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ORIGINAL ARTICLE

LRS Bianchi type-II universe with cosmic strings and bulk viscosity in a modified theory of gravity

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Abstract A locally rotationally symmetric (LRS) Bianchi type-II space-time is considered in the frame work of a modified theory of gravitation proposed by Harko et al. (Phys. Rev. D 84:024020, 2011) when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings. A barotropic equation of state is assumed to get a determinate solution of the field equations. Also, the bulk viscous pressure is assumed to be proportional to the energy density. The physical behavior of the model is also discussed.

Keywords Bianchi type universe · Cosmic strings · Bulk viscosity · Modified gravity

1 Introduction

Modifications of Einstein's theory of gravitation are attracting more and more attention, in recent years, to explain late time acceleration and dark energy confirmed by the high red shift supernova experiments and cosmic microwave background radiation (Riess et al. 1998; Perlmutter et al. 1999; Bennet et al. 2003; Spergel et al. 2003, 2007). During last decade there has been several modifications of general relativity to provide natural gravitational alternative for dark energy. Among them $f(R)$ theory of gravity is treated most suitable due to cosmologically important $f(R)$ gravity models. It has been suggested that cosmic acceleration can be achieved by replacing the Einstein-Hilbert action of general relativity with a general function of Ricci scalar $f(R)$. Nojiri and Odintsov (2007), Multamaki and Vilja (2006, 2007) and Shamir (2010) have obtained physically viable $f(R)$ gravity models which show the unification of early time inflation and late time acceleration of the universe. A comprehensive review on modified $f(R)$ gravity is given by Copeland et al. (2006).

Recently, Harko et al. (2011) proposed another modification of Einstein's theory of gravitation which is known as $f(R,T)$ theory of gravity wherein the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor T_{ij}. They have derived the field equations of $f(R,T)$ gravity from Hilbert-Einstein type variational principle by taking the action

$$S = \frac{1}{16\pi}\int\left[f(R,T) + L_m\right]\sqrt{-g}d^4(x) \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, T is the trace of energy tensor of the matter T_{ij} and L_m is the matter Lagrangian density. By varying the action S of the gravitational field with respect to the metric tensor components g^{ij}, they have obtained the field equations of $f(R,T)$ gravity, with the special choice of $f(R,T)$ (Harko et al. 2011) given by

$$f(R,T) = R + 2f(T) \tag{2}$$

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as (for a detailed derivation of the field equations one can refer to Harko et al. 2011)

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(t)T_{ij} + \left[2pf'(T) + f(T)\right]g_{ij} \tag{3}$$

where the overhead prime indicates derivative with respect to the argument and T_{ij} is given by

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij} \tag{4}$$

ρ and p being energy density and isotropic pressure respectively.

Caroll et al. (2004), Nojiri and Odintsov (2003, 2004, 2007) and Chiba et al. (2007) are some of the authors who have investigated several aspects of $f(R)$ gravity. Very recently, Adhav (2012) has obtained Bianchi type-I cosmological model in $f(R,T)$ gravity. Reddy et al. (2012a, 2012b) have discussed Bianchi type-III and Kaluza-Klein cosmological models in $f(R,T)$ gravity while Reddy and Shantikumar (2013) studied some anisotropic cosmological models and Bianchi type-III dark energy model, respectively, in $f(R,T)$ gravity.

String cosmological models have received considerable attention, during the past two decades, of research workers because of their importance in structure formation in the early stages of evolution of the universe. During the phase transition in the early universe, spontaneous symmetry breaking gives rise to a random network of stable line like topological defects known as cosmic strings. It is well known that massive strings serve as seeds for the large structures like galaxies and cluster of galaxies in the universe. Letelier (1983), Stachel (1980), Vilenkin et al. (1987), Banerjee et al. (1990), Tripathy et al. (2009), Reddy (2003a, 2003b), Katore and Rane (2006) and Sahoo (2008) are some of the authors who have investigated several important aspects of string cosmological models either in the frame work of general relativity or in modified theories of gravitation.

It is well known that viscosity plays an important role in cosmology (Singh and Devi 2011; Singh and Kale 2011; Setare and Sheykhi 2010 and Misner 1969). Also, bulk viscosity appears as the only dissipative phenomenon occurring in FRW models and has a significant role in getting accelerated expansion of the universe popularly known as inflationary space. Bulk viscosity contributes negative pressure term giving rise to an effective total negative pressure stimulating repulsive gravity. The repulsive gravity overcomes attractive gravity of matter and gives an impetus for rapid expansion of the universe hence cosmological models with bulk viscosity have gained importance in recent years. Barrow (1986), Pavon et al. (1991), Martens (1995), Lima et al. (1993), and Mohanty and Pradhan (1992) are some of the authors who have investigated bulk viscous cosmological models in general relativity. Johri and Sudharsan (1989), Pimental (1994), Banerjee and Beesham (1996), Singh et al. (1997), Rao et al. (2011, 2012), Naidu et al. (2012) and Reddy et al. (2012c) have studied bulk viscous and bulk viscous string cosmological models in Brans-Dicke and other modified theories of gravity.

Motivated by the above investigations in modified theories of gravity, we have investigated, in this paper, LRS Bianchi type-II cosmological model, in the modified $f(R,T)$ gravity proposed by Harko et al. (2011), in the presence of cosmic strings and bulk viscosity. In Sect. 2, explicit field equations $f(R,T)$ gravity for LRS Bianchi type-II metric are obtained in the presence of bulk viscous fluid containing one dimensional strings. In Sect. 3 the cosmological model is presented by solving the field equations. Physical and kinematical properties of the model are discussed in Sect. 4. The last section contains some conclusions

2 Metric and field equations

We consider a homogeneous LRS Bianchi type-II space-time given by

$$\begin{aligned} ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - 2B^2xdydz \\ - \left(B^2x + A^2\right)dz^2 \end{aligned} \tag{5}$$

where A and B are functions of cosmic time t.

We consider the energy momentum tensor for a bulk viscous fluid containing one dimensional cosmic strings as

$$T_{ij} = (\rho + \overline{p})u_i u_j + \overline{p}g_{ij} - \lambda x_i x_j \tag{6}$$

and

$$\overline{p} = p - 3\zeta H \tag{7}$$

where ρ is the rest energy density of the system, $\zeta(t)$ is the coefficient of bulk viscosity, $3\zeta H$ is usually known as bulk viscous pressure, H is Hubble's parameter and λ *is string tension density*.

Also, $u^i = \delta^i_4$ is a four-velocity vector which satisfies

$$g_{ij}u^i u_j = 1, \qquad x^i x_j = 1 \quad \text{and} \quad u^i x_i = 0 \tag{8}$$

Here we also consider $\rho, \overline{p}$ and λ are functions of time t only.

Using co moving coordinates and a particular choice of the function given by (Harko et al. 2011)

$$f(T) = \mu T, \quad \mu \text{ is a constant} \tag{9}$$

the field equations (3), for the metric (5) with the help of (6)–(9) can be written, explicitly, as

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{1}{4}\frac{B^2}{A^4}=(\overline{p}+\lambda)(8\pi+3\mu)-\rho\mu \tag{10}$$

$$2\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2-\frac{3}{4}\frac{B^2}{A^4}=\overline{p}(8\pi+3\mu)-\rho\mu \tag{11}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\left(\frac{\dot{A}}{A}\right)^2-\frac{1}{4}\frac{B^2}{A^4}=-\rho(8\pi+3\mu)+\overline{p}\mu \tag{12}$$

where an overhead dot indicates differentiation with respect to time t.

The spatial volume is given by

$$V=A^2B=a^3 \tag{13}$$

where $a(t)$ is the scale factor of the universe.

The scalar expansion θ and the shear scalar σ^2 in the model are defined by

$$\theta=2\frac{\dot{A}}{A}+\frac{\dot{B}}{B} \tag{14}$$

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)^2 \tag{15}$$

3 Solutions and the model

The field equations (10)–(12) are a system of three independent equations in five unknowns $A, B, \overline{p}, \rho$ and λ. Also the field equations are highly non-linear in nature and therefore we use the following plausible physical conditions to find determinate solution.

(i) The shear scalar σ^2 is proportional to scalar expansion θ so that we can take (Collins et al. 1980)

$$A=B^m \tag{16}$$

where $m\neq 1$ is a constant and it takes care of the anisotropic nature of the model.

(ii) A more general relationship between the proper rest energy density ρ and string tension density λ is taken to be

$$\rho=r\lambda \tag{17}$$

where r is an arbitrary constant which can take both positive and negative values. The negative values of r lead to the absence of strings in the universe and the positive values show the presence of one dimensional strings in the cosmic fluid. The energy density of the particles attached to the strings is

$$\rho_p=\rho-\lambda=(r-1)\lambda \tag{18}$$

(iii) For a barotropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\overline{p}=p-3\varsigma H=\varepsilon\rho \tag{19}$$

where

$$\varepsilon=\varepsilon_0-\alpha\quad(0\le\varepsilon_0\le 1),\qquad p=\varepsilon_0\rho,\qquad H=\frac{\dot{a}}{a} \tag{20}$$

and α *is an arbitrary constant.*

Now using the above conditions the field equations (10)–(12) reduce to the equation

$$\frac{\ddot{B}}{B}+d\left(\frac{\dot{B}}{B}\right)^2=0 \tag{21}$$

where we have put the constants

$$d=\frac{6m^2+m(8k-5)}{m(5-2k)+3},\quad k=\frac{a}{b},$$
$$a=\left(4\in+\frac{1}{\gamma}\right)(8\pi+3\mu)-4\mu,$$
$$b=8\left[\mu+(3+\varepsilon)\pi\right] \tag{22}$$

Integrating Eq. (21) and using (16) we obtain the metric coefficients as

$$A=(d+1)^{m/d+1}(c_1t+c_2)^{m/d+1},$$
$$B=(d+1)^{1/d+1}(c_1t+c_2)^{1/d+1} \tag{23}$$

where $c_1\neq 0$ and c_2 are constants of integration. Now by a suitable choice of coordinates and integration constants (i.e. $c_1=1$ and $c_2=0$) the metric (5), with the help of Eq. (23), can be written as

$$ds^2=dt^2-\left[(d+1)t\right]^{\frac{2m}{d+1}}dx^2-\left[(d+1)t\right]^{\frac{2}{d+1}}dy^2$$
$$-2\left[(d+1)t\right]^{\frac{2}{d+1}}xdydz$$
$$-\left\{\left[(d+1)t\right]^{\frac{2}{d+1}}x+\left[(d+1)t\right]^{\frac{2m}{d+1}}\right\}dz^2 \tag{24}$$

4 Physical properties of the model

Equation (24) represents LRS Bianchi type-II bulk viscous string cosmological model in $f(R,T)$ gravity with the following physical and kinematical parameters which are very important for physical discussion of the model.

Spatial volume of the model from Eq. (13) is

$$V=\left[(d+1)t\right]^{\frac{2m+1}{d+1}} \tag{25}$$

Scalar of expansion from Eq. (14) is

$$\theta = \frac{2m+1}{(d+1)t} \tag{26}$$

Shear scalar from Eq. (15) is

$$\sigma^2 = \frac{(m-1)^2}{3[(d+1)t]^2} \tag{27}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(2m+1)^2} \neq 0, \quad \text{since } m > 1 \tag{28}$$

The Hubble's parameter is

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{m+2}{3(d+1)t} \tag{29}$$

The energy density in the model is

$$\rho = \frac{1}{\varepsilon(8\pi+3\mu)-\mu}\left\{\frac{3m^2-2m(1+d)}{(d+1)^2}\frac{1}{t^2} - \frac{3}{4}\frac{1}{(d+1)^{\frac{4m-2}{d+1}}}\frac{1}{t^{\frac{4m-2}{d+1}}}\right\} \tag{30}$$

The string tension density is

$$\lambda = \frac{1}{r[\varepsilon(8\pi+3\mu)-\mu]}\left\{\frac{3m^2-2m(1+d)}{(d+1)^2}\frac{1}{t^2} - \frac{3}{4}\frac{1}{(d+1)^{\frac{4m-2}{d+1}}}\frac{1}{t^{\frac{4m-2}{d+1}}}\right\} \tag{31}$$

Coefficient of bulk viscosity

$$\zeta = \frac{(\varepsilon_0-\varepsilon)(d+1)}{(m+2)[\varepsilon(8\pi+3\mu)-\mu]}\left\{\frac{3m^2-2m(1+d)}{d+1}\frac{1}{t} - \frac{3}{4}\frac{1}{(d+1)^{\frac{4m-2}{d+1}-1}}\frac{1}{t^{\frac{4m-2}{d+1}-1}}\right\} \tag{32}$$

Energy density of the particles attached to the string

$$\rho_p = \frac{(r-1)}{r[\varepsilon(8\pi+3\mu)-\mu]}\left\{\frac{3m^2-2m(1+d+d^2)}{(d+1)^2}\frac{1}{t^2} - \frac{3}{4}\frac{1}{(d+1)^{\frac{4m-2}{d+1}}}\frac{1}{t^{\frac{4m-2}{d+1}}}\right\} \tag{33}$$

Scale factor of the model is

$$a(t) = \left(A^2B\right)^{\frac{1}{3}} = \left[(d+1)t\right]^{\frac{2m+1}{3(d+1)}} \tag{34}$$

The deceleration parameter in this model is

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left[1 - \frac{3(d+1)}{2m+1}\right] = \text{a constant} \tag{35}$$

It is obvious that q depends on the constants d and m and it may be noted that in order to get accelerated expansion model, one should have $1 - \frac{3(d+1)}{2m+1} > 0$.

From the above results it can be observed that the model (24) has no initial singularity and the spatial volume increases as t increases giving the accelerated expansion of the universe since. In this model, we also observe that $\theta, \sigma^2, H, \overline{p}, p, \rho, \lambda, \zeta$ and ρ_p all diverge at the initial epoch, i.e. at $t = 0$ while they vanish for infinitely large t. Also $\frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy throughout the evolution of the universe. However when $m = 1$ the model becomes isotropic.

5 Conclusions

It is well known that $f(R,T)$ gravity has been proposed to explain the recent scenario of accelerated expansion of the universe. In spite of the fact the present day universe is well represented by the spatially homogeneous isotropic FRW model, experiments reveal that there is certain amount of anisotropy in our universe. Hence, in this paper, we present a spatially homogeneous and anisotropic LRS Bianchi type-II model in $f(R,T)$ gravity proposed by Harko et al. (2011) when the source of energy momentum tensor is a viscous fluid containing one dimensional cosmic strings. A barotropic cosmic fluid is considered for this study. A general equation of state for the energy density is assumed. We have also assumed that the scalar expansion of the space-time is proportional to shear scalar to get a determinate solution. The model presented will help to discuss the role of bulk viscosity in getting an inflationary model and to understand structure formation in the universe.

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