Astrophys Space Sci (2013) 344:269–274
DOI 10.1007/s10509-012-1315-z

ORIGINAL ARTICLE

Reconstruction of $f(T)$ gravity from the Holographic Dark Energy

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Received: 9 November 2012 / Accepted: 20 November 2012 / Published online: 29 November 2012

Abstract Among different candidates to play the role of Dark Energy (DE), modified gravity has emerged as offering a possible unification of Dark Matter (DM) and DE. The purpose of this work is to develop a reconstruction scheme for the modified gravity with $f(T)$ action using holographic energy density. In the framework of the said modified gravity we have considered the equation of state of the Holographic DE (HDE) density. Subsequently we have developed a reconstruction scheme for modified gravity with $f(T)$ action. Finally we have obtained a modified gravity action consistent with the HDE scenario.

Keywords $f(T)$ gravity · Holographic Dark Energy · Reconstruction

1 Introduction

Cosmological observations obtained with Supernovae Ia (SNeIa), the Cosmic Microwave Background (CMB) radiation anisotropies, the Large Scale Structure (LSS) and X-ray experiments have well established the accelerated expansion of our universe (Perlmutter et al. 1999; Bennett et al. 2003; Spergel et al. 2003; Tegmark et al. 2004; Abazajian et al. 2004, 2005; Allen et al. 2004). A missing energy component also known as Dark Energy (DE) with negative pressure is widely considered by scientists as responsible of this accelerated expansion. Recent analysis of cosmological observations indicates that the two-thirds of the total energy of the universe is been occupied by the DE whereas DM occupies almost the remaining part (the baryonic matter represents only a few percent of the total energy density of the universe) (Bennett et al. 2003). The contribution of the radiation is practically negligible.

The nature of DE is still unknown and many candidates have been proposed in order to describe it (Peebles and Ratra 2003; Padmanabhan 2003, 2005; Carroll 2001, 2004; Bean et al. 2005; Sahni and Starobinsky 2000; Weinberg 1989). The simplest one is represented by a tiny positive cosmological constant, with a negative constant equation of state (EoS) parameter ω, i.e. $\omega = -1$. However, cosmologists know that the cosmological constant suffers from two well-known difficulties, the fine-tuning and the cosmic coincidence problems: the former asks why the vacuum energy density is so small (of the order of 10^{-123} smaller than what we observe) and the latter says why vacuum energy and DM are nearly equal today (Copeland et al. 2006; Weinberg 1989).

As possible alternative to cosmological constant, dynamical scalar field models have been proposed some of which are quintessence (Wetterich 1988; Ratra and Peebles 1988; Zlatev et al. 1999; Doran and Jaumlckel 2002), phantom (Caldwell 2002; Nojiri and Odintsov 2003), f-essence (Setare 2006; Jamil et al. 2012c) and K-essence (Armendariz-Picon et al. 2000, 2001; Chiba et al. 2000).

An important advance in the studies of black hole theory and string theory is the suggestion of the so called holographic principle which was proposed by Fischler and Susskind (1998). According to the holographic principle, the

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number of degrees of freedom of a physical system should be finite and should scale with its bounding area rather than with its volume ('t Hooft 1993) and it should be constrained by an infrared cut-off (Cohen et al. 1999). The Holographic DE (HDE), based on the holographic principle, is one of the most studied models of DE (Huang and Li 2004; Jamil et al. 2009, 2012d; Hsu 2004; Pasqua et al. 2012a, 2012b). The HDE models have also been constrained and tested by various astronomical observations (Zhang and Wu 2005, 2007).

Applying the holographic principle to cosmology, the upper bound of the entropy contained in the universe can be obtained. Following this line, Li (2004) suggested as constraint on the energy density of the universe $\rho_\Lambda \leq 3\gamma M_p^2 L^{-2}$, where γ is a numerical constant, L is the IR cut-off radius and M_p is the reduced Planck mass. The equality sign holds when the holographic bound is saturated.

Importance of modified gravity for late acceleration of the universe has been reviewed (Nojiri and Odintsov 2009; Clifton et al. 2012). Holographic reconstruction of DE models has been attempted in a handful of works. Some of them in this direction are reviewed below. Setare (2007a) studied cosmological application of holographic dark energy density in the Brans-Dicke framework and suggested a correspondence between the holographic dark energy scenario in flat universe and the phantom dark energy model in framework of Brans–Dicke theory with potential. In another work, Setare (2007b) reconstructed the potential and the dynamics of the scalar field which describe the Chaplygin cosmology. Setare (2007c) considered a correspondence between the holographic dark energy density and tachyon energy density in FRW universe. Subsequently, they reconstructed the potential and the dynamics of the tachyon field which describe tachyon cosmology. A correspondence between the holographic dark energy density and interacting generalized Chaplygin gas energy density in FRW universe in the work of Setare (2007d), where they reconstructed the potential of the scalar field which describe the generalized Chaplygin cosmology. A holographic model of Chaplygin gas in the framework of DGP cosmology was considered in Setare (2009), where it was shown that the holographic Chaplygin gas can mimic a phantom fluid and cross the phantom divide in a DGP brane-world setup. Setare (2007e) considered holographic model of interacting dark energy in non-flat universe and reconstructed the potential of the phantom scalar field.

Various modified gravity theories have been proposed so far: some of the most studied include $f(R)$ (Nojiri and Odintsov 2006), $f(G)$ (Myrzakulov et al. 2011; Banijamali et al. 2012), Horava-Lifshitz (Kiritsis and Kofinas 2009) and Gauss-Bonnet (Nojiri and Odintsov 2005; Li et al. 2007) theories. Recently, a new theory of gravity known as $f(T)$ gravity, which is formulated in a space-time possessing absolute parallelism (Cai et al. 2011; Ferraro and Fiorini 2007), has been proposed.

Fundamental aspects of $f(T)$ gravity have been recently studied (Li et al. 2011; Sotiriou et al. 2011). In the $f(T)$ theory of gravity, the teleparallel Lagrangian density described by the torsion scalar T has been promoted to be a function of T, i.e. $f(T)$, in order to account for the late time cosmic acceleration (Bamba and Geng 2011). In a recent work, Jamil et al. (2012b) examined the interacting DE model in $f(T)$ cosmology assuming DE as a perfect fluid and choosing a specific cosmologically viable form $f(T)=\beta\sqrt{T}$. Statefinder diagnostic of $f(T)$ gravity has been studied in Jamil et al. (2012a). Purpose of the present work is to develop a reconstruction scheme for the modified gravity with $f(T)$ action using holographic energy density.

2 Reconstruction of $f(T)$ gravity

In the framework of $f(T)$ theory, the action of modified teleparallel action is given by (Myrzakulov 2011):

$$I=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\big[f(T)+L_m\big], \tag{1}$$

where L_m is the Lagrangian density of the matter inside the universe, G is the gravitational constant and g is the determinant of the metric tensor $g^{\mu\nu}$. We consider a flat Friedmann-Robertson-Walker (FRW) universe filled with the pressureless matter. Choosing ($8\pi G=1$), the modified Friedmann equations in the framework of $f(T)$ gravity are given by (Ferraro and Fiorini 2007; Myrzakulov 2011):

$$H^2=\frac{1}{3}(\rho+\rho_T), \tag{2}$$

$$2\dot{H}+3H^2=-(p+p_T), \tag{3}$$

where

$$\rho_T=\frac{1}{2}(2Tf_T-f-T), \tag{4}$$

$$p_T=-\frac{1}{2}\big[-8\dot{H}Tf_{TT}+(2T-4\dot{H})f_T-f+4\dot{H}-T\big], \tag{5}$$

and (Ferraro and Fiorini 2007):

$$T=-6\big(H^2\big). \tag{6}$$

Earlier, Setare (2008) reconstructed $f(R)$ gravity from HDE. In this section, we describe a reconstruction of $f(T)$ gravity in HDE scenario. We must emphasize that reconstruction of $f(T)$ gravity has been already studied in Bamba et al. (2012a) and a general DE review which discusses the reconstruction in $f(T)$ gravity can be found in Bamba et al. (2012b).

Following Setare (2008), the HDE density is chosen as:

$$\rho_\Lambda=\frac{3c^2}{R_h^2}, \tag{7}$$

where R_h represents the future event horizon and c is a constant.

The expression of R_h is given by:

$$R_h = a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2}. \tag{8}$$

The dimensionless DE is defined by using the critical energy density $\rho_{cr} = 3H^2$ as follow:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{c^2}{R_h^2 H^2}. \tag{9}$$

The time derivative of the future horizon is given by:

$$\dot{R}_h = R_h H - 1 = \frac{c}{\sqrt{\Omega_\Lambda}} - 1. \tag{10}$$

Using the conservation equation, the EoS parameter ω_Λ for HDE has been obtained by Setare (2008) as follow:

$$\omega_\Lambda = -\left(\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c}\right). \tag{11}$$

In order to reconstruct $f(T)$ gravity in HDE scenario, we replace the energy density of Eq. (2) with ρ_Λ and hence we get:

$$\rho_\Lambda = 6H^2 f_T + \frac{1}{2} f(T). \tag{12}$$

Using Eq. (9) in Eq. (12), we can write:

$$6H^2\Omega_\Lambda = 12H^2 f_T + f(T). \tag{13}$$

Hence, we get

$$f(T) = 6H^2\Omega_\Lambda - 12H^2 f_T = -T(\Omega_\Lambda - 2f_T). \tag{14}$$

We now consider Eq. (3), where p would be replaced by p_Λ. As $\omega_\Lambda = p_\Lambda/\rho_\Lambda$, we can write

$$p_\Lambda = 3H^2\omega_\Lambda\Omega_\Lambda, \tag{15}$$

where ω_Λ is given in Eq. (11). Using Eqs. (3), (5) and (15), we get:

$$3\omega_\Lambda H^2\Omega_\Lambda = -4\dot{H}T f_{TT} + (T - 2\dot{H}) f_T - \frac{1}{2} f(T). \tag{16}$$

Subsequently, using $T = -6H^2$, we can write:

$$3\omega_\Lambda\Omega_\Lambda = 24\dot{H} f_{TT} - \left(6 + \frac{2\dot{H}}{H^2}\right) f_T - \frac{1}{2H^2} f(T). \tag{17}$$

For scale factor $a(t)$ we shall consider the solution (Setare 2008):

$$a(t) = a_0(t_s - t)^n, \tag{18}$$

where a_0 is the present value of $a(t)$ and t_s and n are constants. Hence:

$$H = -\frac{n}{t_s - t}, \tag{19}$$

$$T = -\frac{6n^2}{(t_s - t)^2}, \tag{20}$$

$$\Omega_\Lambda = c^2\left(1 - \frac{1}{n}\right)^2. \tag{21}$$

The solution of Eq. (14) is given by:

$$f(T) = \left[\frac{c(n-1)}{n}\right]^2 (T - 2) + C_1 e^{-\frac{T}{2}}, \tag{22}$$

where C_1 is a constant.

Instead, the solution of Eq. (17) is given by:

$$f(T) = \frac{\alpha}{\beta_1 - 3} T + T^{\frac{\beta_2 - \beta_1 - \sqrt{(\beta_1-\beta_2)^2 + 12\beta_2}}{2\beta_2}} \times \left(T^{\frac{\sqrt{(\beta_1-\beta_2)^2+12\beta_2}}{\beta_2}} C_2 + C_3\right), \tag{23}$$

where

$$\alpha = \frac{c^2(n-1)^2(2-3n)}{n^3}, \tag{24}$$

$$\beta_1 = -6 + \frac{2}{n}, \tag{25}$$

$$\beta_2 = \frac{4}{n}, \tag{26}$$

and C_2 and C_3 are two constants.

The solutions for $f(T)$ are now studied graphically. In Fig. 1, we plot the solution $f(T)$ given in Eq. (22) against t and n. The three-dimensional plot shows a decaying behavior of $f(T)$ with evolution of the universe. Furthermore, the rate of decay is increasing with increase in the value of n. Similarly, when we plot the expression of $f(T)$ given in Eq. (23) in Fig. 2, we find a decaying pattern of $f(T)$ with passage of cosmic time. However, contrary to what happened in the previous case, the rate of decay is less in the case of higher values of n. In Figs. 3 and 4, we have studied the behavior of $f(T)$ with variation in T. We observe that for

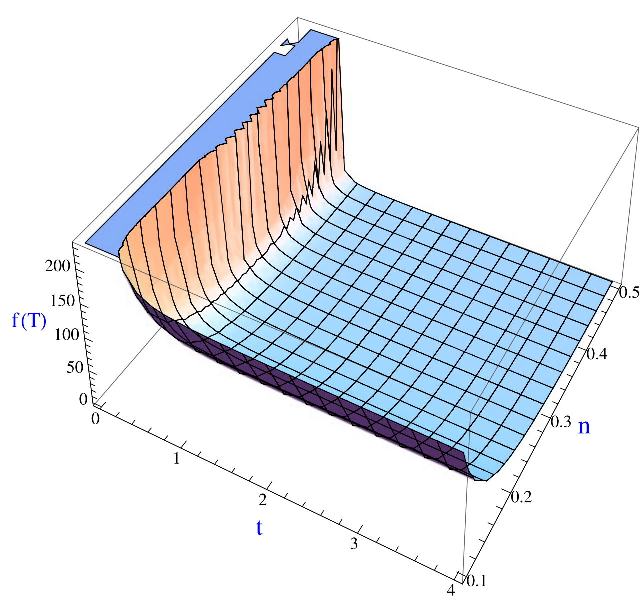

Fig. 1 This figure plots evolution of reconstructed $f(T)$ for the solution given in Eq. (22) and we find $f(T) \geq 0$ with the evolution of the universe for a range of n. The x-axis plots t and y-axis plots n

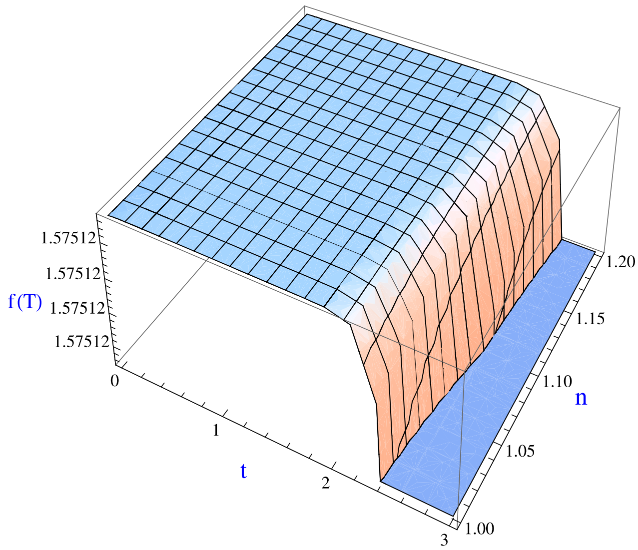

Fig. 2 This figure plots evolution of reconstructed $f(T)$ for the solution given in Eq. (23) and we find $f(T) \geq 0$ with the evolution of the universe for a range of n. The x-axis plots t and y-axis plots n

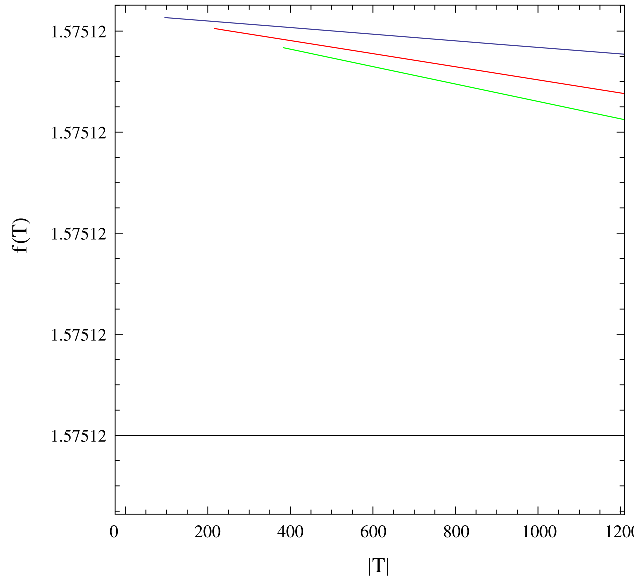

Fig. 4 This figure plots $f(T)$ for solution given in Eq. (23) against $|T|$ for different values of n

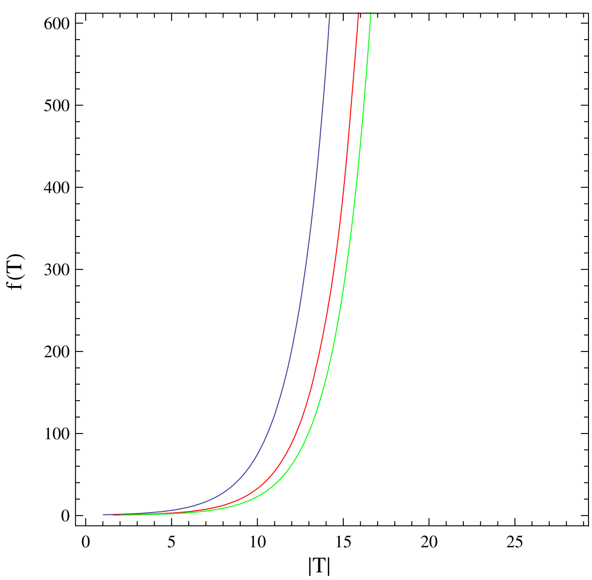

Fig. 3 This figure plots $f(T)$ for solution given in Eq. (22) against $|T|$ for different values of n

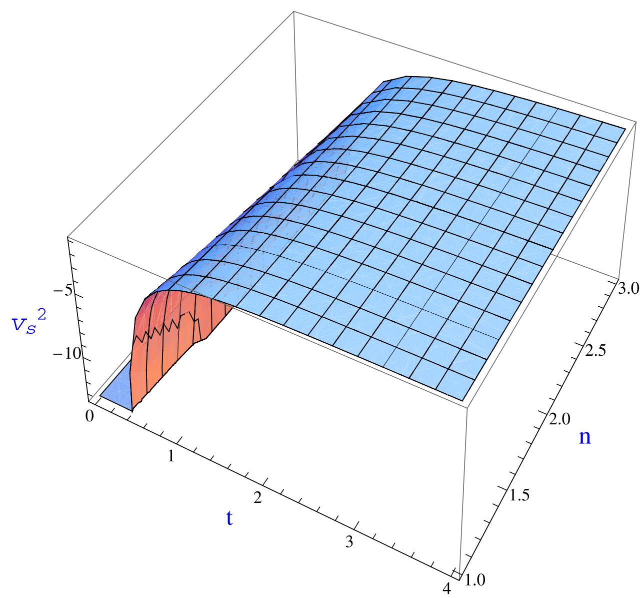

Fig. 5 This figure plots v_s^2 for solution of $f(T)$ given in Eq. (22) against cosmic time t for different values of n

the solution corresponding to Eq. (22), the $f(T)$ increases with increase in absolute value of T. On the other hand, the expression $f(T)$ obtained from Eq. (23) decays with $|T|$. It has been discussed in Rastkar et al. (2012) that satisfaction of the above condition is a sufficient condition for a realistic model. From Fig. 3, we understand that $f(T) \to 0$ as $T \to 0$ for the solution obtained from Eq. (22). However, this does not occur for the solution obtained in Eq. (23). Thus, it may be stated that the solution obtained in Eq. (22) is a more realistic model than that obtained in Eq. (23).

We now want to study an important quantity, namely the squared speed of sound, defined as:

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}}. \tag{27}$$

The sign of v_s^2 is crucial for determining the stability of a background evolution. If this is negative, it implies a classical instability of a given perturbation (Sharif and Jawad 2012; Kim et al. 2008). Sharif and Jawad (2012) have shown that interacting new holographic dark energy is characterized by negative v_s^2. Myung (2007) has observed that the

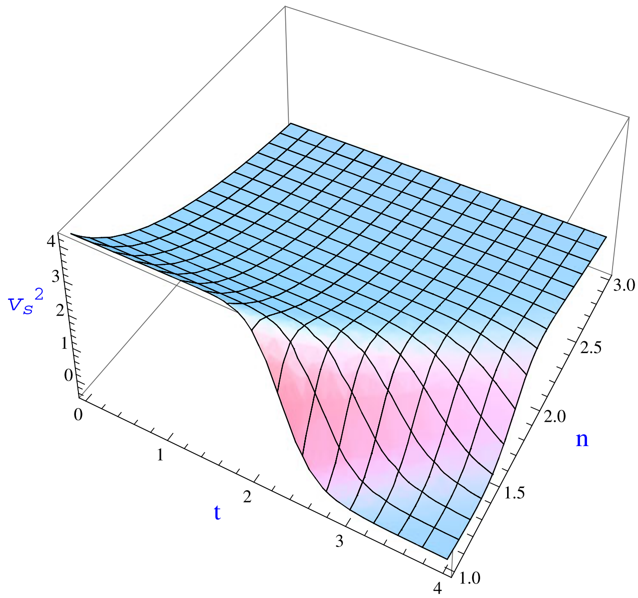

Fig. 6 This figure plots v_s^2 for solution of $f(T)$ given in Eq. (23) against cosmic time t for different values of n

squared speed for holographic DE is always negative when choosing the future event horizon as the IR cut-off, while those for Chaplygin gas and tachyon are non-negative. Kim et al. (2008) found that the squared speed for agegraphic DE is always negative and this indicates that the perfect fluid for agegraphic dark energy is classically unstable. In this work, we have reconstructed $f(T)$ gravity in HDE scenario with future event horizon as the IR cut-off. We have obtained two solutions for $f(T)$ in Eq. (22) and Eq. (23). In Figs. 5 and 6, we plot v_s^2 for the solutions given in Eq. (22) and Eq. (23), respectively for a set of values of the parameters. In Fig. 5, we see that v_s^2 is staying at negative level and in Fig. 6 we find that v_s^2 is staying at positive level. Thus, the model represented by Eq. (22) is classically unstable, whereas, the model represented by Eq. (23) is stable.

3 Conclusion

Among different candidates to play the role of DE, modified gravity has emerged as a possible unification of DM and DE. The present work aims at a cosmological application of the HDE density in the framework of a modified gravity, named as $f(T)$ gravity. In the framework of the said modified gravity, we have considered the equation of state of the HDE density. Subsequently, we have developed a reconstruction scheme for modified gravity with $f(T)$ action. Considering the DE energy density given in Eq. (4) in holographic form and then by assuming a simple solution for the scale factor $a(t)$ as given in Eq. (18), we have obtained a solution for differential equation for $f(T)$ in Eq. (22). Again, by considering the DE pressure in holographic form, we have created a differential equation for $f(T)$ in Eq. (17) and using the same simple solution for the scale factor, we derived $f(T)$. It has been revealed that the solution of Eq. (22) seems to be more realistic than that of Eq. (23). In this, way we get a modified gravity action consistent with the HDE scenario. Finally, the stability of the two reconstructed $f(T)$ models is examined by means of the squared speed of sound. It is observed that although the solution of Eq. (23) is stable, the solution of Eq. (22) is classically unstable. It is, therefore, noted that although holographic reconstructed $f(T)$ gravity implied by Eq. (22) is apparently more realistic than that by Eq. (23), the stability analysis proves that it is classically unstable.

Acknowledgements The authors express sincere thanks to the anonymous reviewer for constructive suggestions. The first author sincerely acknowledges the facilities provided to him by the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India, during his visit in November, 2012 under the Visitor Associateship Programme. Financial support from DST, Govt. of India under project SR/FTP/PS-167/2011 is duly acknowledged by the first author.

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