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ORIGINAL ARTICLE

Nonextensive collisioneless dust-acoustic shock waves in a charge varying dusty plasma

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Abstract Nonlinear dust-acoustic (DA) shock waves are addressed in a nonextensive dusty plasma exhibiting self-consistent nonadiabatic charge variation. Our results reveal that the amplitude, strength and nature of the DA shock waves are extremely sensitive to the degree of ion nonextensivity. Significant differences in the potential function occur for very small changes in the value of the nonextensive parameter. Stronger is the ions correlation, more important is the charge variation induced nonlinear wave damping.

Keywords Dusty plasmas · Dust acoustic waves · Shock waves · Anomalous damping · Nonadiabatic charge variation · Nonextensive theory

1 Introduction

Nonlinear phenomena in dusty plasma attracted much attention during the last two decades. A dusty plasma is a three component plasma consisting of electrons, ions, and very massive solid grains. Dusty plasma coexist in a wide variety of cosmic and laboratory environments. It is ubiquitous in different parts of our solar system, namely, in planetary rings, in the interplanetary medium, in cometary comae and tails, in asteroid zones, in the Earth's ionosphere and magnetosphere, in interstellar molecular clouds (Verheest 2000; Shukla and Mamun 2002)... etc. Beside these, dust particles have been observed in low temperature plasmas, like those used in plasma processing and plasma crystal. Unique and novel features of dusty plasmas when compared with the usual electron-ion plasmas are the existence of a new, ultra-low frequency regime for wave propagation and the highly charging of the grains which can fluctuate due to the collection of plasma currents onto the dust surface. Dust grains become charged due to different processes, such as collection of charged particles from the surrounding plasma, photoionization, secondary electron emission, sputtering by energetic ions, etc. There has been a great deal of interest in understanding different types of collective processes in dusty plasmas (Goertz 1989; Mendis and Rosenberg 1994; Horanyi 1996; Lin and Zhang 2007; Mamun and Cairns 2009; Mamun and Shukla 2009; Mamun et al. 2009; Pakzad 2009, 2010; Alinejad 2010, 2011a, 2011b, 2011c; El-Labany et al. 2010; Shalaby et al. 2010; Barman and Talukdar 2011; Eslami et al. 2011; Mayout and Tribeche 2011; Tribeche and Benzekka 2011). It has been shown both theoretically and experimentally that the presence of extremely massive and highly charged dust grains in a plasma can either modify the behavior of the usual waves and instabilities or introduce new eigenmodes. The most well studied of such modes is the so-called "Dust Acoustic Wave" (DAW) (Rao et al. 1990) which arises due to the restoring force provided by the plasma thermal pressure (electrons and ions) while the inertia is due to the dust mass. However, in a real dusty plasma, the dust charge may fluctuate becoming therefore a new dynamical variable, and the electron behavior can be strongly modified by external or self-consistent plasma fields.

Over the last two decades, a great deal of attention has been devoted to the nonextensive generalization of the Boltzmann–Gibbs–Shannon (BGS) entropy, first recognized by Renyi (1955) and subsequently proposed by Tsallis (1988). Owing to an increasing amount of experimental and theoretical evidence showing that the BGS formalism fails to describe systems with long range interactions

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and memory effects, Tsallis proposed a new entropy (Tsallis 1988). The latter and the ensuing generalized statistics have been employed with success in plasma physics (Lima et al. 2000; Du 2004; Liyan and Du 2008, 2009; Liu et al. 2009; Tribeche et al. 2010; Amour and Tribeche 2010; Tribeche and Djebarni 2010; Ait Gougam and Tribeche 2011; Pakzad and Tribeche 2011). Recently and because of a lack of formal derivation, a nonextensive approach to kappa-distributions has been suggested (Leubner 2004a). It has been shown that distributions very close to kappa-distributions are a consequence of the generalized entropy favored by nonextensive statistics.

Recently (Amour and Tribeche 2011), we focused on the effects of ion nonextensivity on nonlinear dust acoustic solitary waves. It has been demonstrated that the presence of such non-thermal ions may significantly modify the wave propagation characteristics in collisionless charge varying dusty plasmas. However, we mainly dealt with the adiabatic dust charging case, viz., $I_e + I_i = 0$, ruling out the possibility of existence of dissipative structures. To complement and provide new insight into our previously published work on this problem, we propose here to revisit the nonlinear dust acoustic waves in a nonadiabatic charge varying dusty plasma with nonextensive ions. The outline of the paper is the following. In the next section, we present the basic equations of the theoretical model, our numerical results. A summary of our results and conclusions is given in Sect. 3.

2 Theoretical model

Let us consider a collisionless, unmagnetized three component dusty plasma having electrons, positive ions, and dust grains of density n_e, n_i, and n_d, respectively. Although the size (and thus the charge) of the dust grains varies from one grain to another, we assume for simplicity that all the grains have the same negative charge, $q_d = -eZ_d$, where Z_d is the number of charges residing on the dust grain. On the dust time scale, the electrons are assumed to be in thermal equilibrium, with the density given by

$$n_e = n_{e0}\exp(e\phi/T_e) \tag{1}$$

Here and in the following, $j = e, i, d$ denote electrons, ions and dust grains, respectively, ϕ is the electrostatic potential, $q_{j=i,e} = \pm e$ are the charges, T_j the temperatures, and m_j the masses. Charge neutrality at equilibrium requires $f = n_{i0}/n_{e0} = 1 + Z_{d0}n_{d0}/n_{e0}$. The subscript "0" stands for equilibrium values. We assume that the variable charge dust component is a cold beam of particles, each particle having the same speed at a given position. Thus, we choose (Tribeche et al. 2002)

$$f_d(x, v_d) = n_{d0}\frac{v_{d0}}{\widetilde{v}_d}\delta(v_{d0} - \widetilde{v}_d) \tag{2}$$

where

$$\widetilde{v}_d = v_{d0}\left(1 - \frac{2}{m_d v_{d0}^2}\int_0^\phi q_d\, d\phi\right)^{1/2} \tag{3}$$

Integrating f_d over all velocity space, we get

$$n_d = n_{d0}\frac{v_{d0}}{\widetilde{v}_d} = n_{d0}\left(1 - \frac{2}{m_d v_{d0}^2}\int_0^\phi q_d\, d\phi\right)^{-1/2} \tag{4}$$

To model the effects of ion nonextensivity, we refer to the following q-distribution function given by Silva et al. (1998)

$$f_i = C_q\left\{1 - (q-1)\left[\frac{m_i v^2}{2T_i} + \frac{e\phi}{T_i}\right]\right\}^{\frac{1}{q-1}} \tag{5}$$

The constant of normalization is (Amour and Tribeche 2011)

$$C_q = \begin{cases} n_{i0}\frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q}-\frac{3}{2})}[\frac{m_i(1-q)}{2\pi T_i}]^{3/2}, & \text{for } -1 < q < 1 \\ n_{i0}\frac{(3q-1)}{2}\frac{\Gamma(\frac{1}{q-1}+\frac{3}{2})}{\Gamma(\frac{1}{q-1})}[\frac{m_i(q-1)}{2\pi T_i}]^{3/2}, & \text{for } q > 1 \end{cases} \tag{6}$$

Here, the parameter q stands for the strength of nonextensivity and the quantity Γ for the standard gamma function. It may be useful to note that for $q < -1$, the q-distribution (5) is unnormalizable. In the extensive limiting case ($q \to 1$), distribution (5) reduces to the well-known Maxwell–Boltzmann velocity distribution. Integrating f_i over the velocity space and noting that for $q > 1$, the distribution function (5) exhibits a thermal cutoff on the maximum value allowed for the velocity of the particles, given by

$$v_{\max} = \sqrt{\frac{2T_i}{m_i(q-1)} - \frac{2e\phi}{m_i}} \tag{7}$$

we get

$$\begin{aligned} n_i(\phi) &= \begin{cases} 4\pi\int_0^{+\infty} v^2 f_i(v)\,dv, & \text{for } -1 < q < 1 \\ 4\pi\int_0^{+v_{\max}} v^2 f_i(v)\,dv, & \text{for } q > 1 \end{cases} \\ &= n_{i0}\left\{1 - (q-1)\frac{e\phi}{T_i}\right\}^{\frac{1}{q-1}+\frac{3}{2}} \end{aligned} \tag{8}$$

This nonextensive ion density is valid for $-1 < q < 1$ as well as for $q > 1$. In each case, we get the same expression for the ion density. It has been written in such manner to draw the reader attention to the fact that for $q > 1$, the ion q-distribution function exhibits a thermal cutoff on the maximum value allowed for the velocity of the particles.

Adopting the nondimensional variables $\Psi = e\phi/T_i$, $Q_d = eq_d/r_d T_e$, and $X = x/\lambda_D$, where $\lambda_D = (T_i/4\pi n_{d0} Z_d e^2)^{1/2}$ and r_d is the grain radius, the Poisson equation can be written as

$$\frac{d^2\Psi}{dX} = N_e - fN_i + (f-1)\frac{Q_d}{Q_{d0}}N_d \tag{9}$$

where

$$N_e = \frac{n_e}{n_{e0}} = \exp(\sigma\Psi)$$

$$N_i = \frac{n_i}{n_{i0}} = \{1-(q-1)\Psi\}^{\frac{1}{q-1}+\frac{3}{2}} \tag{10}$$

$$N_d = \frac{1}{(1-\gamma_1\chi)^{1/2}}$$

$$\chi = \int_0^\Psi Q_d d\Psi \tag{11}$$

with $\gamma_1 = \frac{2r_d\sigma T_e^2}{m_d e^2 v_{d0}^2}$, and $\sigma = T_i/T_e$.

In the standard orbit-limited probe model for the dust grain (Allen 1992), the latter is charged by the plasma currents at the grain surface. The charging current originates from electrons and ions hitting the grain surface. Accordingly, the variable dust charge $q_d = -eZ_d$ is determined self-consistently by

$$\frac{dq_d}{dt} = \widetilde{v}_d\frac{\partial q_d}{\partial x} = I_e + I_i \tag{12}$$

where I_e and I_i are the average microscopic electron and ion currents entering the dust grains. The grain current from the thermal electrons is (Shukla and Mamun 2002)

$$I_e = -\pi r_d^2 e\left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_{e0}\exp(\sigma\Psi)\exp(Q_d) \tag{13}$$

For non-isothermal distributions such as (5), one should first rederive the ion dust charging current. The latter is obtained by averaging the effective collision cross section $\sigma_i(v, q_d) = \pi r_d^2(1 - 2eq_d/m_i C v^2)$ for charged ions impacting the dust grains over the ion distribution, where $C = r_d(1 + r_d/\lambda_{De}) \simeq r_d$ is the effective grain capacitance

$$I_i(\phi) = \begin{cases} e\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \int_0^{+\infty} \sigma_i v^3 f_i(v)dv, \\ \quad \text{for } -1<q<1 \\ e\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \int_0^{v_{\max}} \sigma_i v^3 f_i(v)dv, \\ \quad \text{for } q>1 \end{cases} \tag{14}$$

After performing the integrals in (14), one can obtain the following ion charging current (Amour and Tribeche 2011)

$$I_i = \pi r^2 e\left(\frac{8T_i}{\pi m_i}\right)^{1/2} B_q\left\{\frac{1}{(2q-1)}\left[1-(q-1)\Psi\right]^{\frac{1}{q-1}+2} - \frac{Q_d}{\sigma}\left[1-(q-1)\Psi\right]^{\frac{1}{q-1}+1}\right\} \tag{15}$$

where

$$B_q = \begin{cases} \frac{(1-q)^{3/2}\Gamma(\frac{1}{1-q})}{q\Gamma(\frac{1}{1-q}-\frac{3}{2})}, & \text{for } -1<q<1 \\ \frac{(3q-1)(q-1)^{3/2}\Gamma(\frac{1}{q-1}+\frac{3}{2})}{2q\Gamma(\frac{1}{q-1})}, & \text{for } q>1 \end{cases} \tag{16}$$

The latter reduces to the well-known thermal ion current in the extensive case ($q \to 1$). After rearranging the terms in (12), one can obtain the following normalized charging equation

$$\frac{dQ_d}{dX} = kN_d\left\{-\sqrt{\mu/\sigma}\exp(Q_d+\sigma\Psi) + fB_q\left[\frac{1}{(2q-1)}\left[1-(q-1)\Psi\right]^{\frac{1}{q-1}+2} - \frac{Q_d}{\sigma}\left[1-(q-1)\Psi\right]^{\frac{1}{q-1}+1}\right]\right\}, \tag{17}$$

with

$$k = \left(\frac{2n_{e0}e^2r^2\sigma^2}{m_i v_{d0}^2}\right)^{1/2} \tag{18}$$

At equilibrium ($\Psi = 0$, $Q_d = Q_{d0}$), (17) requires the following constraint

$$f = \frac{\sqrt{\mu/\sigma}\exp(Q_{d0})}{B_q[1/(2q-1) - Q_{d0}/\sigma]} = \begin{cases} \frac{\sqrt{\mu/\sigma}\exp(Q_{d0})}{[1/(2q-1)-Q_{d0}/\sigma]}\left\{\frac{(1-q)^{3/2}\Gamma(\frac{1}{1-q})}{q\Gamma(\frac{1}{1-q}-\frac{3}{2})}\right\}^{-1}, \\ \quad \text{for } -1<q<1 \\ \frac{\sqrt{\mu/\sigma}\exp(Q_{d0})}{[1/(2q-1)-Q_{d0}/\sigma]}\left\{\frac{(3q-1)(q-1)^{3/2}\Gamma(\frac{1}{q-1}+\frac{3}{2})}{2q\Gamma(\frac{1}{q-1})}\right\}^{-1}, \\ \quad \text{for } q>1 \end{cases} \tag{19}$$

It may be useful to note that the dust charging equation $\frac{dq_d}{d}t = I_e + I_i$ can be rewritten as

$$\frac{\omega_{pd}}{\nu_{ch}}\frac{dq_d}{d(\omega_{pd}t)} = \frac{I_e+I_i}{\nu_{ch}} \tag{20}$$

where

$$\nu_{ch} = -\frac{e}{r_d T_e}\left[\frac{\partial(I_e+I_i)}{\partial Q_d}\right]_{\Psi=0,\ Q_d=Q_{d0}} = \left(\frac{8\pi e^4 r_d^2 n_{i0}^2}{m_i T_i}\right)^{1/2}\left[\frac{B_q}{\sigma} + \frac{\sqrt{\sigma\mu}\exp(Q_{d0})}{f}\right] \tag{21}$$

is the nonextensive dust charging frequency and

$$\omega_{pd} = \left(\frac{4\pi n_{d0}q_{d0}^2}{m_d}\right)^{1/2} = \left(\frac{r_d^2T_e^2}{e^2}\frac{4\pi n_{d0}Q_{d0}^2}{m_d}\right)^{1/2} \tag{22}$$

is the dust plasma frequency. Making use of (19), the nonextensive dust charging frequency may be rewritten as

$$\nu_{ch} = \left(\frac{8\pi e^4 r_d^2 n_{i0}^2}{m_i T_i}\right)^{1/2} B_q \left(\frac{1}{\sigma} + \frac{\sigma}{2q-1} - Q_{d0}\right) \tag{23}$$

The latter depends sensitively on the nonextensive parameter q. Note that Rosenberg and Mendis (1992) were among the first to point out the necessity of reexamining the dust grain charging in space plasmas most of which are observed to have non-Maxwellian tails.

The time scale for the charging of dust particles τ_{ch} ($\sim 1/\nu_{ch}$) is the response time of the dust charge due to oscillations in the surrounding plasma. In a laboratory plasma, the charging time is much less than the time scale of the motion of dust particles τ_{pd} (proportional to the inverse dust plasma frequency, ω_{pd}). In a space plasma, however, τ_{ch} may be of the order of or even larger than τ_{pd}. In the theory of nonadiabatic dust charge variation (Ghosh et al. 2004; Ghosh and Gupta 2005), it is assumed that the ratio ω_{pd}/ν_{ch} is relatively small and finite, i.e., $\omega_{pd}/\nu_{ch} \neq 0$. Dust grains immersed in a plasma exhibit self-consistent charge variations in response to the surrounding plasma oscillations and thus become a time dynamical variable. The propagation of low-frequency dust acoustic (DA) waves in such plasma may be strongly influenced by the dust charging rates (high or low) compared to the dust oscillation frequency. Under the assumption that the dust hydrodynamical time scale τ_{pd} ($\sim 1/\omega_{pd}$) is much greater than that of the dust charging time scale τ_{ch} ($\sim 1/\nu_{ch}$), the dust charge varies but instantaneously reaches equilibrium value at each space-time point determined by the local electrostatic potential $\phi(x,t)$. Such charge variation is termed adiabatic ($\omega_{pd}/\nu_{ch} \sim \tau_{ch}/\tau_{pd} \approx 0$). The scenario changes drastically for a non-negligible value of τ_{ch}/τ_{pd}. In this case, there is an extra contribution from the dust grain charge, which is known as nonadiabatic variation. This causes an oscillatory decay of the dust charge magnitude and consequently plays the role of a dissipative mechanism on the dust fluid by decreasing the driving force causing dust motion. In fact, the nonsteady dust charge variation due to collective perturbations modifies the dust floating potential which in turn self-consistently opposes the buildup of the plasma currents at the dust grains and consequently causes dissipative effects in dusty plasmas which lead ultimately to the formation of collisionless shock waves (Ghosh et al. 2002a, 2002b, 2006; Ghosh 2005). Let, therefore, consider the situation in which

$$\omega_{pd}/\nu_{ch} = \left(\frac{T_e^2 n_{d0} Q_{d0}^2 m_i T_i}{2 n_{i0}^2 m_d e^6}\right)^{1/2} \Big/ \left[B_q\left(\frac{1}{\sigma} + \frac{\sigma}{2q-1} - Q_{d0}\right)\right] \tag{24}$$

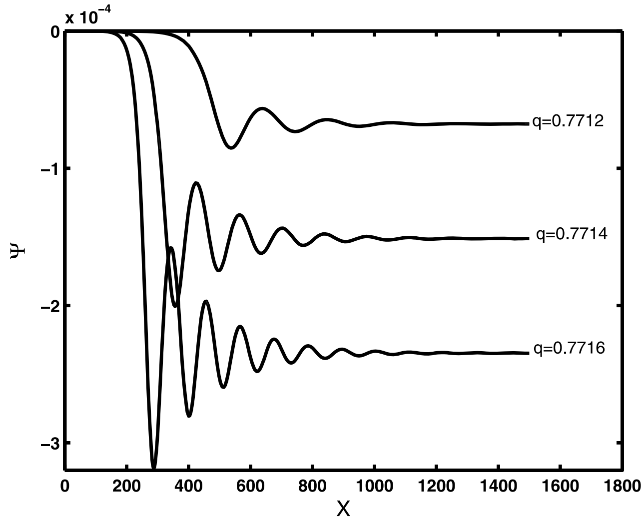

Fig. 1 Dust acoustic shock-like solution for the electrostatic potential Ψ in the case of variable charge dust grain for different values of the ion nonextensive parameter $q = 0.7712$ ($\omega_{pd}/\nu_{ch} \sim 1.3529$, $f = 1.1515$, $k = 0.5595$), 0.7714 ($\omega_{pd}/\nu_{ch} \sim 1.3522$, $f = 1.1514$, $k = 0.5595$), and 0.7716 ($\omega_{pd}/\nu_{ch} \sim 1.3515$, $f = 1.1513$, $k = 0.5595$). The remaining parameters are $Q_{d0} = -2.31$, $T_e = 1$ eV, $n_{i0} = 5 \times 10^4$ cm^{-3}, $v_{d0} = 85$ cm s^{-1}, $\sigma = 0.42$, $r_d = 10$ μm, and $m_d = 10^{13} m_i$

has a non-negligible value. Next, (9), (11), and (17) are integrated numerically. The following parameters, $Q_{d0} = -2.31$, $T_e = 1$ eV, $n_{i0} = 5 \times 10^4$ cm^{-3}, $v_{d0} = 85$ cm s^{-1}, $\sigma = 0.42$, $r_d = 10$ μm, and $m_d = 10^{13} m_i$, have been chosen so that the condition $\omega_{pd}/\nu_{ch} \neq 0$ is now fulfilled. Figure 1 indicates that under certain conditions, the dust charge variation induced nonlinear wave damping leads to the development of a DA shock wave. This is a collisionless shock wave in the sense that no viscous or damping effects resulting from collisions between dust and plasma particles are involved. In contrast to the classical collisionless shock waves, where the dissipation is only from the turbulent wave-particle interactions, the dissipation due to the dust charging involves the interaction of the electrons and ions with dust grains in the form of microscopic grain currents. The influence of the ion nonextensivity on the shock front height is clearly displayed (Fig. 1). For the sake of comparison, we have plotted the nonlinear potential Ψ for different values of the nonextensive parameter $q = 0.7712$ ($\omega_{pd}/\nu_{ch} \sim 1.3529$), 0.7714 ($\omega_{pd}/\nu_{ch} \sim 1.3522$), and 0.7716 ($\omega_{pd}/\nu_{ch} \sim 1.3515$). Our results reveal that the amplitude, strength and nature (oscillatory or monotonic) of these DA shock waves are extremely sensitive to the degree of ion nonextensivity. The effect of separation of charges, which is manifested by the appearance of some oscillations in the shock wave profile, may decrease as the value of the q-nonextensive parameter decreases (i.e., the ions evolve far away from their Maxwellian equilibrium). This means that stronger is the ions correlation, more important is the charge variation induced nonlinear wave damping (anomalous dissipation may even prevail over dispersion).

3 Conclusion

To conclude, we have extended our recent analysis to situations in which dust grains exhibit self-consistent nonadiabatic charge variation. The plasma consists of cold fluid dust grains, thermal electrons, and nonextensive ions. Interestingly and because of the nonadiabatic dust charge variation, our plasma model may admit collisionless dust acoustic shock waves the nature and amplitude of which depend sensitively on the parameter q. The latter underpins the generalized entropy of Tsallis and is linked to the underlying dynamics of our plasma system. The "maxwellianization" process of the ions ($q \to 1$) may weaken the charge variation induced anomalous dissipation. As is well-known, Maxwellian distribution in Boltzmann–Gibbs statistics is believed valid universally for the macroscopic ergodic equilibrium systems. However, for the systems with the long-range interactions, such as plasma and gravitational systems, where the non-equilibrium stationary states exist, Maxwellian distribution might be inadequate for the description of the systems. A few examples of physical systems where the standard Boltzmann–Gibbs approach seems to be inadequate are self-gravitating systems and some kinds of plasma turbulence. A growing body of evidence suggests that the q-entropy may provide a convenient frame for the analysis of many astrophysical scenarios, such as stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy clusters. It has been shown that the experimental results, for electrostatic plane-wave propagation in a collisionless thermal plasma, point to a class of Tsallis's velocity distribution described by a nonextensive q-parameter smaller than unity (Lima et al. 2000). In the light of the used parameters, our results may be relevant to low-temperature space dusty plasmas containing micrometer-sized and highly negative charged grains. Note that the transformation linking q-statistics and kappa(κ)-distributions (which were extensively criticized because of lack of profound derivation in view of fundamental physics) was first provided by Leubner (2004b) as

$$-\kappa = \frac{1}{q-1} \tag{25}$$

In view of the following values of the spectral index $\kappa = 3$–6 usually assigned in the literature to fitting procedures, the nonextensive parameter q may range from 0.66 to 0.83 (that is why our numerical results are provided with q around 0.7). Moreover, our study may be viewed as a first step towards a more comprehensive dissipative structures in stationary nonequilibrium plasmas.

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