\}^2 \\ \times (dx^2 + dy^2 + dz^2) \\ - \left( k^{\frac{m}{k}} t^{\frac{k^2-2m^2-2m+km}{k(k-2m-2)}} \right)^2 d\psi^2. \tag{35}$$

The metric (35) represents the five-dimensional Kaluza-Klein cosmological model in  $f(R, T)$  gravity theory with the following physical and kinematical parameters (Figs. 4, 5 and 6)

$$V = k^{\frac{m+3}{k}} \\ \times [t]^{\frac{3k(k-2m-2)}{2m(k^2-2m^2-2m+mk)+(m-4)k(k-2m-2)} + \frac{k^2-2m^2-2m+km}{k(k-2k-2)}}, \tag{36}$$

$$H = \left[ \frac{1}{8t(m(k^2 - 2m^2 - 2m + mk) + km(k - 2m - 2) - 4k^2(k - 2m - 2)^2)} \right] \\ \times \left[ ((m - 4)k(k - 2m - 2)(1 + (k - 2m - 2)((m - 4)k + 3mk^2))) \right. \\ \left. + [(k^2 - 2m^2 - 2m + km)(2km(k - 2m - 2) + 2m^2(k^2 - 2m^2 - 2m + km))] \right], \tag{37}$$

$$\theta = \left[ \frac{3}{8t(m(k^2 - 2m^2 - 2m + mk) + km(k - 2m - 2) - 4k^2(k - 2m - 2)^2)} \right] \\ \times \left[ ((m - 4)k(k - 2m - 2)(1 + (k - 2m - 2)((m - 4)k + 3mk^2))) \right. \\ \left. + [(k^2 - 2m^2 - 2m + km)(2km(k - 2m - 2) + 2m^2(k^2 - 2m^2 - 2m + km))] \right], \tag{38}$$

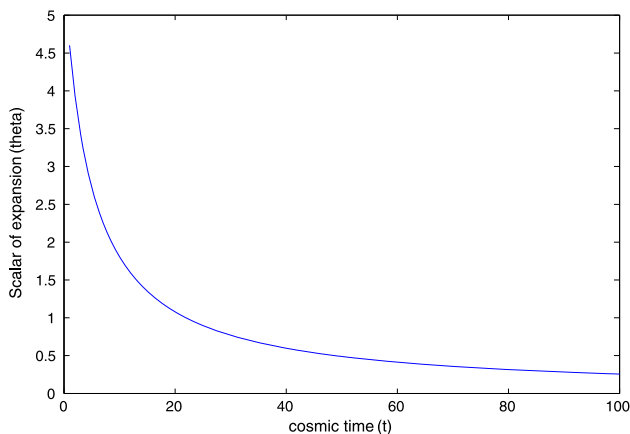
$$\sigma^2 = \frac{3}{4}\theta^2, \tag{39}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left[ 1 + (E_3^3 E_4)^{\frac{1}{2}} \right], \tag{40}$$

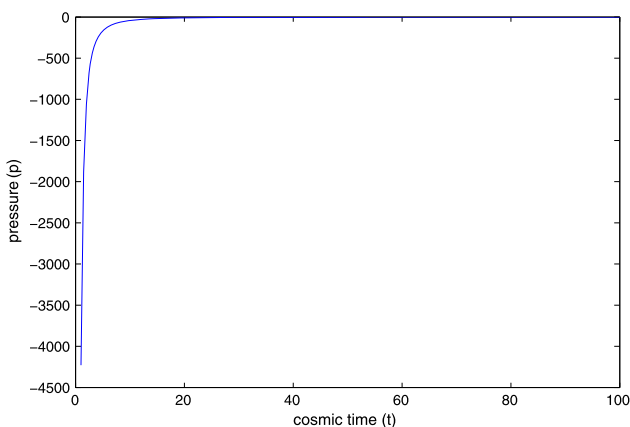
$$p = -\left[ \frac{(16\pi + 3\lambda)(E_3^2 + E_3 E_4) + (8\pi + 3\lambda)(E_4^2 - (E_3 + E_4))}{t^2(8\pi + 4\lambda)(8\pi + 2\lambda)} \right], \tag{41}$$

$$\rho = \left[ (3E_3 E_4) - \frac{\lambda}{(8\pi + 4\lambda)(8\pi + 2\lambda)}(16\pi + 3\lambda) \right. \\ \left. \times (E_3^2 + E_3 E_4) + (8\pi + 3\lambda)(E_4^2 - (E_3 + E_4)) \right] \\ \times \left[ \frac{1}{t^2(8\pi + 3\lambda)} \right] \tag{42}$$

where



**Fig. 4** The plot of  $\theta$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$



**Fig. 5** The plot of pressure  $p$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$

$$E_3 = \left[ \frac{k^2(k - 2m - 2)}{2(m(k^2 - 2m^2 - 2m + mk) + mk(k - 2m - 2) - 4k(k - 2m - 2))} + \frac{1}{k} \right],$$

$$E_4 = \frac{k^2 - 2m^2 - 2m + mk}{k(k - 2m - 2)}$$

From the Fig. 5, it is clear that the model (35) represents a five-dimensional Kaluza-Klein accelerating cosmological model. The other physical and kinematical behaviors of the model are same as Model I.

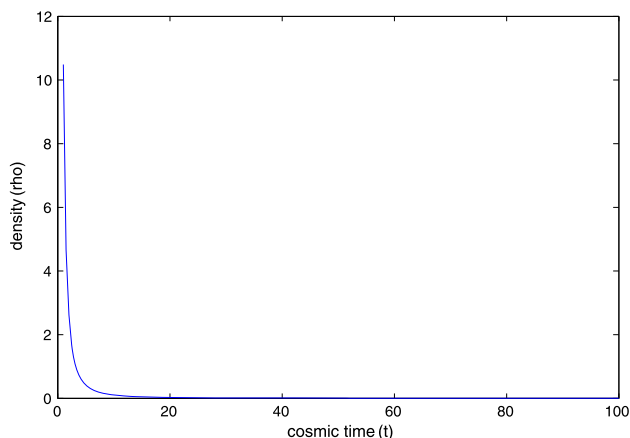
**Model III**

We now use formula (18) to generate new metric function A by taking

$$A_0 = (kt)^{\frac{1}{k}}, B = (kt)^{\frac{m}{k}}$$

Then performing integration in (18), we obtain

$$A = k_2 k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2 - 4k + m)} + \frac{1}{k}}, \tag{43}$$



**Fig. 6** The plot of density  $\rho$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$

assuming  $k_1 = 0$ . Then the metric (1) can be written in the form (44) where  $k_5$  is integration constant.

$$B = (kt)^{\frac{m}{k}}.$$

The metric can be written as

$$ds^2 = dt^2 - \left( k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2 - 4k + m)} + \frac{1}{k}} \right)^2 (dx^2 + dy^2 + dz^2) - (kt)^{\frac{2m}{k}} d\Psi^2 \tag{44}$$

The model (44) represents the five-dimensional Kaluza-Klein cosmological with perfect fluid in  $f(R, T)$  gravity theory. The physical and the kinematical parameters of the model (44) are given as follows (Figs. 7, 8 and 9).

$$V = (k)^{\frac{m+3}{k}} [t]^{\frac{3+m(2m^2-5k+2m)}{2k(m^2-4k+m)}}, \tag{45}$$

$$H = \frac{1}{4tk} (3kE_5 + m), \tag{46}$$

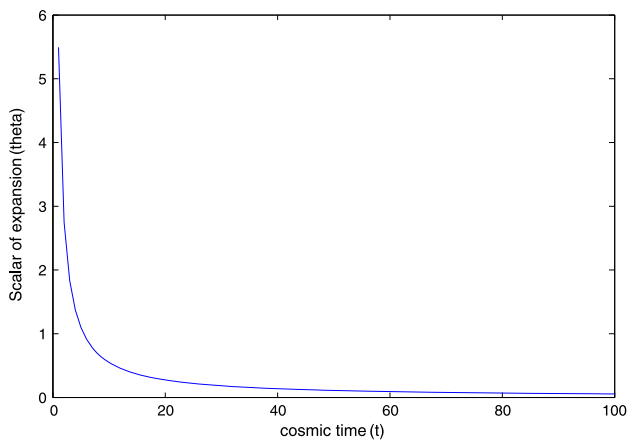
$$\theta = 3H = \frac{3}{4kt} (3kE_5 + m), \tag{47}$$

$$\sigma^2 = \frac{27}{64t^2k^2} (3kE_5 + m)^2, \tag{48}$$

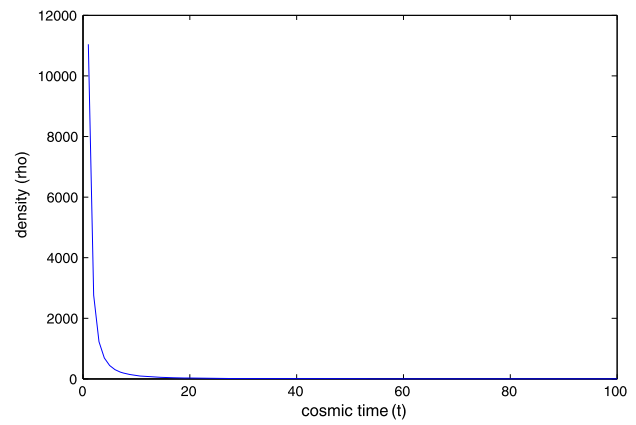
$$q = -1 + \left[ \frac{8(m^2 - 4k - m)k}{t(3k^2m + 2m(m^2 - 4k + m))} \right], \tag{49}$$

$$p = [E_5((8\pi + 3\lambda)k^2 + 3\lambda km^2) - (8\pi + 3\lambda)(E_5(2mk - E_5) + m(m - k)) - (8\pi + 2\lambda)k^2 E_5^2] [(8\pi + 4\lambda)(8\pi + 2\lambda)k^2 t^2]^{-1}, \tag{50}$$

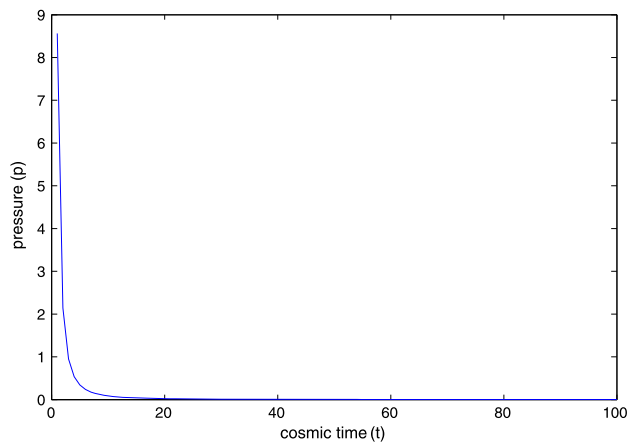
$$\rho = \left[ \frac{1}{(8\pi + 4\lambda)(8\pi + 3\lambda)(8\pi + 2\lambda)k^2 t^2} \right] \times [((3E_5^2(E_5 - 1)^2 k^2 + 3E_5 m)(8\pi + 4\lambda)(8\pi + 2\lambda)) + [\lambda E_5(8\pi + 3\lambda)k^2 + 3\lambda km^2 - \lambda(8\pi + 3\lambda)(m - k)m - (8\pi + 2\lambda)k^2 E_5]] \tag{51}$$



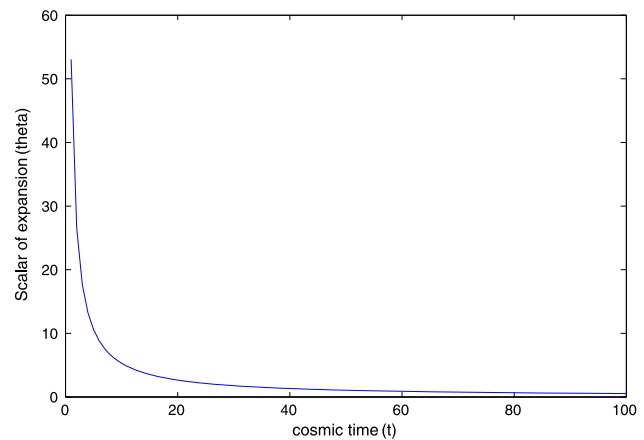
**Fig. 7** The plot of scalar expansion  $\theta$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$



**Fig. 9** The plot of density  $\rho$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$



**Fig. 8** The plot of pressure  $p$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$



**Fig. 10** The plot of Scalar of expansion  $\theta$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$

where

$$E_5 = \frac{k}{2(m^2 - 4k + m)} + \frac{1}{k}.$$

For the metric (44) the spatial volume is zero at  $t = 0$  if  $k < \frac{m(m+1)}{4}$ . The physical and kinematical properties same as perfect fluid Model I.

**Model IV**

We use the formula (22) for the metric (44) to generate the new function B by setting

$$A = k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2-4k+m)} + \frac{1}{k}}, \quad B = (kt)^{\frac{m}{k}}.$$

Then, from Eq. (23), the new function B is obtained as:

$$B = k_4 k^{\frac{m}{k}} [t]^{\frac{mk(m^2-4k+m)}{km^2+2(m^2-4k+m)+(3k+2m)(m^2-4k+m)+mk(m^2-4k+m)}} \quad (52)$$

taking  $k_3 = 0$ . The metric of the solution can be written in the form

$$ds^2 = dt^2 - \left[ k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2-4k+m)} + \frac{1}{k}} \right]^2 (dx^2 + dy^2 + dz^2) - \left[ (k)^{\frac{m}{k}} [t]^{\frac{km(m^2-4k+m)}{km^2+2(m^2-4k+m)(1+m+3km)}} \right]^2 d\psi^2 \quad (53)$$

The metric (53) represents five-dimensional Kaluza-Klein cosmological model in  $f(R, T)$  gravity with the following physical and kinematic parameters in the model (Figs. 10, 11 and 12).

$$V = k^{\frac{m+3}{k}} \left[ t^{\frac{3km}{2(m^2-4k+m)} + \frac{3}{k} + \frac{m}{k} + \frac{(km(m^2-4k+m))}{km^2+(2+m(4+k))(m^2-4k+m)}} \right], \quad (54)$$

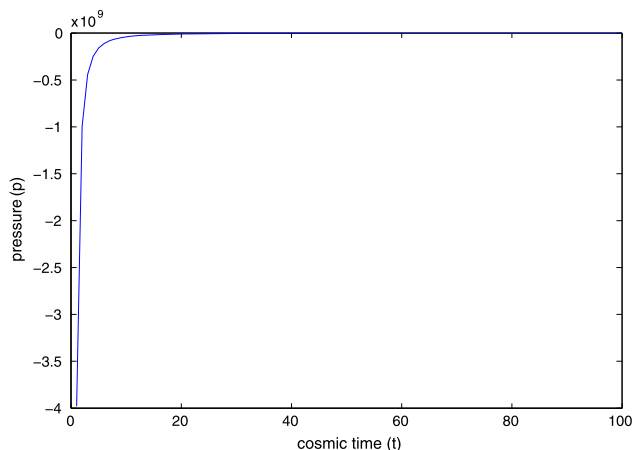
$$H = \frac{(3(km^2 + 1) + 2m(m^2 - 4k + m)E_6)}{8tm(m^2 - 4k + m)}, \quad (55)$$

$$\theta = 3H = 3 \left[ \frac{(3(km^2 + 1) + 2m(m^2 - 4k + m)E_6)}{8tm(m^2 - 4k + m)} \right], \quad (56)$$

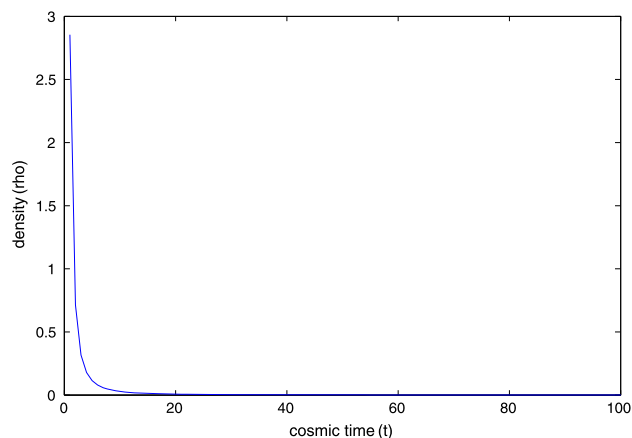
$$\sigma^2 = \frac{27}{256} \left[ \frac{(3(km^2 + 1) + 2m(m^2 - 4k + m)E_6)}{8tm(m^2 - 4k + m)} \right]^2, \quad (57)$$

$$q = - \left( 1 - \frac{1}{E_7} \right), \quad (58)$$





**Fig. 11** The plot of pressure  $p$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$



**Fig. 12** The plot of density  $\rho$  versus cosmic time  $t$ ,  $m = 0.5$ ;  $\lambda = 1$ ;  $k = 1$

$$p = - \left[ \frac{1}{4m^2(m^2 - 4k + m)^2 t^2 (8\pi + 4\lambda)(8\pi + 2\lambda)} \right] \times [((km^2 + 1)^2(16\pi + 3\lambda) + (32\pi + 6\lambda)E_6 + [(km^2 + 1)m(m^2 - 4k + m)]) + [(8\pi + 3\lambda)m(m^2 - 4k + m)(km^2 + 1)] + [(8\pi + 3\lambda)E_6(E_6 - 1)m(m^2 - 4k + m)]], \quad (59)$$

$$\rho = \left[ \frac{3(km^2 + 1)^2 E_6(km^2 + 1)}{4m^2(m^2 - 4k + m)^2 t^2 (8\pi + 3\lambda)} \right] - \left[ \frac{\lambda}{(8\pi + 4\lambda)(8\pi + 3\lambda)(8\pi + 2\lambda)t^2 4m^2(m^2 - 4k + m)^2} \right] \times [((km^2 + 1)^2(16\pi + 3\lambda) + (32\pi + 6\lambda) \times E_6(km^2 + 1)m(m^2 - 4k + m)] + [(8\pi + 3\lambda)E_6(E_6 - 1)m(m^2 - 4m + m)] \quad (60)$$

where

$$E_6 = \frac{m}{k} + \frac{m^2 - 4k + m}{(km^2 + (2 + 4km + m)(m^2 - 4k + m))},$$

$$E_7 = \left[ \frac{3km}{2(m^2 - 4k + m)} + \frac{m}{k} + \frac{km(m^2 - 4k + m)}{km^2 + (m^2 - 4k + m)(2 + 4m + km)} \right].$$

We observe that the spatial volume of the model (53) is zero at  $t = 0$  and increases with time if  $k < \frac{m(m+1)}{4}$ . Therefore the model has a point type singularity at  $t = 0$  where  $\theta$ ,  $\sigma^2$ ,  $H$ ,  $p$  and  $\rho$  diverge. These parameters are decreasing function of time and ultimately tend to zero for large time. The negative pressure, as shown by Fig. 11, indicates that the model is accelerating.

### 4 Conclusions

The higher dimensional cosmological models are of considerable importance because of the underlying idea that cosmos in early stages of evolution might have had a higher dimensional era. The extra space reduces to a volume with the passage of time, which is beyond the ability of experimental observation at the moment (Reddy et al. 2009). It is well known that Kaluza-Klein models represent the cosmos in its early stages of evolution. In the present work, we have derived algorithms for generating new solutions of the field equations with a perfect fluid for a five dimension Kaluza-Klein space-time within the framework of  $f(R, T)$  gravity theory proposed by Harko et al. (2011). Starting from the model obtained by Reddy et al. (2012b), we have presented new cosmological models of the present-day accelerating universe. These models are expanding, shearing and accelerating which have point-type singularity at  $t = 0$ . All the physical and kinematical parameters, being infinite at the initial singularity, are decreasing functions of time which ultimately tend to zero for large time. The anisotropy in the cosmological models are maintained throughout the passage of time.

Nojiri and Odintsov (2003b) studied a modify theory of gravity where the universe inturns inflates, decelerates and then accelerates in early times, radiation dominated era. Our models are similar to the case of five dimensional  $f(R)$  gravity except the decelerating behavior in the presence of a perfect fluid source discussed by Huang et al. (2010) and Agmohammadi et al. (2009). It has been observed that in the five dimensional  $f(R)$  and  $f(R, T)$  gravity theories the expansion and contraction of the extra dimension could result in the present accelerated expansion of other spatial dimensions. This is possible by cosmic re-collapse of the universe in the finite future. It follows that the the present accelerating models of the universe are consistent with the recent



observation of type-Ia supernovae (Permuter et al. 1999; Riess et al. 1998, and the references therein).

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