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ORIGINAL ARTICLE

Positron acoustic solitary waves interaction in a four-component space plasma

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Abstract The characteristics of the head-on collision (HOC) between two positron acoustic solitary waves (PASWs) in a four component electron-positron-ion (EPI) space plasma have been investigated theoretically, using the extended Poincaré-Lighthill-Kuo (PLK) method. The analytical phase shifts after the collision of the two solitary waves occurs are derived. Numerically, the influences of the cold/hot positron parameters on the phase shifts are explicitly investigated. The present theory is applied to analyze the formation and the interaction of localized coherent PASWs structures in space plasmas (pulsar environments).

Keywords Head-on collision · Extended Poincaré-Lighthill-Kuo (PLK) method · Positron acoustic solitary wave · Two-temperature positron · Electron-positron-ion plasma

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1 Introduction

During the last few years, there have been a lot of activities in the field of pair (electron–positron) plasma. Electron–positron plasmas have received a great deal of attention due to its importance not only in plasma astrophysics (i.e., in early universe, in active galactic nuclei (Miller and Witta 1987; Wang and Durouchoux 2011), in magnetospheres of pulsars (Shukla 1985), polar regions of neutron stars (Michel 1991), intense laser fields (Berezhiani et al. 1992)), but also in laboratory experiments in which the positrons can be used to probe the particle transport in tokamak plasmas (Greaves et al. 1994; Helander and Ward 2003). In fact, positrons are created in the interstellar medium when the atoms become interacted by the cosmic ray nuclei (Adrani et al. 2009). The annihilation, which takes place in the interaction of matter (electrons) and anti-matter (positrons), usually occurs at much longer characteristic time scales compared with the time in which the collective interaction between the charged particles takes place (Surko and Murphy 1990).

On the other hand, a great deal of attention has been devoted to the study of different types of collective processes and instabilities in electron-positron-ion (EPI) plasmas (Verga and Ferro Fontán 1984; Shukla et al. 1986;

Rizzato 1988; Tajima and Taniuti 1990; Popel et al. 1995; Jammalamadaka et al. 1996; Nejoh 1996; Lakhina and Verheest 1997; El-Taibany et al. 2008; Tribeche et al. 2009; Pakzad 2009; Mahmood and Ur-Rehman 2009; Tribeche 2010; El-Shamy and El-Bedwehy 2010; Alinejad 2010; El-Bedwehy and Moslem 2011). It is well known that when positrons are introduced into electron- ion plasma the response of the latter changes significantly. In contrast to the usual two component plasma, it has been observed that the nonlinear waves in plasmas having an additional positron component behave differently. Popel et al. (1995) investigated the nonlinear propagation of ion-acoustic solitons in a three-component plasma, whose constituents are electrons, ions, and positrons. They found that the presence of the positron component leads to the reduction of the soliton amplitude. Later, Nejoh (1996) considered a plasma consisting of electrons, positrons, and a beam of electrons to study one-dimensional propagation of large amplitude positron acoustic solitary waves (PASWs) associated with such plasma model. By using the pseudopotential approach, he identified the propagation domain of the allowable Mach numbers. The latter enlarges as the positron temperature increases. Recently, El-Shamy and El-Bedwehy (2010) studied the linear and nonlinear characteristics of electrostatic solitary waves propagating in magnetized EPI plasmas. They found that the inclusion of the background ions leads to a modification of the nonlinear characteristics of electrostatic solitary waves. On one hand, a number of investigators (Mace et al. 1991; El-Labany et al. 2005) exerted a lot of efforts for explaining the broadband electrostatic noise (a common wave activity in the plasma sheet boundary layer of the Earth's magnetotail region) as being solitary electron-acoustic structures with negative potential in two temperature electron plasma released. On the other hand, there are few investigators tried to studying the solitary structures with a positive potential. For example, Verheest et al. (2005) have showed that if hot electron inertia is retained, there exists a parameter range where positive potential solitary waves are formed, which can help in interpretation of several astrophysical. In analogue to their model (Verheest et al. 2005; Tribeche et al. 2009) reported that due to outflows of electron-positron plasma from pulsars entering an interstellar cold, low density electron-ion plasma, may form two-temperature positron electron-ion plasmas. Accordingly, Tribeche et al. (2009) have investigated nonlinear small-amplitude PASWs involving the dynamics of cold positrons in a four component plasma model consisting of two-temperature positron, isothermal electrons, and immobile ions. The domain of their allowed Mach numbers enlarges when a relative amount of hot positrons is added. Furthermore, Tribeche (2010) investigated the small amplitude positron double layer for the same theoretical model (Tribeche et al. 2009). Currently, the excitation, propagation, and interaction of solitary waves are important issues in our theoretical researches. The interesting features of the collision between solitary waves have been revealed; when two solitary waves approach closely, they interact, exchange their energies, and positions with each other and then separate off, regaining their original wave forms. Throughout the whole process of the collision, the solitary waves are remarkably stable entities, preserving their identities through interaction; the unique effect due to the collision is their phase shifts (Zabusky and Kruskal 1965). So, we focus our attention to study the phase shifts and the trajectories of the two solitary waves after a collision occurs. It is well known that, in a one dimensional system, the solitons may interact each other in two different ways. One is overtaking collision and it can be studied by the inverse scattering transformation method (Gardner et al. 1967). Another one is the head on collision (HOC) (Su and Miura 1980) where the angle between two propagation directions of two solitons is π. For such collision, the solution of two solitary waves, via two Korteweg-de Vries (KdV) equations, can explain resonance phenomena and these have been observed in shallow water wave experiments (Maxworthy 1980), in plasmas experiments (Lonngren 1998), in two core optical fiber (Tsang et al. 2004) and in fluid filled elastic tubes (Demiray 2005). For a HOC between two solitary waves traveling to positive and negative directions, one must search their evolution and so he needs to employ a suitable asymptotic expansion to solve original fluid equations. There are a number of investigations for the HOC of two solitary waves in different plasma models using extended Pioncaré-Lighthill-Kuo (PLK) method (Huang and Velarde 1996; Han et al. 2008a, 2008b; El-Labany et al. 2010a, 2010b). This method is a combination of the standard perturbation method with the technique of strained coordinates. The main idea of this method is as follows. In the limit of the long-wavelength approximation, asymptotic expansions for both the flow field variables and the spatial or time coordinates are used. This makes a uniformly valid asymptotic expansion (i.e., eliminates secular terms) and at the same time obtains the change of the trajectories (i.e., phase shifts) of the solitary waves after their collision. Recently, there are researchers (Han et al. 2008b; El-Shamy et al. 2009; El-Labany et al. 2010a, 2010b; Chatterjee et al. 2010) who have used the PLK method to investigate HOC for two nonlinear solitary waves in various plasma combinations. For instance, Han et al. (2008b) have studied the HOC of two ion-acoustic solitary waves in an EPI plasma. They illustrated that the temperature ratio of electron to positron, and the number density ratio of positrons to electrons have significant influence on the phase shifts of the produced solitons. El-Shamy (2010) has investigated the characteristics of the HOC between two electrostatic solitary waves in unmagnetized EPI plasmas. Moreover, El-Shamy illustrated that the positron-to-electron temperature ratio have strong effects on the phase shifts. In order to study the effect of superthermal distributed electrons, Chatterjee et al. (2010) have discussed the HOC of ion-acoustic solitary waves in an EPI plasma. Up to the best of our knowledge, the HOCs of PASWs have never been addressed in the plasma literature. Therefore, it is worthwhile to present a first study for the HOC of two electrostatic solitary waves (ESWs) corresponding to PASWs involving the dynamics of cold positron fluids in a four-component EPI space plasma.

This paper is organized in the following fashion: In Sect. 2, the basic set of fluid equations are presented, we derive the KdV equations for the EPI plasma. In addition, we will estimate both of the phase shifts and the trajectories after the interaction between two PAWs. Section 3 is devoted to the numerical results and the discussion.

2 Basic equations

Let us consider a four-component plasma consists of cold positrons, immobile positive ions, and Boltzmann distributed electrons and hot positrons of density n_{pc}, n_i, n_e, and n_{ph}, respectively. Thus, at equilibrium, we have $n_{e0} = n_{pc0} + n_{ph0} + n_{i0}$, where the subscript "0" stands for unperturbed quantities. The dynamics of one-dimensional positron oscillations is governed by the following normalized equations (Tribeche et al. 2009)

$$\frac{\partial n_{pc}}{\partial t} + \frac{\partial (n_{pc} u_{pc})}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_{pc}}{\partial t} + u_{pc} \frac{\partial u_{pc}}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \alpha n_e - n_{pc} - \beta n_{ph} - \alpha + \beta + 1, \tag{3}$$

where $j =$ pc, ph, i, and e denotes cold positrons, hot positrons, ions, and electrons, respectively, ϕ is the electrostatic potential, n_{pc} (u_{pc}) is the cold positron number density (velocity). $n_{i0}/n_{pc0} = \alpha - \beta - 1$. The hot positrons and the electrons are assumed to be in thermal equilibrium with the densities

$$n_{ph} = \exp[-\sigma \phi/(\alpha + \beta\sigma)], \tag{4}$$

$$n_e = \exp[\phi/(\alpha + \beta\sigma)]. \tag{5}$$

Here, the following normalization are used: $n_j = N_j/n_{j0}$, $u_{pc} = V_{pc}/\omega_{pc}\lambda_{Dp}$, $\phi = e\varphi/T_{eff}$, $x = X/\lambda_{Dp}$ and $t = T\omega_{pc}$, where $\omega_{pc} = \sqrt{4\pi e^2 n_{pc0}/m_p}$ and $\lambda_{Dp} = \sqrt{T_{eff}/4\pi e^2 n_{pc0}}$ with $T_{eff} = T_e/(\alpha + \beta\sigma)$, $\alpha = n_{e0}/n_{pc0}$, $\beta = n_{ph0}/n_{pc0}$ and $\sigma = T_e/T_{ph}$. T_e and T_{ph} represents, respectively, the electron and hot positron temperatures. m_p is the positron mass.

To investigate the collision of two ESWs, we consider two solitons S_1 and S_2 in the plasma, which are, asymptotically, far apart in the initial state and travel toward each other. After some time they interact, collide, and then depart. We also assume that these solitary waves have small amplitudes $\sim \varepsilon$ (where ε is a small, formal perturbation parameter characterizing the strength of nonlinearity) and the interactions between solitons are weak. Hence, we expect that the collision will be quasielastic so it will cause only shifts of the post-collision trajectories (phase shift). According to the extended PLK perturbation method, the dependent variables are expanded as Han et al. (2008b), El-Shamy (2010), El-Labany et al. (2010a, 2010b)

$$\psi = \Psi^{(0)} + \sum_{n=1}^{\infty} \varepsilon^{(n+1)} \Psi^{(n)}, \tag{6}$$

where $\psi = [n_{pc}, u_{pc}, \phi]$ and $\Psi^{(0)} = [1, 0, 0]$.

Now let us introduce the stretched coordinates as follows,

$$\xi = \varepsilon(x - \lambda t) + \varepsilon^2 P^{(0)}(\eta, \tau) + \varepsilon^3 P^{(1)}(\xi, \eta, \tau) + \cdots, \tag{7}$$

$$\eta = \varepsilon(x + \lambda t) + \varepsilon^2 Q^{(0)}(\xi, \tau) + \varepsilon^3 Q^{(1)}(\xi, \eta, \tau) + \cdots, \tag{8}$$

$$\tau = \varepsilon^3 t, \tag{9}$$

where ξ and η denotes the trajectories of two solitons traveling toward each other (i.e., to the right and to the left, respectively), λ is the unknown phase velocity of PASWs (to be determined later). The variables of $P^{(0)}(\eta, \tau)$ and $Q^{(0)}(\xi, \tau)$ will be calculated also.

Substituting (6)–(9) into (1)–(5) and equating the quantities with equal powers of ε, we obtain coupled equations in different orders of ε. To the leading order, we have

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) n_{pc}^{(1)} + \left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) u_{pc}^{(1)} = 0, \tag{10}$$

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) u_{pc}^{(1)} + \left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) \phi^{(1)} = 0, \tag{11}$$

$$-n_{pc}^{(1)} + \phi^{(1)} = 0. \tag{12}$$

Solving this system of (10)–(12), we get the following relations among these different physical quantities:

$$n_{pc}^{(1)} = \phi^{(1)} = \phi_1^{(1)}(\xi, \tau) + \phi_2^{(1)}(\eta, \tau), \tag{13}$$

$$u_{pc}^{(1)} = \phi_1^{(1)}(\xi, \tau) - \phi_2^{(1)}(\eta, \tau), \tag{14}$$

and with the solvability condition (i.e., the condition to obtain a uniquely defined $n_{pc}^{(1)}$ and $u_{pc}^{(1)}$ from (13) and (14) when $\phi^{(1)}$ is given by (13), the wave velocity can be estimated as $\lambda = 1$. It is remarked that the wave velocity in dimensional form can be expressed as $\bar{\lambda} = \sqrt{T_e/[m_p(\alpha + \beta\sigma)]}$.

It is obvious that the wave velocity is reduced as the density percentage of hot/cold positron, βincreases. Furthermore, the PASW velocity decreases as the temperature ratio of hot electron-to-positron, σ, increases.

However, the unknown functions $\phi_1^{(1)}(\xi,\tau)$ and $\phi_2^{(1)}(\eta,\tau)$ will be calculated from the next orders. Equations (13) and (14) imply that, at the leading order, we have two waves, one of which; $\phi_1^{(1)}(\xi,\tau)$, is traveling to the right, and the other one; $\phi_2^{(1)}(\eta,\tau)$, is traveling to the left. For the higher order, the solutions also have the following forms:

$$n_{pc}^{(2)} = \phi^{(2)} = \phi_1^{(2)}(\xi,\tau) + \phi_2^{(2)}(\eta,\tau), \tag{15}$$

$$u_{pc}^{(2)} = \phi_1^{(2)}(\xi,\tau) - \phi_2^{(2)}(\eta,\tau). \tag{16}$$

Furthermore, for the next order, we get

$$\begin{aligned}
-2\frac{\partial^2 u_{pc}^{(3)}}{\partial\xi\partial\eta} &= \frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1^{(1)}}{\partial\tau} + A\phi_1^{(1)}\frac{\partial\phi_1^{(1)}}{\partial\xi} + B\frac{\partial^3\phi_1^{(1)}}{\partial\xi^3}\right] \\
&+ \frac{\partial}{\partial\eta}\left[\frac{\partial\phi_2^{(1)}}{\partial\tau} - A\phi_2^{(1)}\frac{\partial\phi_2^{(1)}}{\partial\eta} - B\frac{\partial^3\phi_2^{(1)}}{\partial\eta^3}\right] \\
&+ \left(C\frac{\partial P^{(0)}}{\partial\eta} - D\phi_2^{(1)}\right)\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2} \\
&- \left(C\frac{\partial Q^{(0)}}{\partial\xi} - D\phi_1^{(1)}\right)\frac{\partial^2\phi_2^{(1)}}{\partial\eta^2}.
\end{aligned} \tag{17}$$

Integrating (17) with respect to the variables ξ and η, yields

$$\begin{aligned}
-2u_{pc}^{(3)} &= \int\left[\frac{\partial\phi_1^{(1)}}{\partial\tau} + A\phi_1^{(1)}\frac{\partial\phi_1^{(1)}}{\partial\xi} + B\frac{\partial^3\phi_1^{(1)}}{\partial\xi^3}\right]d\eta \\
&+ \int\left[\frac{\partial\phi_2^{(1)}}{\partial\tau} - A\phi_2^{(1)}\frac{\partial\phi_2^{(1)}}{\partial\eta} - B\frac{\partial^3\phi_2^{(1)}}{\partial\eta^3}\right]d\xi \\
&+ \int\int\left(C\frac{\partial P^{(0)}}{\partial\eta} - D\phi_2^{(1)}\right)\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2}d\xi\, d\eta \\
&- \int\int\left(C\frac{\partial Q^{(0)}}{\partial\xi} - D\phi_1^{(1)}\right)\frac{\partial^2\phi_2^{(1)}}{\partial\eta^2}d\xi\, d\eta,
\end{aligned} \tag{18}$$

where

$$A = \frac{1}{2}\left[3 + \frac{\beta\sigma^2 - \alpha}{(\beta\sigma + \alpha)^2}\right],$$

$$B = \frac{1}{2}, \qquad C = 2 \quad \text{and} \quad D = 2 - A.$$

It is noted here that the first (second) term in the right hand side of (18) will be proportional to $\eta(\xi)$ because the integrated function is independent of $\eta(\xi)$, respectively. Thus the first two terms of (18) are all secular terms, which must be eliminated in order to avoid spurious resonances. Hence we have

$$\frac{\partial\phi_1^{(1)}}{\partial\tau} + A\phi_1^{(1)}\frac{\partial\phi_1^{(1)}}{\partial\xi} + B\frac{\partial^3\phi_1^{(1)}}{\partial\xi^3} = 0, \tag{19}$$

$$\frac{\partial\phi_2^{(1)}}{\partial\tau} - A\phi_2^{(1)}\frac{\partial\phi_2^{(1)}}{\partial\eta} - B\frac{\partial^3\phi_2^{(1)}}{\partial\eta^3} = 0. \tag{20}$$

The third and the fourth terms in the R.H.S. of (18) are not secular terms in this order, but they will become secular in the next order (Han et al. 2008b; El-Labany et al. 2010a, 2010b; Chatterjee et al. 2010). Hence we have

$$C\frac{\partial P^{(0)}}{\partial\eta} = D\phi_2^{(1)}, \tag{21}$$

$$C\frac{\partial Q^{(0)}}{\partial\xi} = D\phi_1^{(1)}. \tag{22}$$

Equations (19) and (20) are the two-side traveling wave solutions corresponding to these two KdV equations in the reference frames of ξ and η, respectively. Their solutions are

$$\phi_1^{(1)} = \phi_1\,\mathrm{sech}^2\left[\sqrt{\frac{A\phi_1}{12B}}\left(\xi - \frac{A}{3}\phi_1\tau\right)\right], \tag{23}$$

$$\phi_2^{(1)} = \phi_2\,\mathrm{sech}^2\left[\sqrt{\frac{A\phi_2}{12B}}\left(\xi + \frac{A}{3}\phi_2\tau\right)\right], \tag{24}$$

where ϕ_1 and ϕ_2 are the amplitudes of the two solitons S_1 and S_2 in their initial positions. The leading phase which released due to the collision can be calculated from (21) and (22). So, they are given as,

$$P^{(0)}(\eta,\tau) = \frac{D}{C}\sqrt{\frac{12B\phi_2}{A}} \times\left[\tanh\sqrt{\frac{A\phi_2}{12B}}\left(\eta + \frac{A}{3}\phi_2\tau\right) + 1\right], \tag{25}$$

$$Q^{(0)}(\eta,\tau) = \frac{D}{C}\sqrt{\frac{12B\phi_1}{A}} \times\left[\tanh\sqrt{\frac{A\phi_1}{12B}}\left(\xi - \frac{A}{3}\phi_1\tau\right) - 1\right]. \tag{26}$$

Therefore, up to $O(\varepsilon^2)$, the trajectories of the two solitary waves for weak HOCs are

$$\xi = \varepsilon(x - \lambda t) + \varepsilon^2\frac{D}{C}\sqrt{\frac{12B\phi_2}{A}} \times\left[\tanh\sqrt{\frac{A\phi_2}{12B}}\left(\eta + \frac{A}{3}\phi_2\tau\right) + 1\right] + \cdots, \tag{27}$$

$$\eta = \varepsilon(x + \lambda t) + \varepsilon^2\frac{D}{C}\sqrt{\frac{12B\phi_1}{A}} \times\left[\tanh\sqrt{\frac{A\phi_1}{12B}}\left(\xi - \frac{A}{3}\phi_1\tau\right) - 1\right] + \cdots. \tag{28}$$

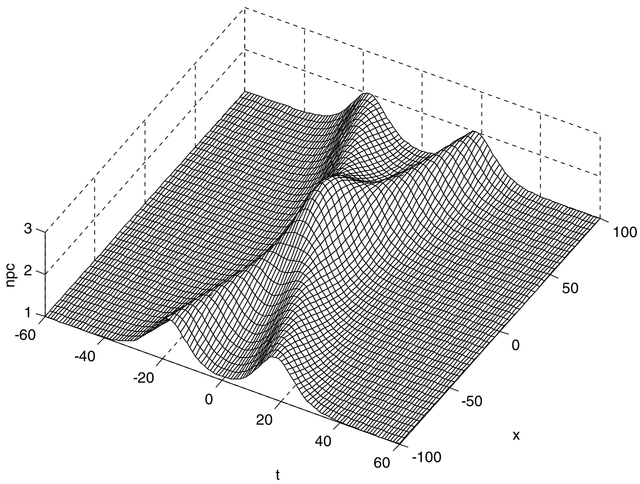

Fig. 1 The variation of n_{pc} with $\beta = 0.05, \alpha = 3, \sigma = 2, \varepsilon = 0.1$ and $\phi_1 = \phi_2 = 1$

To obtain the phase shifts after a HOC of these two solitons, we assume that the two solitons; S_1 and S_2 are, asymptotically, far from each other at the initial time ($t = -\infty$), i.e., soliton S_1 is at $\xi = 0, \eta = -\infty$ and soliton S_2 is at $\eta = 0, \xi = +\infty$, respectively. After the collision ($t = +\infty$), the soliton S_1 is far to the right of soliton S_2 , i.e., soliton S_1 is at $\xi = 0, \eta = +\infty$ and soliton S_2 is at $\eta = 0, \xi = -\infty$. Using (27) and (28), we obtain the corresponding phase shifts $\Delta P^{(0)}$ and $\Delta Q^{(0)}$ as follows (El-Shamy 2010):

$$\Delta P^{(0)} = -2\varepsilon^2 \frac{D}{C}\sqrt{\frac{12B\phi_2}{A}}, \tag{29}$$

$$\Delta Q^{(0)} = 2\varepsilon^2 \frac{D}{C}\sqrt{\frac{12B\phi_1}{A}}. \tag{30}$$

3 Numerical investigations and conclusions

Let us now illustrate the dependence of the soliton phase shift, released after HOC occurred, on the system physical parameter variations. For the case at hand, soliton S_1 is traveling to the right and soliton S_2 is traveling to the left, we see from (29) and (30) that due to collision, each soliton has a negative phase shift in its traveling direction. Physically, the negative phase shift means that PASWs reduce their velocities during the collision stage. The magnitudes of the phase shifts which are related to the physical parameters, i.e., $\varepsilon, \alpha, \sigma, \beta, \phi_1$, and ϕ_2 are calculated. Before going to the discussion, it should be mentioned here we have used typical physical parameter values of EPI space plasmas (pulsar environments) in the present numerical investigations. Proceeding through (23) and (24), one can obtain the cold positron density up to $O(\varepsilon^2)$ order from (6) and (13). Figure 1 demonstrates the variation of the cold positron density

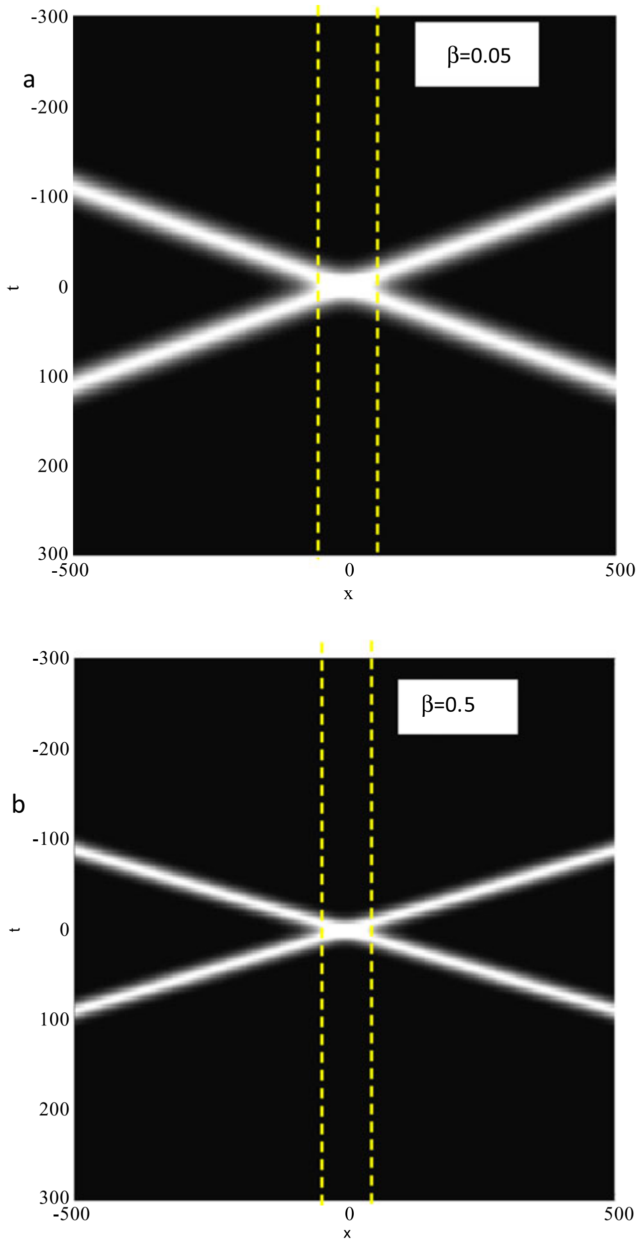

Fig. 2 (Color online) Space-time plots of two colliding PASWs. Degree of brightness indicates n_{cp}. The *vertical lines* represent the begin and end spaces of the collision, when the solitons appear merged together. The selected parameters are $\varepsilon = 0.1$ and $\phi_1 = \phi_2 = 1, \alpha = 1, \sigma = 1$ and (**a**) $\beta = 0.05$ and (**b**) $\beta = 0.5$

n_{pc} against the space coordinate x and the time variable t. Moreover, Fig. 1 illustrates that two compressive PASWs propagate in opposite directions approach to each other, collide, and asymptotically separate away. We observe that during collision one practically motionless composite structure forms for some time interval appeared. After the collision, it is easy to find that the compressive PASWs propagate along the trajectories deviated from the initial trajectories. In fact, these deviations are just the phase shifts for two PASWs. Now, we try to discuss the effects of physical pa-

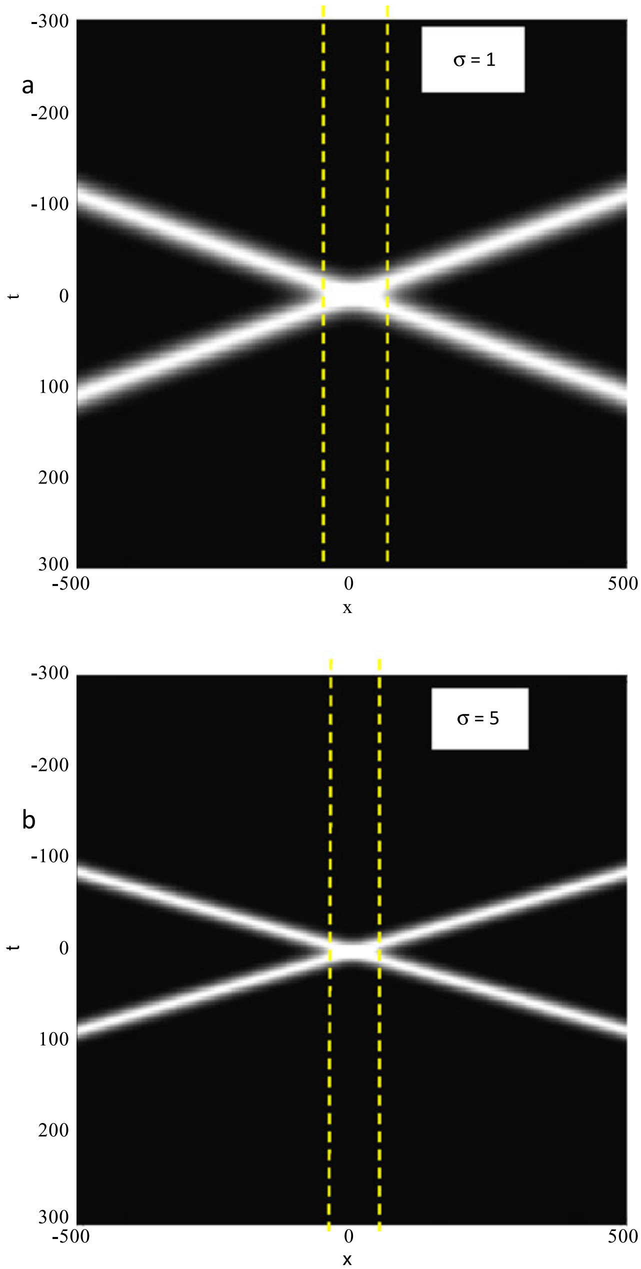

Fig. 3 (Color online) Space-time plots of two colliding PASWs for two different values of σ. The selected parameters are the same as of Fig. 2 with $\beta = 0.05$ and for $\sigma = 1$ in panel (**a**) but for $\sigma = 5$ in panel (**b**)

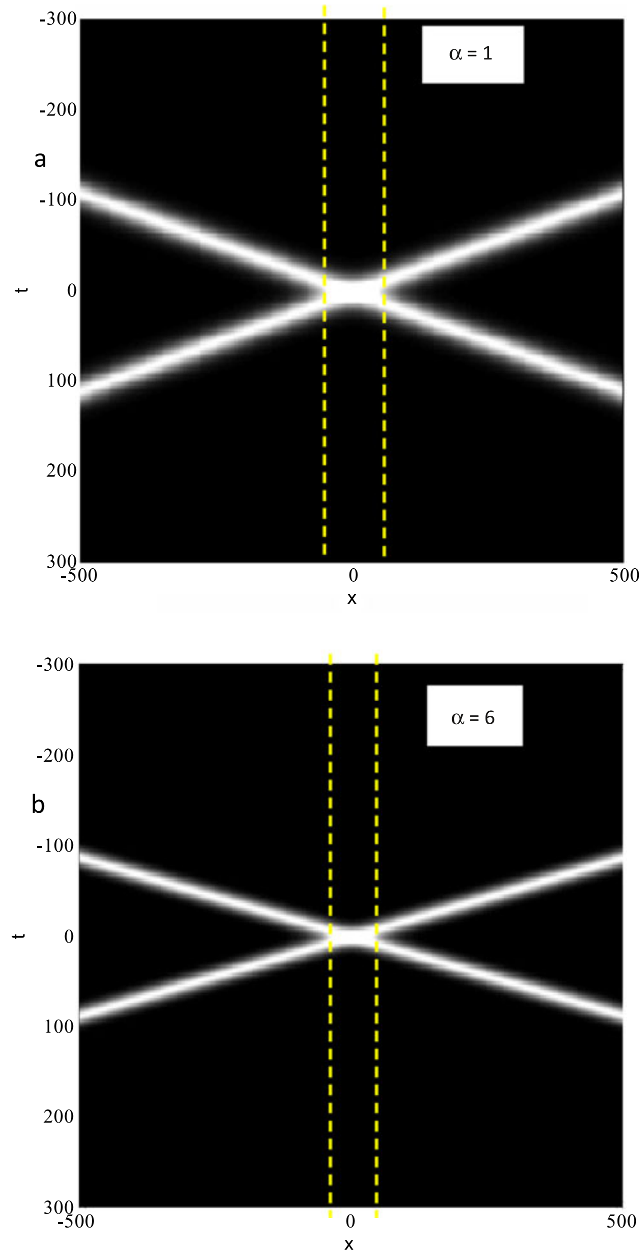

Fig. 4 (Color online) Space-time plots of two colliding PASWs for two different values of α. Here $\beta = 0.05$ and $\alpha = 1$ in panel (**a**) but for $\alpha = 6$ in panel (**b**). The remainder parameters are the same as of Fig. 2

rameter changes on the phase shifts of PASWs after collision. In Figs. 2, 3, 4, we illustrate the space-time plots of two PASWs collision and their phase shifts. The vertical lines represent the begin and end spaces of the collision, when the solitons appear merged together. This space is a measure of the propagation delay (negative phase shift) of PASWs after collision. Figures 2–4 show that the phase shifts decrease as β, σ and α increase, respectively. In other words, the increase of the cold positron number density results as an increment of the solitary wave phase shift. Moreover, the soliton phase shift increases as the temperature ratio of hot electron-to-positron increases. Therefore, we can conclude that the cold/hot positron physical parameters (number density and temperature) have significant influence on the phase shifts and the trajectories (see (27) and (28)) of two PASWs after collision. Introducing a small percentage of hot positrons leads to reduction in phase shifts. This response can be managed by controlling the hot positron temperature.

The current study presents a first investigation for the HOC of two PASWs in an EPI plasma. It should be mentioned here the current findings would be applied to different regions of space. For example, we recall that when a high energy cosmic ray interacts with the Earth's atmosphere, it may produce an electron-positron pair with enormous velocities. The data obtained during the alpha magnetic spectrometer flight permitting to probe the radiation belts in the Earth's innermost magnetosphere provided an evidence of the presence of positrons (Fiandrini et al. 2004; Plyaskin 2008). A recent study on the electron acoustic solitary waves revealed that the results of the EPI plasmas agree very well with the satellite observations in the auroral acceleration region. On another side, our results should help to understand the localized structures that may occur in laboratory plasmas as new sources of colds positrons which are now available and well developed (Abdullah et al. 1995; Kurz et al. 1998; Greaves et al. 2002).

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