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ORIGINAL ARTICLE

Non-existence of Bianchi type-III bulk viscous string cosmological model in $f(R,T)$ gravity

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Abstract A spatially homogeneous and anisotropic Bianchi type-III space-time is considered in the presence of bulk viscous fluid containing one dimensional cosmic strings in the frame work of $f(R,T)$ gravity proposed by Harko et al. (Phys. Rev. D 84:024020, 2011). To get a determinate solution of the field equations of this theory, we have used (i) a barotropic equation of state for the pressure and density and (ii) the bulk viscous pressure is proportional to the energy density. It is interesting to observe that, in this case, Bianchi type-III bulk viscous string cosmological model does not exist and degenerates into vacuum model of general relativity.

Keywords Cosmic strings · Bulk viscosity · Bianchi type-III model · $f(R,T)$ gravity

1 Introduction

The current cosmological observations suggest to us that the present observable universe is undergoing an accelerating expansion (Riess et al. 1998; Perlmutter et al. 1999 and Bennet et al. 2003). The source driving this acceleration is known as 'dark energy' whose origin is still a mystery in modern cosmology. This is because of the fact that we do not have, so far, a consistent theory of quantum gravity. The accelerated expansion of the universe is driven by the negative pressure of the dark energy. 'The cosmological constant' is the most simple and natural candidate for explaining cosmic acceleration, but it faces serious problems and a large discrepancy between theory and observations (Copeland et al. 2006; Nojiri and Odintsov 2007). Hence, in recent years, there has been lot of interest in constructing dark energy models by modifying the geometrical part of Einstein–Hilbert action of general relativity. This approach is called as the modified gravity which can successfully explain the rotation curves of galaxies and the motion of galaxy clusters in the universe. Among the various modifications of general relativity $f(R)$ theory of gravity (Caroll et al. 2004) and $f(R,T)$ gravity (Harko et al. 2011) are attracting more and more attention during the last decade. It has been suggested that cosmic acceleration can be achieved by replacing Einstein–Hilbert action of general relativity with a general function $f(R)$ where R is a Ricci scalar. Chiba et al. (2007), Nojiri and Odintsov (2007), Multamaki and Vilja (2006, 2007) and Shamir (2010) are some of the authors who have investigated several aspects of $f(R)$ gravity models which show the unification of early time inflation and late time acceleration.

The analysis of the large scale cosmic microwave background fluctuations confirm that our present day physical universe is isotropic, homogeneous, expanding and is well represented by FRW model. But other analyses reveal some inconsistency.Analysis of WMAP data sets show us that the universe could have a preferred direction. Hence, the study the anisotropies Bianchi models are important. Harko et al. (2011) have investigated several aspects of $f(R,T)$ gravity including FRW dust model. The investigation of Bianchi type models in $f(R,T)$ gravity is also stimulating growing interest in anisotropic cosmological models of the universe. Adhav (2012) has obtained Bianchi type-I cosmological model in $f(R,T)$ gravity. Reddy et al. (2012a) have discussed Bianchi type-III cosmological model in $f(R,T)$ gravity while Reddy et al. (2013a), Reddy and Shanthiku-

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mar (2013a) studied Bianchi type-III dark energy model and some anisotropic cosmological models, respectively, in $f(R,T)$ gravity. Very recently, Reddy and Shanthikumar (2013b) have obtained Bianchi type-II cosmological model in this modified theory of gravity.

During the past two decades, string cosmological models have received considerable attention of research workers because of their importance in structure formation in the early stages of evolution of the universe. During the phase transition in the early universe, spontaneous symmetry breaking gives rise to a random network of stable line like topological defects known as cosmic strings. It is well known that massive strings serve as seeds for the large structures like galaxies and cluster of galaxies in the universe. Letelier (1983), Stachel (1980), Vilenkin (1987), Banerjee et al. (1990), Tripathy et al. (2009), Reddy (2003a, 2003b), Katore and Rane (2006) and Sahoo (2008) are some of the authors who have investigated several important aspects of string cosmological models either in the frame work of general relativity or in modified theories of gravitation.

It is well known that viscosity plays an important role in cosmology (Singh and Devi 2011; Singh and Kale 2011; Setare and Sheykhi 2010 and Misner 1969). Also, bulk viscosity appears as the only dissipative phenomenon occurring in FRW models and has a significant role in getting accelerated expansion of the universe popularly known as inflationary space. Bulk viscosity contributes negative pressure term giving rise to an effective total negative pressure stimulating repulsive gravity. The repulsive gravity overcomes attractive gravity of matter and gives an impetus for rapid expansion of the universe hence cosmological models with bulk viscosity have gained importance in recent years. Barrow (1986), Pavon et al. (1991), Maartens et al. (1995), Lima et al. (1993), and Mohanthy and Pradhan (1992) are some of the authors who have investigated bulk viscous cosmological models in general relativity. Johri and Sudharsan (1989), Pimental (1994), Banerjee and Beesham (1996), Singh et al. (1997), Rao et al. (2012a, 2012b), Naidu et al. (2012) and Reddy et al. (2012b) have studied bulk viscous string cosmological models in Brans and Dicke (1961) and other modified theories of gravity. Very recently, Reddy et al. (2013b) and Reddy et al. (2013c) have discussed five dimensional Kaluza-Klein universe with bulk viscous cosmic strings in Saez and Ballester (1986) scalar-tensor theory and $f(R,T)$ theory of gravity respectively.

Motivated by the above discussion and investigations in modified theories of gravity, we have taken up the study of Bianchi type-III bulk viscous cosmic string cosmological model in $f(R,T)$ gravity and, interestingly, it is observed this model does not survive in this theory. This paper is organized as follows: Sect. 2 presents a brief description of $f(R,T)$ gravity. Explicit field equations in $f(R,T)$ gravity are derived with the particular form of $f(T)$ used by Harko et al. (2011) with the help of Bianchi type-III metric in the presence of bulk viscous fluid with one dimensional cosmic strings in Sect. 3. From the field equations, it is shown, in Sect. 4, that bulk viscous cosmic string cosmological with barotopic equation of state does not exist. Section 5 contains some conclusions

2 $f(R,T)$ theory of gravity

In $f(R,T)$ gravity, the field equations are derived from a variational, Hilbert-Einstein type, principle.

The action for the modified $f(R,T)$ gravity is

$$S=\frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar, R, T is the trace of stress-energy tensor of the matter, T_{ij} and L_m is the matter Lagrangian density. We define the stress-energy tensor of matter as

$$T_{ij}=\frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{2}$$

and its trace by $T=g^{ij}T_{ij}$ respectively. By assuming that L_m of matter depends only on the metric tensor components g_{ij}, and not on its derivatives, we obtain

$$T_{ij}=g_{ij}L_m-2\frac{\partial L_m}{\partial g^{ij}} \tag{3}$$

Now by varying the action S of the gravitational field with respect to the metric tensor components g^{ij}, we obtain the field equations of $f(R,T)$ gravity as

$$\begin{aligned} &f(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}\Box-\nabla_i\nabla_j)f_R(R,T)\\ &\quad=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\theta_{ij} \end{aligned} \tag{4}$$

where

$$\theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \tag{5}$$

Here $f_R=\frac{\delta f(R,T)}{\delta R}$, $f_T=\frac{\delta f(R,T)}{\delta T}\Box=\nabla^i\nabla_i$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy-momentum tensor derived from the Lagrangian L_m. It may be noted that when $f(R,T)\equiv f(R)$ Eqs. (4) yield the field equations of $f(R)$ gravity.

The problem of the perfect fluids described by an energy density ρ, pressure p and four velocity u^i is complicated since there is no unique definition of the matter Lagrangian. However, here, we assume that the stress energy tensor of the matter is given by

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij} \tag{6}$$

and the matter Lagrangian can be taken as $L_m=-p$ and we have

$$u^i\nabla_j u_i=0,\quad u^iu_i=1 \tag{7}$$

Then with the use of Eqs. (5) we obtain for the variation of stress-energy of perfect fluid the expression

$$\theta_{ij} = -2T_{ij} - pg_{ij} \tag{8}$$

Generally, the field equations also depend through the tensor θ_{ij}, on the physical nature of the matter field. Hence in the case of $f(R,T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of $f(R,T)$. Assuming

$$f(R,T) = R + 2f(T) \tag{9}$$

as a first choice (for other choices see Harko et al. 2011) where $f(T)$ is an arbitrary function of the trace of stress-energy tensor of matter, we get the gravitational field equations of $f(R,T)$ gravity from Eq. (4) as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij} \tag{10}$$

where the prime denotes differentiation with respect to the argument.

If the matter source is a perfect fluid,

$$\theta_{ij} = -2T_{ij} - pg_{ij}$$

then the field equations become

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f'(T)T_{ij} + \left[2pf'(T) + f(T)\right]g_{ij} \tag{11}$$

3 Metric and field equations

We consider the spatially homogeneous and anisotropic Bianchi type-III metric given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2qx}B^2(t)dy^2 - C^2(t)dz^2 \tag{12}$$

where A, B, C are cosmic scale factors and q is a positive constant.

We consider the energy momentum tensor for a bulk viscous fluid containing one dimensional cosmic strings as

$$T_{ij} = (\rho + \overline{p})u_i u_j - \overline{p}g_{ij} - \lambda x_i x_j \tag{13}$$

and

$$\overline{p} = p - 3\zeta H \tag{14}$$

where ρ is the rest energy density of the system, $\zeta(t)$ is the coefficient of bulk viscosity, $3\zeta H$ is usually known as bulk viscous pressure, H is Hubble's parameter and λ *is string tension density*.

Also, $u^i = \delta^i_4$ is a four-velocity vector which satisfies

$$g_{ij}u^i u_j = 1, \qquad x^i x_j = 1 \quad \text{and} \quad u^i x_i = 0 \tag{15}$$

Here we also consider ρ, $\overline{p}$ and λ are functions of time t only.

Using co moving coordinates and a particular choice of the function given by (Harko et al. 2011)

$$f(T) = \mu T, \qquad \mu \text{ is a constant,} \tag{16}$$

the $f(R,T)$ gravity field equations, with the help of Eqs. (13)–(15) for the metric (12), can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = \overline{p}(8\pi + 3\mu) + \lambda(8\pi + 3\mu) - \rho\mu \tag{17}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} = \overline{p}(8\pi + 3\mu) - (\rho - \lambda)\mu \tag{18}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{q^2}{A^2} = \overline{p}(8\pi + 3\mu) - (\rho - \lambda)\mu \tag{19}$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{A_4}{A}\frac{C_4}{C} + \frac{B_4}{B}\frac{C_4}{C} - \frac{q^2}{A^2} = -\rho(8\pi + 3\mu) + \overline{p}\mu + \lambda\mu \tag{20}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0 \tag{21}$$

where a suffix 4 indicates differentiation with respect to t.

4 Non-existence of Bianchi type-III model

Integrating Eq. (21), we get

$$A = \alpha B \tag{22}$$

where α is a constant of integration which can be taken as unity without any loss of generality so that we have

$$A = B. \tag{23}$$

Using Eq. (23), the field equations (17)–(20) reduce to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = \overline{p}(8\pi + 3\mu) + \lambda(8\pi + 3\mu) - \rho\mu \tag{24}$$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = \overline{p}(8\pi + 3\mu) - (\rho - \lambda)\mu \tag{25}$$

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{q^2}{B^2} = \overline{p}(8\pi + 3\mu) - (\rho - \lambda)\mu \tag{26}$$

$$\frac{B_4^2}{B^2} + 2\frac{B_4}{B}\frac{C_4}{C} - \frac{q^2}{B^2} = -\rho(8\pi + 3\mu) + \overline{p}\mu + \lambda\mu \tag{27}$$

From Eqs. (24) and (25) we, immediately, obtain

$$(8\pi + 3\mu)\lambda = 0 \tag{28}$$

which implies

$$\lambda = 0 \tag{29}$$

i.e., tension density of the string vanishes.

Now, using Eq. (29) and the fact that for a barotropic fluid the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\overline{p} = p - 3\varsigma H = \varepsilon\rho \quad \text{where} \\ \varepsilon = \varepsilon_0 - \beta \quad (0 \le \varepsilon_0 \le 1), \; p = \varepsilon_0 \rho, \tag{30}$$

and β is an arbitrary constant, it is a simple matter to see, from Eqs. (24)–(27), that

$$\rho = 0, \quad \overline{p} = 0 \quad \text{and} \quad p = 0 \tag{31}$$

Thus in view of Eqs. (29) and (31) we observe that the field equations (17)–(20) reduce to Bianchi type-III vacuum field equations in general relativity and hence we conclude that, in this case, Bianchi type-III bulk viscous cosmic string model does not exist in $f(R,T)$ gravity.

5 Conclusions

$f(R,T)$ gravity is proposed by modifying general relativity to explain mysterious problem of late time acceleration of the universe in modern cosmology substantiated by Supernova 1a experiment. Since CMB data favors minor anisotropies in the universe and since anisotropic models are well represented by Bianchi models,lot of research is going on to find out which model is favored by astrophysical data. In this process we have made an attempt in $f(R,T)$ gravity and interestingly enough we have naturally arrived from the field equations that Bianchi type-III bulk viscous string cosmological model does not survive in this theory. Research in this direction continues to decide about the suitable model of our universe. It may be noted that Reddy et al. (2013c) have presented Bianchi type-II bulk viscous cosmic string cosmological model while Reddy et al. (2013d) have discussed five dimensional Kaluz-Klein bulk viscous cosmic string model in $f(R,T)$ gravity. It is quite interesting to note that Bulk viscous cosmic string model does not exist in Bianchi type-III space-time in this theory. It is known that current universe is isotropic and accelerating (for a recent review see Bamba et al. 2012). Thus, the absence of the specific anisotropic universe suggests that one can prefer isotropic models in spite of the fact that there is a small amount of anisotropy in the universe.

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