Astrophys Space Sci (2012) 341:99–104
DOI 10.1007/s10509-012-0988-7

ORIGINAL ARTICLE

Problems and possibilities in fine-tuning of the Cepheid P-L relationship

L. Szabados · P. Klagyivik

Received: 31 August 2011 / Accepted: 9 January 2012 / Published online: 17 January 2012

Abstract Factors contributing to the scatter around the ridge-line period-luminosity relationship are listed, followed by a discussion how to eliminate the adverse effects of these factors (mode of pulsation, crossing number, temperature range, reddening, binarity, metallicity, non-linearity of the relationship, blending), in order to reduce the dispersion of the P-L relationship.

Keywords Stars: variables: Cepheids · Stars: distances

1 Introduction

The century-long history of the period-luminosity (P-L) relationship of Cepheids can be characterised as a permanent attempt at improving the calibration of this relationship fundamental for securing the cosmic distance scale. In spite of tremendous effort, the distribution of the points in the P-L plots derived for any galactic system shows a wider dispersion along the ridge-line approximation than hoped for (Fig. 1). The current situation is summarized concisely by Marconi (2009) and, in a broader context—from the point of view of determining the Hubble constant—by Freedman and Madore (2010).

Lately, most works on the P-L relationship deal with the metallicity dependence of the luminosity of Cepheids and the possible non-linearity of the relationship. There are, however, other factors contributing to the dispersion of the empirical P-L relation. The aim of this paper is to list all these factors and discuss the elimination of their adverse effect.

L. Szabados (✉) · P. Klagyivik
Konkoly Observatory, Budapest, Hungary
e-mail: szabados@konkoly.hu

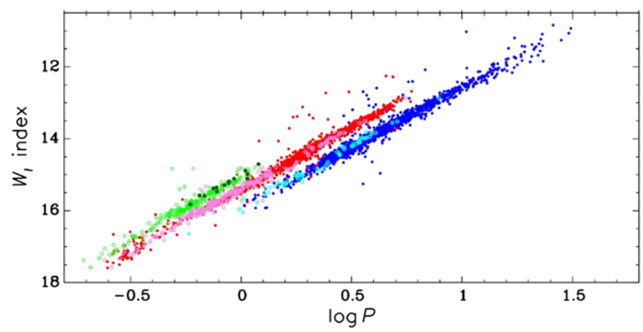

Fig. 1 Empirical P-L relationship for Cepheids in the Large Magellanic Cloud. To decrease the scatter, Wesenheit magnitudes are indicated along the vertical axis (Soszyński et al. 2008). *Blue* and *cyan points* show fundamental-mode pulsators, *red* and *magenta* denote first-overtone, *green* characters second overtone pulsators. *Solid dots* are single-mode Cepheids, *empty circles* represent the respective mode of double-mode Cepheids. The dispersion is still about $0\overset{m}{.}5$

2 Causes of the dispersion of the P-L relationship

The following factors contribute to the width of the empirical P-L plot:

- pulsation mode,
- crossing number (evolutionary stage),
- effective temperature of the star,
- interstellar reddening,
- binarity,
- blending,
- metallicity,
- helium content,
- nonlinearity of the relationship,
- other (magnetic field; overshooting; depth effect in the host galaxy; etc.).

For individual factors, the amount of the effect can vary from negligible to considerable and can depend on the pulsation period.

3 Reduction of the dispersion

Existence of the P-L relationship follows from basic physics (eigenfrequency of a radial oscillator and Stefan-Boltzmann law—see Freedman and Madore (2010) and references therein) which makes Cepheids standard candles for establishing the cosmic distance scale. The reduction of the scatter in the empirically determined P-L relationship is, therefore, an obvious goal in the calibration procedure. However, each 'widening' effect has to be dealt with separately.

3.1 Pulsation mode

For a given star performing radial pulsation, the oscillation in the fundamental mode has the longest period, while oscillations in overtones are characterised by shorter period—without any change in stellar luminosity. This behaviour gives rise to a separate sequence for each pulsation mode which is a conspicuous feature in the P-L plot of extragalactic Cepheids (see Fig. 1).

In the case of external galaxies, the different sequences show up provided the plot is based on a rich Cepheid sample. Assuming a common distance for all Cepheids in a given galaxy, the pulsation mode of the individual Cepheids can be inferred from the period vs. apparent brightness diagram plotted for Cepheids because this diagram reflects the structure of the P-L plot (see Fig. 1).

In the case of Galactic Cepheids, however, the period vs. apparent brightness diagram is senseless because of the huge range of distance of Cepheids involved. The pulsation mode has to be determined before deriving the luminosity. Fourier decomposition of the photometric and radial velocity phase curves can be efficiently used for the mode determination: values of the Fourier parameters R_{21}, ϕ_{21}, R_{31}, and ϕ_{31} determined from the brightness variations are sufficient to assign the pulsation mode of any Cepheid. In those period intervals where ϕ_{21} values overlap for different modes, ϕ_{31} values offer good selection criteria (Soszyński et al. 2010).

If reliable radial velocity phase curve is also available for a Cepheid, then the period dependent difference of the Fourier phases $\Delta\phi_1 = \phi_{21}^{V_{rad}} - \phi_{21}^{phot}$ serves as an indicator of the pulsation mode (Ogłoza et al. 2000). Usefulness of this empirical phase lag parameter for mode identification (see Fig. 2) was confirmed theoretically by Szabó et al. (2007).

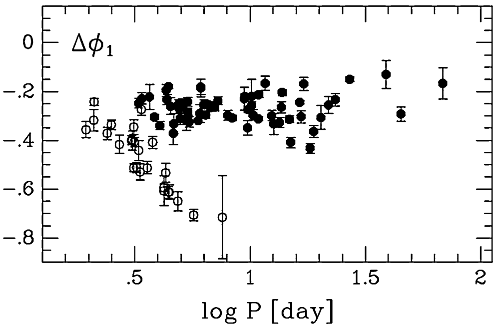

Fig. 2 The phase lag as defined in the text is a reliable criterion for the mode identification (Ogłoza et al. 2000)

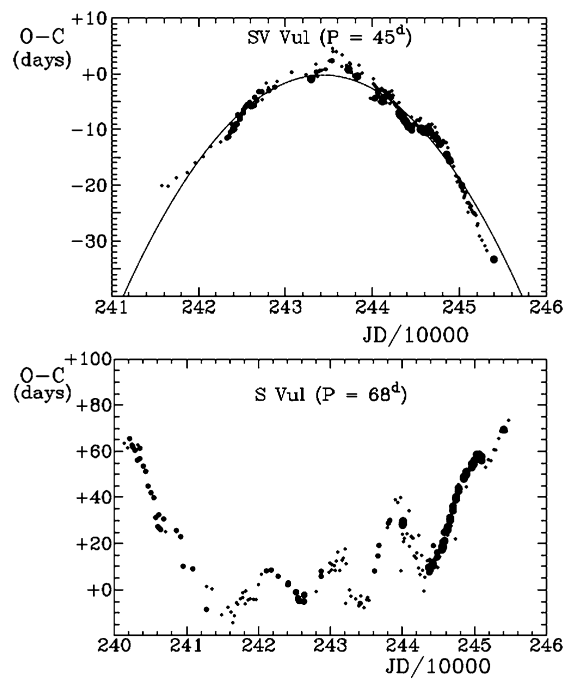

Fig. 3 Example for secular period decrease (SV Vul) and increase (S Vul) (Turner et al. 2009)

3.2 Crossing number

Intermediate-mass stars cross the classical instability region up to 5 times during their evolution and the subsequent crossings occur at increasing luminosities. Due to the slope of the lines of constant period in the Hertzsprung-Russell diagram, odd numbered crossings result in a secular period increase, while during even numbered crossings the pulsation period of Cepheids is decreasing on a long time scale (Fig. 3). The 2nd crossing is the slowest one, thus majority of Cepheids are in this evolutionary phase. The observed rates of period change in over 200 Milky Way Cepheids were studied by Turner et al. (2006) for assigning the crossing number.

Post-main sequence evolution of stars is accompanied with changes in the surface chemical composition, so in principle, the abundance of elements derived from the observed spectra supplemented with information on the direction of the secular period change is sufficient for assigning the crossing mode unambiguously. Viability of such procedure has been presented by the study of SV Vul (Turner and Berdnikov 2004). Applicability of this method is, however, encumbered by the fact that the CNO-processed material can be dredged up into the stellar atmosphere before the red supergiant phase as a consequence of meridional mixing in rapidly rotating B stars, progenitors of Cepheids (Turner et al. 2006).

3.3 Temperature range

In fact, the P-L relationship is the two-dimensional projection of the underlying period-luminosity-colour (P-L-C) relationship. A part of the intrinsic scatter of the P-L relationship is caused by the neglect of the colour (temperature) dependence.

Knowledge of T_{eff} is essential for removing the reddening effect and correcting for the interstellar extinction.

3.4 Reddening

Extensive lists of spectroscopically determined T_{eff} values (and the $E(B-V)$ colour excess derived) have been compiled by Andrievsky et al. (2005) and quite recently by Luck et al. (2011) and Luck and Lambert (2011) involving about 400 Galactic Cepheids. For individual calibrating Cepheids in our Galaxy, the intrinsic colour and apparent brightness can be determined from this data base.

The effect of line-of-sight extinction can be practically 'removed' by using the reddening-free Wesenheit function for any pair of bands ij:

$$W_{ij} = m_i - R_{ij} \times (m_i - m_j),$$

where $R_{ij} = R_i/(R_i - R_j)$. The dereddened W function is unaffected by extinction in case the ratio of total-to-selective absorption, R_{ij}, is universal and known. Such Wesenheit function can be defined for any combination of optical and near-IR bandpasses.

This formalism can account for the differential reddening as long as R_{ij} does not vary from star to star but it cannot account for the varying amount of dust along the line of sight to each Cepheid.

3.5 Binarity

Companion stars effectively contribute to the observed width of the Cepheid P-L relationship. Unresolved companions (either physical or optical) falsify the brightness and colours of the Cepheid to an extent that depends on the luminosity and temperature differences between the Cepheid and its companion(s). Most of the bright outliers in Fig. 1 can be unrecognized binaries in the Large Magellanic Cloud.

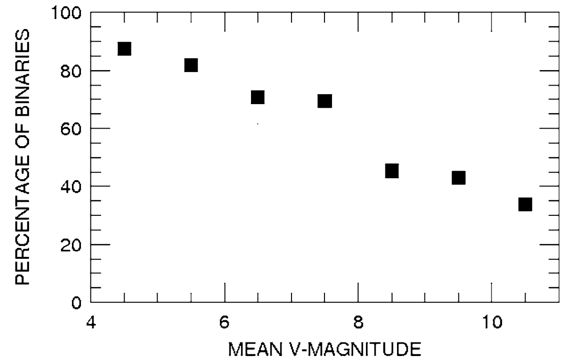

Fig. 4 Observed frequency of occurrence of binaries among Galactic Cepheids as a function of the mean apparent brightness (Szabados et al. 2011). The trend seen in the diagram is the result of observational selection effects

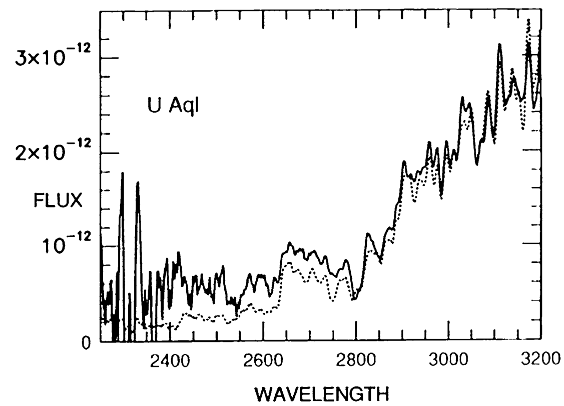

Fig. 5 Blue companions can be readily detected from ultraviolet spectra. U Aql has a B9.8V type secondary (Evans 1992). The *dotted curve* is the flux from the Cepheid component

As to our Galaxy, an in-depth survey and analysis of the available observational data indicates that at least 50 per cent of Cepheids are not single stars (Szabados 2003).

Considering such high frequency of occurrence of binaries among Cepheids, it is essential to remove photometric and other effects of companions when using individual Cepheids for calibrating the P-L relationship.

In fact, it is an observational challenge to reveal binarity of fainter Cepheids (cf. the selection effect seen in Fig. 4). There are, however, various methods for pointing out faint companions.

Spectroscopy, especially in the UV region, is instrumental in revealing faint blue secondaries (see Fig. 5). Radial velocity phase curves obtained in different seasons can be used for discovering spectroscopic binary nature of a Cepheid by detecting the orbital effect superimposed on the radial velocity variations due to pulsation.

Multicolour *photometric* data can also indicate presence of a companion. Klagyivik and Szabados (2009) devised dif-

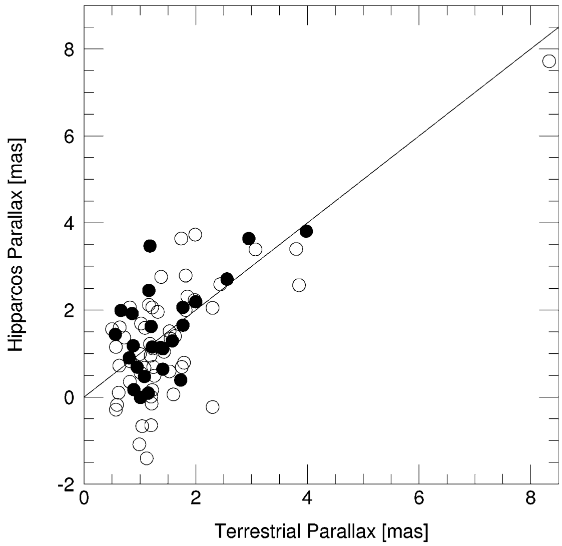

Fig. 6 Revised Hipparcos parallax of the nearest Cepheids as a function of their astrophysically determined distances converted into parallax ('terrestrial parallax'). *Filled circles* correspond to Cepheids without known companions, *empty circles* represent Cepheids with companions. It is remarkable that all negative Hipparcos parallaxes correspond to Cepheids belonging to binary (multiple) systems (Szabados et al. 2011)

ferent parameters based on the wavelength dependence of the pulsational amplitude which hint at the presence of a photometric (i.e., not necessarily physical) companion.

Companions to Cepheids can be revealed by *astrometric* methods, as well. The Gaia astrometric space probe (to be launched in 2013) will be able to resolve dozens of visual binaries among Cepheids. For some of these systems, even orbital elements can be determined from the Gaia astrometric measurements spanning a 5-year-long interval. (The shortest orbital period of a binary system involving a supergiant Cepheid component is about a year.)

The angular sensitivity of about 0.001 arc second of the previous astrometric satellite, Hipparcos, was not sufficient to resolve the orbits of binaries containing a Cepheid component. The adverse effect of the unrecognised orbital motion is, however, obvious even in the revised Hipparcos parallaxes (Fig. 6): all negative parallax values for Cepheids within 2 kpc belong to Cepheids with known companions, i.e., neglect of the subtle orbital motion falsified the derived parallax in each case (and in either sense). If the orbital period exceeds 6–8 years, the astrometric contribution of the orbital motion is negligible during the 3–5 year measurement period of astrometric space missions like Hipparcos or Gaia. Some Cepheids marked as single stars in Fig. 6 may also have unresolved physical companions as implied by Fig. 4.

If the binary system cannot be resolved even by Gaia astrometry, its binarity can be revealed from data to be obtained by the Gaia radial velocity spectrometer (to the limiting integral brightness of about 13th magnitude).

3.6 Blending

Angular proximity of unrelated stars mainly occurs in crowded stellar fields, typical of locations of extragalactic Cepheids. Blending of light from a Cepheid with that of a star along the same line-of-sight results in a bright outlier in the P-L relationship. If the disturbing point source cannot be separated from the Cepheid, the bright outlier can be considered to be a binary source involving a Cepheid component (see Sect. 3.5). Ngeow and Kanbur (2010) presents some examples for both cases among Cepheids in the Small Magellanic Cloud.

Persson et al. (2004) list three methods of handling the problem of field-star crowding. In extreme cases, however, the photometric contamination cannot be corrected for, and the given Cepheid has to be removed from the sample of calibrating stars.

Because long-period Cepheids are more luminous, their photometry is less affected by the blending effect.

3.7 Metallicity

The metallicity sensitivity of stellar luminosity is an obvious and long-discussed cause of the dispersion of the P-L relationship. The main problem here is that contradictory results have been achieved from both theoretical calculations and observational data. For an excellent summary, see Romaniello et al. (2008).

Although the debate on the influence of chemical composition on the P-L relationship is unsettled yet, it is undoubted that practically all pulsational properties (amplitudes, amplitude ratios) of Cepheids depend on the atmospheric metal content (Szabados and Klagyivik 2012). It is remarkable that some of these dependences cannot be explained by the differential line blanketing. Such an effect is seen in Fig. 7: even the peak-to-peak radial velocity amplitude slightly depends on the iron content, [Fe/H], of the Cepheid. An additional interesting fact is that the limiting period that separates short- and long-period Cepheids appears at 10.47 days, instead of the conventional value of 10 days (see Sect. 3.9).

3.8 Helium content

In addition to metallicity, the helium abundance also makes influence on the luminosity of the star. Unlike metallicity, the He content cannot be determined from spectral observations, so its effect cannot be studied observationally. Theoretical computations, however, indicate that varying helium/metal abundance ratio results in different synthetic P-L relationships, and the effect of compositional differences is larger for longer pulsation periods (Marconi et al. 2005).

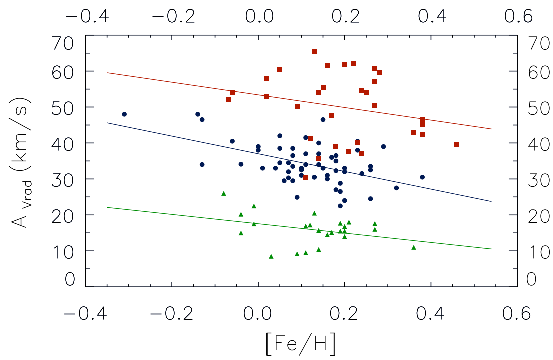

Fig. 7 Metallicity has an effect on most physical properties of Cepheids: the peak-to-peak amplitude of radial velocity variations during the pulsational cycle also depends on the iron abundance, [Fe/H]. Data are plotted only for single Cepheids: *triangles* represent s-Cepheids, *circles* short-period Cepheids of normal amplitude, *squares* long-period Cepheids (Szabados and Klagyivik 2012). Short- and long-period Cepheids are separated at the pulsation period of $10^{\mathrm{d}}.47$

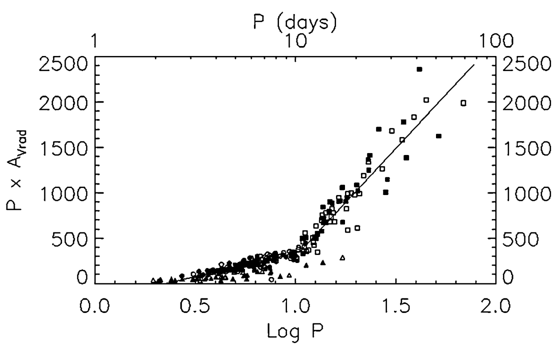

Fig. 8 This diagram shows a sharp break separating short- and long-period Cepheids. The dichotomy appears at $\log P = 1.02$. The quantity along the vertical axis, product of the pulsation period and peak-to-peak radial velocity amplitude, is a measure of the radius variation during a pulsational cycle (Klagyivik and Szabados 2009)

3.9 Non-linearity

There is growing evidence of the existence of a break in the ridge-line P-L relationship. The dichotomy between short- and long-period Cepheids was pointed out first in the LMC. Sandage et al. (2004) found different slopes of the P-L relationships of Cepheids with period shorter and longer than 10 days. Deviations from linearity of the LMC Cepheid P-L and P-L-C relationships was confirmed by Koen et al. (2007).

When studying the period dependence of pulsational properties of Galactic Cepheids, a similar dichotomy was found by Klagyivik and Szabados (2009). However, as is seen in Fig. 8, the break does not appear at an exact value of $P = 10$ days, in fact, the dichotomy occurs at 10.47 days ($\log P = 1.02$).

A break was revealed in the P-L relationship of the SMC Cepheids, as well (Ngeow and Kanbur 2010) but, strangely enough, the dividing period is at $\log P = 0.4$.

The presence of such break and the period value separating short- and long-period Cepheids in the three galaxies (Milky Way, LMC, SMC) implies that the dichotomy is not a metallicity effect but the location of the break can be metallicity dependent.

3.10 Miscellaneous other effects

There are other physical effects, e.g., stellar magnetic field, overshooting, etc. that also contribute to the scatter in the P-L plot constructed from observational data.

In the case of the nearest galaxies the line-of-sight extent of the system is an additional factor leading to increased scatter of the points in the P-L diagram. This effect can be corrected for, as was successfully done by Nikolaev et al. (2004) and Persson et al. (2004) in constructing the P-L plot for Cepheids in the Large Magellanic Cloud. This geometric effect of back-to-front distance differences of Cepheids in remote galaxies, especially outside the Local Group can be neglected.

4 Conclusion

More than ten factors contributing to the scatter in the P-L relationship have been listed and removal of some of these effects has been discussed. All these effects are independent of each other. Therefore, neglect of some factors falsify results on the role of other factors in observational studies. For example, when studying the effect of metallicity on stellar luminosity, Cepheids belonging to binary systems have to be excluded. Another example is the determination of the tilt of the Large Magellanic Cloud from the dispersion of the P-L relationship. In this procedure, the deviation of individual Cepheids from the ridge-line P-L relationship is usually attributed solely to the depth effect, neglecting effects of metallicity and binarity, etc.

Reduction of the scatter can be achieved by turning to longer wavelengths. However, moving to infrared spectral region does not remove the whole spread in the P-L relationship. The effect of the interstellar extinction becomes negligible in near-IR but effect of metallicity increases (and reverses) toward longer infrared wavelengths (Freedman and Madore 2011). Moreover, circumstellar envelopes that commonly occur around Cepheids cause an IR excess (Barmby et al. 2011).

In view of the increasing number of extragalactic Cepheids, the non-linearity of the P-L relationship can be and has to be studied beyond the Magellanic Clouds. The behaviour of the long-period Cepheids is especially important because such stars overwhelm among Cepheids known in remote galaxies.

Acknowledgements This research has been supported by the ESA PECS 98090 project. Constructive remarks by the referee are gratefully acknowledged.

References

Andrievsky, S.M., Luck, R.E., Kovtyukh, V.V.: Astron. J. **130**, 1880 (2005)
Barmby, P., Marengo, M., Evans, N.R., Bono, G., Huelsman, D., Su, K.Y.L., Welch, D.L., Fazio, G.G.: Astron. J. **141**, 42 (2011)
Evans, N.R.: Astrophys. J. **384**, 220 (1992)
Freedman, W.L., Madore, B.F.: Annu. Rev. Astron. Astrophys. **48**, 673 (2010)
Freedman, W.L., Madore, B.F.: Astrophys. J. **734**, 46 (2011)
Klagyivik, P., Szabados, L.: Astron. Astrophys. **504**, 959 (2009)
Koen, C., Kanbur, S., Ngeow, C.: Mon. Not. R. Astron. Soc. **380**, 1440 (2007)
Luck, R.E., Lambert, D.L.: Astron. J. **142**, 136 (2011)
Luck, R.E., Andrievsky, S.M., Kovtyukh, V.V., Gieren, W.P., Graczyk, D.: Astron. J. **142**, 51 (2011)
Marconi, M.: Mem. Soc. Astron. Ital. **80**, 141 (2009)
Marconi, M., Musella, I., Fiorentino, G.: Astrophys. J. **632**, 590 (2005)
Ngeow, C.-C., Kanbur, S.M.: Astrophys. J. **720**, 626 (2010)
Nikolaev, S., Drake, A.J., Keller, S.C., Cook, K.H., Dalal, N., Griest, K., Welch, D.L., Kanbur, S.M.: Astrophys. J. **601**, 260 (2004)
Ogłoza, W., Moskalik, P., Kanbur, S.: In: Szabados, L., Kurtz, D. (eds.) Proc. IAU Coll. 176, the Impact of Large-Scale Surveys on Pulsating Star Research. ASP Conf. Ser., vol. 203, p. 235. ASP, San Francisco (2000)
Persson, S.E., Madore, B.F., Krzemiński, W., Freedman, W.L., Roth, M., Murphy, D.C.: Astron. J. **128**, 2239 (2004)
Romaniello, M., Primas, F., Mottini, M., Pedicelli, S., Lemasle, B., Bono, G., François, P., Groenewegen, M.A.T., Laney, C.D.: Astron. Astrophys. **488**, 731 (2008)
Sandage, A., Tammann, G.A., Reindl, B.: Astron. Astrophys. **424**, 43 (2004)
Soszyński, I., Poleski, R., Udalski, A., Szymański, M.K., Kubiak, M., Pietrzyński, G., Wyrzykowski, Ł., Szewczyk, O., Ulaczyk, K.: Acta Astron. **58**, 163 (2008)
Soszyński, I., Poleski, R., Udalski, A., Szymański, M.K., Kubiak, M., Pietrzyński, G., Wyrzykowski, Ł., Szewczyk, O., Ulaczyk, K.: Acta Astron. **60**, 17 (2010)
Szabados, L.: Inf. Bull. Var. Stars No. 5394 (2003)
Szabados, L., Klagyivik, P.: Astron. Astrophys. **537**, A81 (2012). arXiv:1112.0115
Szabados, L., Kiss, Z.T., Klagyivik, P.: EAS Publ. Ser. **45**, 441 (2011)
Szabó, R., Buchler, J.R., Bartee, J.: Astrophys. J. **667**, 1150 (2007)
Turner, D.G., Berdnikov, L.N.: Astron. Astrophys. **423**, 335 (2004)
Turner, D.G., Abdel-Sabour Abdel-Latif, M., Berdnikov, L.N.: Publ. Astron. Soc. Pac. **118**, 410 (2006)
Turner, D.G., Majaess, D.J., Lane, D.J., Szabados, L., Kovtyukh, V.V., Usenko, I.A., Berdnikov, L.N.: In: Stellar Pulsation: Challenges for Theory and Observation. AIP Conf. Proc., vol. 1170, p. 108 (2009)