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ORIGINAL ARTICLE

Five dimensional dark energy model in a scalar-tensor theory of gravitation

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Abstract A five dimensional Kaluza-Klein dark energy model with variable equation of state (EoS) parameter and a constant deceleration parameter is presented in Saez and Ballester (Phys. Lett. A 113:467, 1986) scalar-tensor theory of gravitation. Some physical and kinematical properties of the model are also discussed.

Keywords Five dimensional · Dark energy model · Scalar-tensor theory

1 Introduction

In recent years there is a strong belief among the Cosmologists that our universe is experiencing an accelerated expansion. Analysis of type 1a supernovae (Perlmutter et al. 1997, 1998, 1999; Reiss et al. 1998, 2004), cosmic microwave back ground (CMB) anisotropy (Caldwell 2002; Huange et al. 2006) and large scale structure (Daniel et al. 2008) strongly indicate that dark energy dominates the present universe, causing cosmic acceleration.

Inspite of several attempts to identify the candidates for dark energy still cosmic acceleration is a challenge for modern cosmology. However dark energy has conventionally been characterized by the EoS parameter mentioned by $\omega(t) = \frac{p}{\rho}$ which is not necessarily constant where p is the fluid pressure and ρ is energy density (Carroll and Hoffman 2003). Ray et al. (2010), Akarsu and Kilinc (2010a, 2010b), Yadav et al. (2011), Yadav and Yadav (2011), Pradhan and Amirhashchi (2011), Pradhan et al. (2010) are some of the authors who have obtained dark energy models with variable EoS parameter. Recently Pradhan et al. (2011), Yadav (2011) have discussed some Bianchi type dark energy models in general relativity.

There has been a considerable interest in deriving cosmological models for alternative theories of gravity during past decades. Brans and Dicke (1961) and Saez and Ballester (1986) scalar-tensor theories of gravitation are quite important among them. Brans-Dicke theory includes a long range scalar field interacting equally with all forms of matter (with the exception of electromagnetism) while in Saez-Ballester scalar-tensor theory the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives satisfactory description of weak fields. Inspite of the dimensionless character of the scalar field an antigravity regime appears. The study of cosmological models in the frame work of scalar-tensor theories has been the active area of research for the last few decades. Singh and Rai (1983) gave a detailed discussion of Brans-Dicke cosmological models while Singh and Agarwal (1991), Shri Ram and Tiwari (1998), Reddy and Rao (2001) and Reddy et al. (2006) have studied Saez-Ballester cosmological models in four dimensions.

The study of higher dimensional space-time is important at early stages of evolution of the universe. Witten (1984), Appelquist et al. (1987), Chodos and Detweller (1980), and Marchiano (1984) were attracted to the study of higher dimensional cosmology because it has physical relevance to

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the early times before the universe has undergone compactification transitions. Reddy et al. (2007) and Reddy and Naidu (2007) have discussed five dimensional Kaluza-Klein cosmological models in Brans-Dicke and Saez-Ballester scalar-tensor theories respectively.

In this paper, we present a five dimensional Kaluza-Klein dark energy cosmological model in the presence of perfect fluid with variable equation of state (EoS) parameter in Saez- Ballester scalar-tensor theory of gravitation. This is relevant in view of the fact that scalar fields play a vital role in the discussion of dark energy models.

2 Metric and field equations

We consider five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2 d\psi^2 \tag{1}$$

where A and B are functions of cosmic time t and the fifth coordinate is taken to be space-like. Unlike Wesson (1983). Here the spatial curvature has been taken as zero (Grøn 1988).

The field equations given by Saez and Ballester (1986) for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{2}$$

and the scalar field satisfies the equation

$$\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

Also

$$T^{ij}_{;j} = 0 \tag{4}$$

is a consequence of the field (2) and (3).

Preserving the diagonal form of the energy momentum tensor in a consistent way with the metric (1), the simplest generalization of EoS parameter of perfect fluid is to determine the EoS parameter separately on each spatial axis. Hence the energy momentum tensor of fluid, in five dimensions, is taken as

$$T^i_j = \mathrm{diag}[T^0_0, T^1_1, T^2_2, T^3_3, T^4_4] \tag{5}$$

we can parameterize it as follows

$$\begin{aligned} T^i_j &= \mathrm{diag}[\rho, -p_x, -p_y, -p_z, -p_\psi] \\ &= \mathrm{diag}[1, -\omega_x, -\omega_y, -\omega_z, \omega_\psi]\rho \\ &= \mathrm{diag}[1, -(\omega+\delta), -(\omega+\gamma), -(\omega+\eta), -\omega]\rho \end{aligned} \tag{6}$$

Here ρ is the energy density of the fluid p_x, p_y, p_z, and p_ψ are the pressures and ω_x, ω_y, ω_z and ω_ψ are the directional EoS parameters along the X, Y, Z and ψ axes respectively. ω is the deviation free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_\psi = \omega$ and then introducing skewness parameters δ, γ and η that is deviations from ω along the X, Z and ψ axes respectively.

Also, since for Kaluza-Klein metric $T^1_1 = T^2_2 = T^3_3$, we obtain that

$$\delta = \gamma = \eta \tag{7}$$

Now using co-moving coordinate system the field (2)–(4) with the help of (6) and (7) yield the following independent field equations for the metric (1)

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} - w\phi^n\dot{\phi}^2 = -\rho \tag{8}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} + w\phi^n\dot{\phi}^2 = (\omega+\delta)\rho \tag{9}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} + w\phi^n\dot{\phi}^2 = \omega\rho \tag{10}$$

$$\ddot{\phi} + \dot{\phi}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{11}$$

where an overhead dot indicates differentiation with respect to t.

The spatial volume for the Kaluza-Klein metric is given by

$$V^3 = A^3 B \tag{12}$$

We define $R = (A^3B)^{\frac{1}{3}}$ as the average scale factor of the Kaluza-Klein model (1) so that the Hubble's parameter is given by

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{13}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_x + H_y + H_z + H_\psi) \tag{14}$$

where $H_x = H_x = H_x = \frac{\dot{A}}{A}$ and $H_\psi = \frac{\dot{B}}{B}$ are the directional Hubble's parameter in the directions of X, Y, Z and ψ respectively.

The deceleration parameter q is defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{15}$$

The scalar expansion θ, shear scalar σ^2 and average anisotropy parameter A_m are defined by

$$\theta = 3\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \tag{16}$$

$$\sigma^2 = \frac{1}{3}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)^2 \tag{17}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 \tag{18}$$

where $\Delta H_i = H_i - H$ $(i = 1, 2, 3, 4)$.

3 Solutions of field equations and the model

The field (8)–(11) are a system of four independent equations in six unknowns A, B, ϕ, ρ, ω and δ. Two additional constraints relating these parameters are necessary to obtain explicit solutions of the system.

We shall first, apply the special law of variation for Hubble's parameter proposed by Bermann (1983) which yields constant deceleration parameter model of the universe.

Equation (15) on integration yields the solution

$$R = (A^3 B)^{\frac{1}{3}} = (at + b)^{\frac{1}{1+q}} \tag{19}$$

where a and b are constants of integration. This equation implies that the condition for accelerated expansion of the universe is $1 + q > 0$.

Next, we assume that the scalar of expansion θ is proportional to shear scalar σ^2. This condition leads to

$$A = B^m \tag{20}$$

where m is a constant (Collins et al. 1980).

Solving the field equations (8)–(11) with the help of (19) and (20) we obtain the expressions for the metric coefficients given by

$$A = (at + b)^{\frac{3m}{(3m+1)(1+q)}} \tag{21}$$

$$B = (at + b)^{\frac{3}{(3m+1)(1+q)}} \tag{22}$$

Now, through a proper choice of the coordinates and constants the metric (1), with the help of (21) and (22), can be written as

$$ds^2 = dt^2 - t^{\frac{6m}{(3m+1)(1+q)}} (dx^2 + dy^2 + dz^2) - t^{\frac{6}{(3m+1)(1+q)}} d\psi^2 \tag{23}$$

4 Physical properties of the model

Equation (23) represents five dimensional Kaluza-Klein dark energy model with the following physical and kinematical parameters.

Spatial volume

$$V = A^3 B = t^{\frac{3}{1+q}} \tag{24}$$

Hubble's parameter

$$H = \frac{1}{(1+q)t} \tag{25}$$

The scalar of expansion

$$\theta = \frac{1}{3(1+q)t} \tag{26}$$

Shear scalar

$$\sigma^2 = \frac{1}{27} \cdot \frac{1}{(1+q)^2 t^2} \tag{27}$$

Average anisotropy parameter

$$A_m = \frac{4}{3} \tag{28}$$

Also, we have

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} \neq 0 \tag{29}$$

The scalar field

$$\phi = \left[\frac{2}{n+2}\phi_0 t^{\frac{q-2}{1+q}}\right]^{\frac{2}{n+2}} \tag{30}$$

The energy density

$$\rho = \frac{w\phi_0^2}{t^{\frac{6}{1+q}}} - \frac{27m(m+1)}{(3m+1)^2(1+q)^2} \cdot \frac{1}{t^2} \tag{31}$$

EoS parameter

$$\omega = 1 + \frac{1}{\rho}\left[\frac{27m}{(3m+1)(1+q)^2} - \frac{9m}{(3m+1)(1+q)}\right]\frac{1}{t^2} \tag{32}$$

Skewness parameter

$$\delta = \gamma = \eta = \frac{1}{\rho}\left[\frac{3(m-1)}{(3m+1)(1+q)} - \frac{(27m^2 - 18m - 9)}{(3m+1)^2(1+q)^2}\right]\frac{1}{t^2} \tag{33}$$

From the above results, we observe that the model (23) has no initial singularity. The spatial volume is zero at $t = 0$ and increases with the cosmic time. The parameters H, θ, and σ diverge at $t = 0$. Since A_m is constant, the mean anisotropic parameter is uniform throughout the evolution of the universe and it does not depend on the cosmic time t. Since $\frac{\sigma^2}{\theta^2}$ = constant, the model does not approach isotropy for large values of t. From (32) and (33) it is observed that the EoS parameter ω and δ or γ or η are time dependent. Also from (30) and (31) one can observe that for $t = 0$, the scalar field ϕ vanishes while the energy density $\rho \to \infty$ and for large t, $\phi \to \infty$ while ρ vanishes.

5 Conclusions

We have studied five dimensional cosmological model in the presence of a perfect fluid with a variable equations of state (EoS) parameter in Kaluza-Klein space-time in the framework of Saez and Ballester (1986) scalar-tensor theory of gravitation. The model obtained represents Kaluza-Klein dark energy cosmological model which is expanding and free from initial singularity. It is observed that the dark energy EoS parameter and the skewness parameter are time dependent. The proposed model favors EoS parameter as possible candidate for dark energy in five dimensional space-time. It is to be noted that the study of dark energy and dark energy models has direct relevance to astrophysical problems.

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