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ORIGINAL ARTICLE

Rogue wave in Titan's atmosphere

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Abstract Rogue wave in a collisionless, unmagnetized electronegative plasma is investigated. For this purpose, the basic set of fluid equations is reduced to the Korteweg-de Vries (KdV) equation. However, when the frequency of the carrier wave is much smaller than the ion plasma frequency then the KdV equation is also used to study the nonlinear evolution of modulationally unstable modified ion-acoustic wavepackets through the derivation of the nonlinear Schrödinger (NLS) equation. In order to show that the characteristics of the rogue wave is influenced by the plasma parameters, the relevant numerical analysis of the NLS equation is presented. The relevance of our investigation to the Titan's atmosphere is discussed.

Keywords Plasma · Rogue wave · Titan atmosphere · Envelope solitons

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1 Introduction

The nonlinear Schrödinger (NLS) equation is considering one of the most important equations which governs the movement of the nonlinear structures in many branches of physics ranging from condensed matter, nonlinear optics, plasma, and even biophysics (see e.g., Davydov 1985; Hasegawa 1989; Infeld and Rowlands 1990; Remoissenet 1994; Sulem and Sulem 1999). One of the solutions of the NLS equation is the rational solution that could describe the rogue wave propagation. The latter is been a part of the marine folklore for centuries, while oceanographers did not believe in their existence (Ma 2010). Actually, the first measurement of the rogue wave is taken on the oil platform in Norway in 1995 (Müller et al. 2005). The importance of the rogue wave in the study of the ocean waves are due to the fact that the amplitude of the rogue wave can reach more than twice the value of the surrounding chaotic waves (Bludov et al. 2010). Actually, its appearance can not be predicted, so this wave represents a real danger on the ships and boats and thus many studies dealt with the ocean rogue wave (Janssen 2003; Khaarif et al. 2009; Osborne 2009; Osborne et al. 2009). Understanding the origin of the rogue wave appearing in systems characterized by many waves is currently a matter of debate (Ruban et al. 2010). Clearly, we cannot do much if we leave the creation of rogue wave to chance. So, preparing special initial conditions and understanding the features of rogue wave could be useful either to avoid it or to generate highly energetic pulses. Recently the term of the rogue wave appears in many areas of physics like optical fibers and nonlinear optics (Solli et al. 2007; Yeom and Eggleton 2007; Genty et al. 2010; Bludov et al. 2009a), superfluids (Ganshin et al. 2008), capillary waves (Shatz et al. 2010), finance (Yan 2010), Bose-Einstein condensates (Bludov et al. 2009b), astrophysical environments (Moslem 2011; El-Awady and Moslem 2011), and atmosphere (Stenflo and Marklund 2010).

For more than three decades, negative ion plasmas (electronegative plasma) attracted attention because of their importance in both laboratory (Jacquinot et al. 1977) and industrial use, such as in plasma processing reactors (Gottscho and Gaebe 1986), in neutral beam source (Bascal and Hamilton 1979), and in plasma etching. Also, the electronegative plasmas appear in the universe in some plasma environments, such as in the Earth's ionosphere (Massey 1976). Many theoretical efforts have been paid to understand the basic features of the electronegative plasmas in different systems (see e.g., Das and Singh 1991; El-Labany et al. 2000; Tribeche and Benzekka 2011; El-Shewy et al. 2011; Pakzad 2011; El-Shewy 2011; El-Tantawy and Moslem 2011). Chaizy et al. (1991) reported the presence of the negative ions in the coma of comet Halley, which can easily be destroyed by the solar radiation and they predicted the presence of the negative ions in similar neutral gas and dust environments farther away from the sun like Jupiter's or Saturn's magnetospheres and that negative ions will play a role in a physical processes such as radiative transfer or charge exchange. Recently, Coates et al. (2007) reported the presence of the negative ions in Titan's atmosphere. These ions are presented in a mass groups with high densities, and it is expected that these ions have an important role in the ion chemistry and in forming organic-rich aerosols, which are falling on the surface.

In this paper, we study the ion-acoustic (IA) rogue wave in a positive-negative ion plasma with electrons obeying isothermal distribution. The paper is organized as follows: in Sect. 2 the basic set of fluid equations describing the system is reduced to the Korteweg-de Vries (KdV) equation. When the frequency of the carrier wave is much smaller than the ion plasma frequency, then the KdV equation is transformed to the NLS equation. In Sect. 3, the NLS equation is solved analytically and the effects of the plasma parameters are investigated numerically on the IA rogue electrostatic perturbation. Finally, the results are summarized in Sect. 4.

2 Model equations and derivation of the NLS equation

We consider a nonlinear propagation of the IA rogue electrostatic perturbation mode in a collisionless, unmagnetized, cold plasma consisting of positive and negative ions, as well as isothermal electrons. The nonlinear dynamics of such electrostatic perturbation mode is described by

$$\frac{\partial n_{p,n}}{\partial t} + \frac{\partial}{\partial x}(n_{p,n} u_{p,n}) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + u_p \frac{\partial}{\partial x}\right) u_p + \frac{\partial \varphi}{\partial x} = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + u_n \frac{\partial}{\partial x}\right) u_n - s\frac{\partial \varphi}{\partial x} = 0, \tag{3}$$

and

$$\frac{\partial^2 \varphi}{\partial x^2} - n_e - n_n + n_p = 0. \tag{4}$$

In (1)–(4), the subscripts e, p, and n stand for electrons, positive ions and negative ions, respectively. All the densities are normalized with respect to $n_p^{(o)}$, which is the unperturbed density of the positive ions. The velocities u_p and u_n are normalized with respect to the ion-sound speed $C_{si} = (T_e/m_p)^{1/2}$. The potential φ is normalized by thermal potential T_e/e, the space and time are normalized by positive ion Debye length $\lambda_{Dp} = (T_e/4\pi e^2 n_p^{(o)})^{1/2}$ and positive ion plasma period $\omega_{pp}^{-1} = (m_p/4\pi e^2 n_p^{(o)})^{1/2}$, respectively. Here, e the electron charge, T_e the electrons temperature in electron volt, and $s = m_p/m_n$ the positive-to-negative ions mass ratio. At equilibrium, the quasineutrality condition reads; $N_e = 1 - N_n$, where $N_e = n_e^{(o)}/n_p^{(o)}$ and $N_n = n_n^{(o)}/n_p^{(o)}$ are the unperturbed densities ratios of electrons and negative ions to the unperturbed positive ions density. The electrons obey the Boltzmann distribution as

$$n_e = N_e \exp(\varphi). \tag{5}$$

To examine the one-dimensional electrostatic perturbations propagating in our positive-negative plasma, we analyze the outgoing solutions of (1)–(5) by introducing the stretched coordinates (Washimi and Taniuti 1966)

$$\xi = \epsilon^{1/2}(x - \lambda t) \text{ and } \tau = \epsilon^{3/2} t, \tag{6}$$

where λ is the phase velocity to be determined later and ϵ is a small parameter ($\epsilon < 1$). Furthermore, we expand all the physical quantities in (1)–(4) as follows

$$n_p = 1 + \epsilon n_{p_1} + \epsilon^2 n_{p_2} + \epsilon^3 n_{p_3} + \cdots, \tag{7}$$

$$n_n = N_n + \epsilon n_{n_1} + \epsilon^2 n_{n_2} + \epsilon^3 n_{n_3} + \cdots, \tag{8}$$

$$u_{p,n} = \epsilon u_{p_1,n_1} + \epsilon^2 u_{p_2,n_2} + \epsilon^3 u_{p_3,n_3} + \cdots, \tag{9}$$

$$\varphi = \epsilon\varphi_1 + \epsilon^2\varphi_2 + \epsilon^3\varphi_3 + \cdots. \tag{10}$$

Substituting the stretching (6) and the expansions (7)–(10) into the basic equations (1)–(4), we obtain, for the lowest-order of ϵ, the following relations:

$$\lambda^2 n_{p_1} = \lambda u_{p_1} = \varphi_1, \tag{11}$$

$$-\frac{\lambda^2 n_{n_1}}{sN_n} = -\frac{\lambda u_{n_1}}{s} = \varphi_1, \tag{12}$$

and the compatibility condition gives

$$\lambda = \left(\frac{1+sN_n}{N_e}\right)^{1/2}. \tag{13}$$

The next-order of ϵ gives a system of equations in the second-order perturbed quantities. Eliminating these quantities and with the aid of (11)–(13), we obtain the KdV equation

$$\frac{\partial\phi}{\partial\tau} + A\phi\frac{\partial\phi}{\partial\xi} + B\frac{\partial^3\phi}{\partial\xi^3} = 0. \tag{14}$$

For simplicity, we have assumed that $\varphi_1 \equiv \phi$. The nonlinear and dispersion coefficients are given as

$$A = \frac{\lambda}{2(1+sN_n)}\left[\frac{3}{\lambda^2}\left(1-s^2N_n\right) - N_e\lambda^2\right], \tag{15}$$

and

$$B = \frac{\lambda}{2N_e}. \tag{16}$$

Now, we will study the modulational instability of a weakly nonlinear wave packet described by the KdV (14). We consider the solution of (14) in the form of a weakly modulated sinusoidal wave by expanding ϕ as (Taniuti and Yajima 1969; Asano et al. 1969; Shimizu and Ichikawa 1972; El-Labany et al. 2007)

$$\phi(\xi,\tau) = \sum_{n=1}^{\infty}\varepsilon^n \sum_{l=-\infty}^{\infty}\phi_l^{(n)}(X,T)\exp il(k\xi-\omega\tau), \tag{17}$$

where k is the carrier wavenumber and ω is the frequency for the given dust-ion-acoustic wave. The stretched variables X and T are

$$X = \varepsilon(\xi+\Lambda\tau) \quad \text{and} \quad T = \varepsilon^2\tau, \tag{18}$$

where Λ is the group velocity, which will be determined later.

Assume that all perturbed states depend on the fast scales via the phase $(k\xi-\omega\tau)$ only, while the slow scales (X,T) enter the arguments of the lth harmonic amplitude $\phi_l^{(n)}$. Since $\phi(\xi,\tau)$ must be real, the coefficients in (17) have to satisfy the condition $\phi_{-l}^{(n)} = \phi_l^{(n)*}$, where the asterisk indicates the complex conjugate. The derivative operators appearing in the system of the basic equations become

$$\begin{aligned}&\frac{\partial}{\partial\tau} \longrightarrow \frac{\partial}{\partial\tau} + \varepsilon\Lambda\frac{\partial}{\partial X} + \varepsilon^2\frac{\partial}{\partial T} \quad \text{and}\\ &\frac{\partial}{\partial\xi} \longrightarrow \frac{\partial}{\partial\xi} + \varepsilon\frac{\partial}{\partial X}.\end{aligned} \tag{19}$$

Using (17)–(19) into (14), we obtain

$$\begin{aligned}&\Bigg[-il\omega\phi_l^{(n)} + \Lambda\frac{\partial\phi_l^{(n-1)}}{\partial X} + \frac{\partial\phi_l^{(n-2)}}{\partial T}\\ &\quad + A\sum_{n'=1}^{\infty}\sum_{l'=-\infty}^{\infty}\left(ilk\phi_l^{(n)}\phi_{l-l'}^{(n-n')} + \phi_{l-l'}^{(n-n'-1)}\frac{\partial\phi_l^{(n)}}{\partial X}\right)\\ &\quad + B\bigg(-il^3k^3\phi_l^{(n)} - 3l^2k^2\frac{\partial\phi_l^{(n-1)}}{\partial X}\\ &\quad + 3ilk\frac{\partial^2\phi_l^{(n-2)}}{\partial X^2} + \frac{\partial^3\phi_l^{(n-3)}}{\partial X^3}\bigg)\Bigg] = 0.\end{aligned} \tag{20}$$

The first-order approximation $(n=1)$ with $(l=1)$ yields the linear dispersion relation

$$\omega = -Bk^3. \tag{21}$$

For the first harmonic $(l=1)$ of the second-order approximation $(n=2)$, we find that

$$\Lambda = 3Bk^2, \tag{22}$$

which corresponds to the group velocity. For the second harmonic $(l=2)$, we have

$$\phi_2^{(2)} = \left(A/6Bk^2\right)\phi_1^{(1)2}, \tag{23}$$

whereas for the zeroth harmonic $(l=0)$, we obtain

$$\phi_0^{(2)} = (-A/\lambda)\mid\phi_1^{(1)}\mid^2. \tag{24}$$

Proceeding to the third-order approximation $(n=3)$ and solving for the first harmonic equations $(l=1)$, an explicit compatibility condition will be found, from which we can easily obtain the NLS equation

$$i\frac{\partial\Phi}{\partial T} + \frac{1}{2}P\frac{\partial^2\Phi}{\partial X^2} + Q|\Phi|^2\Phi = 0. \tag{25}$$

For simplicity, we have assumed $\phi_1^{(1)} \equiv \Phi$. The dispersion coefficient P and the nonlinear coefficient Q are given by

$$P = 6Bk, \tag{26}$$

and

$$Q = \frac{A^2}{6Bk}. \tag{27}$$

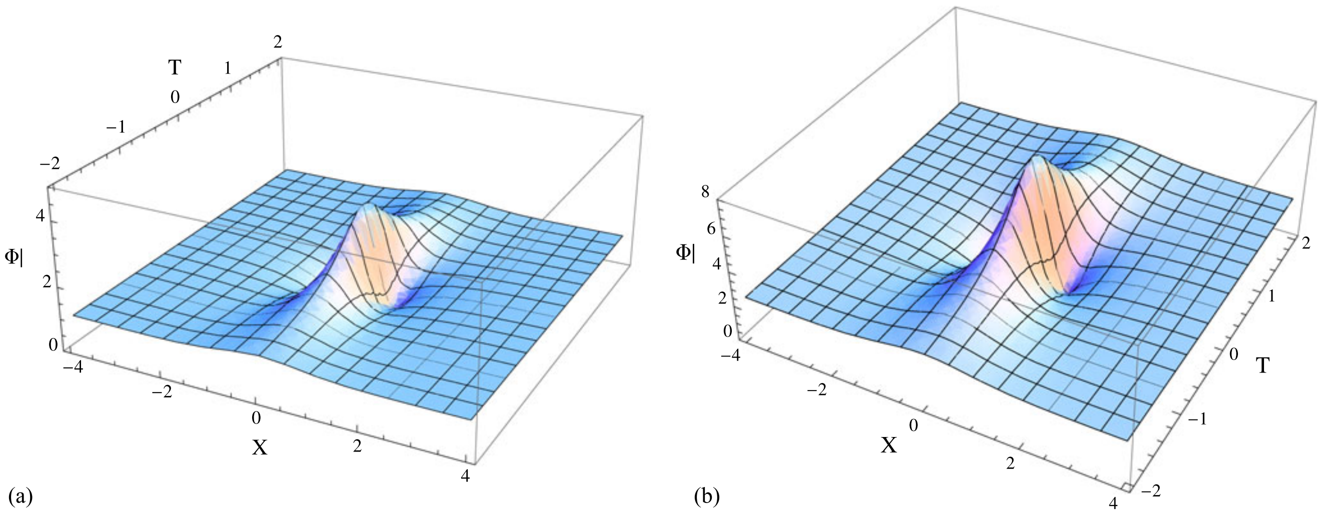

Fig. 1 The absolute of the rogue wave profile Φ for (**a**) $n_n^{(o)} = 1\ \mathrm{cm}^{-3}$ and (**b**) $n_n^{(o)} = 242.1\ \mathrm{cm}^{-3}$. Here, $m_p = 100$ amu, $n_e^{(o)} = 1000\ \mathrm{cm}^{-3}$, and $m_n = 200$ amu

The NLS equation (25) describes the nonlinear evolution of an amplitude modulated IA wave carrier. It can be also derived directly from the system of (1)–(5). In this case, the derivation of the NLS equation has been carried out for an arbitrary frequency of the carrier wave. If we use a similar approach to derive the NLS equation form (1)–(5), then we obtain an equation, but with much more complicated expressions for the nonlinear and dispersion coefficients P and Q. This equation should, in principle, reduce to (25) in the limit of the low wave frequency, i.e. when the frequency of the carrier wave is much smaller than the ion plasma frequency (Ruderman et al. 2008).

3 Rogue wave solution and discussion

The NLS (25) has a rational solution that is located on a nonzero background and localized both in the X and T directions (Ankiewicz et al. 2009; Moslem 2011) as

$$\Phi = \frac{1}{\sqrt{Q}}\left[\frac{4(1+2iT)}{1+4T^2+\frac{4}{P}X^2} - 1\right]e^{iT}, \tag{28}$$

The solution (28) reveals that a significant amount of the IA wave energy is concentrated into a relatively small area in space. This property of the nonlinear solution may serve as the basis for the explanation of the IA rogue wave in positive-negative plasmas. The rogue wave is usually an envelope of a carrier wave with a wavelength smaller than the central region of the envelope.

It is straightforward to see that a negative sign for PQ is required for wave amplitude modulational stability. On the other hand, a positive sign of PQ allows for a random perturbation of the amplitude to grow and thus the IA rogue wave could be created.

Firstly, it is necessary to give a glimpse of the conceptual, as well as a brief description of the observation by Coates et al. (2007). Actually, Titan has a thick atmosphere, principally nitrogen with a few percent by number density of methane. Observations by the Cassini spacecraft have shown that its ionosphere contains a rich positive ion population. The work of Coates et al. (2007) discovered the heavy negative ions in the Titan's upper atmosphere. These ions, with densities up to 100 cm^3, are in mass groups of 10–30, 30–50, 50–80, 80–110, 110–200 and 200+ amu/charge. It is expected that the negative ions play a role in the properties of the electrostatic perturbations that propagate in this region, which is the motives of our study. Therefore, the effects of the mass ratio and the density of the negative ions will investigate on the behavior of the IA rogue wave in the Titan's upper atmosphere.

Now, we numerically analyze the wave envelope Φ and investigate how the negative ion density and mass change the profile expressed by the wave envelope Φ of the IA rogue wave. Figures 1 and 2 depict the profile of the rogue pulses with the negative ions density and mass, respectively. It is seen that increasing the negative ions density would lead to the amplitude to be enhanced two folds. However, the rogue wave amplitude decreases with the increase of the negative ion mass (i.e., decreasing the mass ratio s). Physically, increasing the negative ions density would lead to enhance the nonlinearity and then concentrate a significant amount of energy which makes the pulses taller, but the increasing of the negative ions masses lead to dissipate the energy from the system and reduce the nonlinearity that makes the pulses shorter. Furthermore, it is found numerically that there is a critical value of the negative ions mass (M_c), at which the nonlinear coefficient A vanishes that would lead to disappear the rogue wave. The critical negative ion mass (M_c)

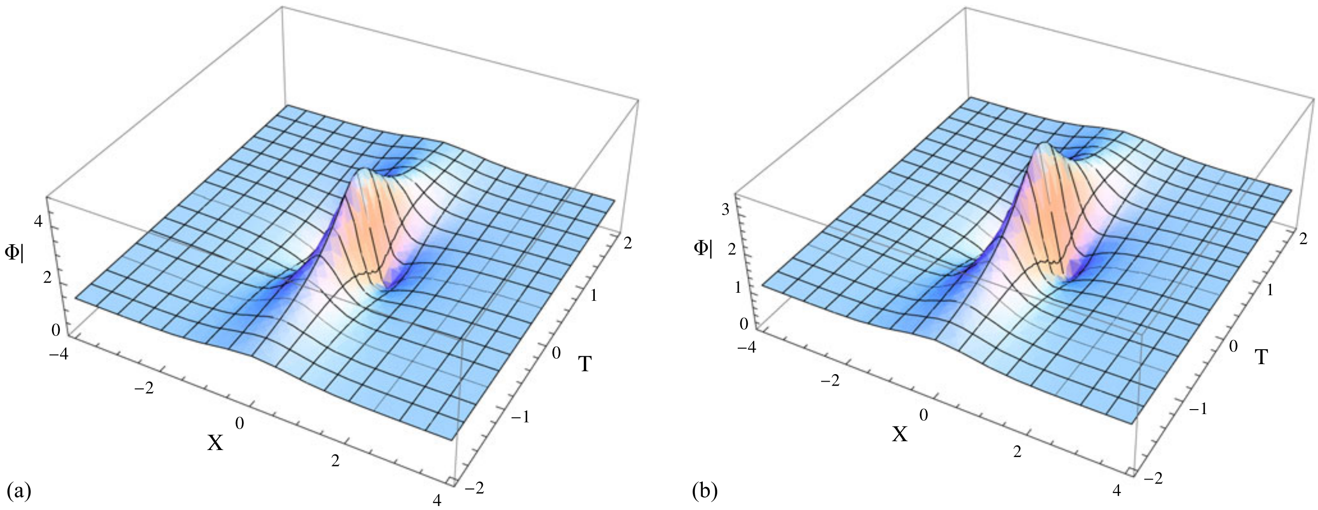

Fig. 2 The absolute of the rogue wave profile Φ for (**a**) $m_n = 30$ amu and (**b**) $m_n = 80$ amu. Here $m_p = 100$ amu, $n_e^{(o)} = 1000\ \mathrm{cm}^{-3}$, and $n_n^{(o)} = 7.815\ \mathrm{cm}^{-3}$

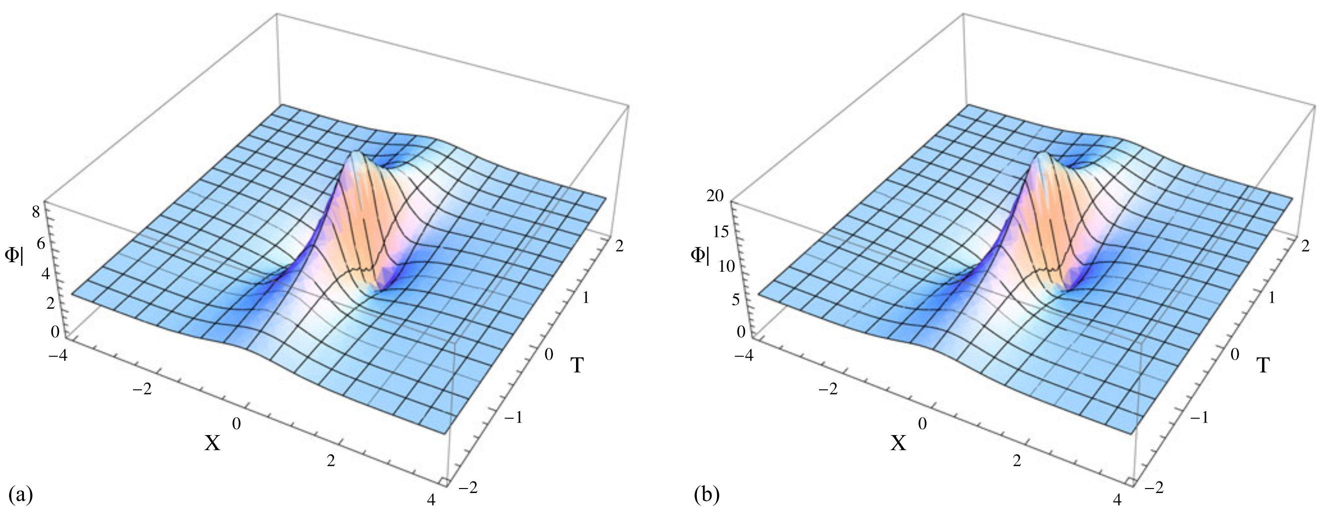

Fig. 3 The absolute of the rogue wave profile Φ for (**a**) $m_n = 10$ amu and (**b**) $m_n = 11$ amu. Here, $m_p = 100$ amu, $n_e^{(o)} = 1000\ \mathrm{cm}^{-3}$, and $n_n^{(o)} = 8.99\ \mathrm{cm}^{-3}$

lies within the mass group 10–30 amu at density 6.64 cm^{-3} and 8.99 cm^{-3}. It is interesting to examine the behavior of the rogue wave near M_c as depicted in Fig. 3. First, for negative ion masses less than M_c, it is obvious that the rogue wave amplitude increases three folds, but for negative ion masses larger than M_c the wave has similar behavior as in Fig. 2. Therefore, the negative ions mass and density play significant roles for maximizing/minimizing of the IA rogue wave energy, as well as in the excitence of rogue wave.

4 Summary

To summarize, we have investigated the properties of the nonlinear IA rogue wave in a three-component plasma composed of positive ions, negative ions, and isothermal electrons. It is found that at certain conditions the modulated IA wavepackets appear in the form of IA rogue wave. The dependence of the rogue wave on negative ions plasma parameters is numerically examined. It is found that within a certain negative ion mass the rogue wave cannot propagate. Also, the negative ions mass and density play significant role in deciding how much energy could be concentrated in the rogue pulses. On the other hand, increasing the negative ions density would lead to the amplitude to be enhanced two folds. In general, the negative ions mass would lead to dissipate the energy and decrease the rogue wave amplitude, but below critical negative ion mass M_c the rogue wave amplitude increases three folds that indicates that the rogue waves gain a significant amount of energy. The present results may be useful in understanding the basic features of the rogue IA perturbations that may explore in the future in the Titan's atmosphere.

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