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ORIGINAL ARTICLE

Axially symmetric radiating cosmological model in a self-creation cosmology

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Abstract An axially symmetric space time is considered in the presence of a perfect fluid source in Barbers (Gen. Relativ. Gravit. 14, 117, 1982) second self creation theory of gravitation. An exact radiating cosmological model is presented using a relation between the metric potentials. Some physical and kinematical properties of the model are also discussed.

Keywords Axially symmetric · Self-creation cosmology · Radiating model

1 Introduction

It is well known that in recent years there has been a considerable interest in alternative theories of gravitation which are viable alternatives to general relativity. Brans and Dicke (1961) formulated a scalar tensor theory of gravitation which incorporates Machs-principle in a relativistic frame work. Barber (1982) produced two continuous creation theories. The first is a modification of Brans Dicke theory and the second is an adoption of general relativity to include continuous creation of matter and is within the limits of observation. These modified theories create the universe out of self contained gravitational and matter fields.

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Several authors have investigated various cosmological models in Barbers second self creation theory Pimentel (1985), Soleng (1987), Singh (1984), Reddy (1987a, 1987b), Reddy et al. (1987), Reddy and Venkateswarlu (1989), Shanti and Rao (1991), Reddy and Naidu (2008), Pradhan and Vishwakarma (2002) are some of the authors who have discussed various cosmological models in second self creation theory. However axially symmetric cosmological models in the presence of radiating perfect fluid have not been investigated in second self creation theory proposed by Barber. Radiating cosmological models are also important to discuss the early stages of evolution of the universe.

In this chapter we obtain axially symmetric radiating cosmological model in the presence of perfect fluid source.

2 Metric and field equations

We consider the uniform, anisotropic and axially symmetric (Bhattacharya and Karade 1993)

$$ds^2 = dt^2 - A^2(t)[d\chi^2 + f^2(\chi)d\phi^2] - B^2(t)dz^2 \qquad (2.1)$$

with the convention $x^1 = \chi$, $x^2 = \phi$, $x^3 = z$, and $x^4 = t$ and A and B are functions of the proper time t alone while f is a function of the coordinate χ alone.

The non-vanishing components of Einstein tensor for the space time (2.1) are

$$G_1^1 = G_2^2 = -\frac{A_{44}}{A} - \frac{B_{44}}{B} - \frac{A_4 B_4}{AB}$$

$$G_3^3 = -2\frac{A_{44}}{A} - \frac{A_4^2}{A^2} + \frac{1}{A^2}\frac{f_{11}}{f}$$

$$G_4^4 = -2\frac{A_4 B_4}{AB} - \frac{A_4^2}{A^2} + \frac{1}{A^2}\frac{f_{11}}{f}$$

Here

$$f_1=\frac{\partial f}{\partial \chi},\qquad A_4=\frac{\partial A}{\partial t}$$

The field equations in Barber's (1982) second self-creation theory are

$$R_{ij}-\frac{1}{2}g_{ij}R=-8\pi\phi^{-1}T_{ij} \tag{2.2}$$

and

$$\Box\phi=\phi^{'k}_{jk}=\frac{8\pi}{3}\alpha T \tag{2.3}$$

where T is the trace of energy-momentum tensor, λ is a coupling constant to be determined from the experiment ($|\lambda|\le 0.1$) and semi-colon denotes covariant differentiation. In the limit as $\alpha\to 0$, this theory approaches the standard general relativity theory in every respect and $G=\emptyset^{-1}$.

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij} \tag{2.4}$$

where ρ is the rest energy density, p is the isotropic pressure, u_i is the four velocity of the matter, so that

$$u^iu_i=1,\qquad u^iu_j=0 \tag{2.5}$$

from (2.4) and (2.5), we have

$$T_1^1=T_2^2=T_3^3=-p,\qquad T_4^4=\rho \tag{2.6}$$

using (2.4) and (2.5), the field equations of Barbers second self creation theory for the metric (2.1) can be written as

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=8\pi\phi^{-1}p \tag{2.7}$$

$$2\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2-\frac{1}{A^2}\left[\frac{f_{11}}{f}\right]=8\pi\phi^{-1}p \tag{2.8}$$

$$\frac{1}{A^2}\left[\frac{f_{11}}{f}\right]-\left(\frac{A_4}{A}\right)^2-2\frac{A_4B_4}{AB}=8\pi\phi^{-1}\rho \tag{2.9}$$

$$\phi_{44}+\phi_4\left(2\frac{A_4}{A}+\frac{B_4}{B}\right)=\frac{8\pi}{3}\phi^{-1}\alpha(\rho-3p) \tag{2.10}$$

Here the suffixes 1 and 4 after an unknown function denote partial differential with respect to X and t respectively.

The function dependence of the metric together with (2.8) and (2.9) imply

$$\frac{f_{11}}{f}=k^2,\quad k^2 \text{ constant} \tag{2.11}$$

If $k=0$, then $f(\chi)=\text{constant}\,\chi$, $0<\chi<\infty$. This constant can be made equal to 1 by suitably choosing units for $\emptyset$. Thus we shell have $f(\chi)=\chi$ resulting in the flat model of the universe (Hawking and Ellis 1976).

Now the field equations (2.7)–(2.10) reduced to

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=8\pi\phi^{-1}p \tag{2.12}$$

$$2\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2=8\pi\phi^{-1}p \tag{2.13}$$

$$\left(\frac{A_4}{A}\right)^2+2\frac{A_4B_4}{AB}=8\pi\phi^{-1}\rho \tag{2.14}$$

$$\frac{\phi_{44}}{\phi_4}+2\frac{A_4}{A}+\frac{B_4}{B}=\frac{8\pi}{3}\phi^{-1}\alpha(\rho-3p) \tag{2.15}$$

Equations (2.12)–(2.15) are four independent equations in five unknowns A, B, ρ, p, $\emptyset$. Hence to find a determinate solution, we use the equation of state $\rho=3p$ which represents disordered radiation of matter distribution. Now the field equation (2.12) to (2.15) take the form

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=8\pi\phi^{-1}p \tag{2.16}$$

$$2\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2=8\pi\phi^{-1}p \tag{2.17}$$

$$\left(\frac{A_4}{A}\right)^2+2\frac{A_4B_4}{AB}=8\pi\phi^{-1}(3p) \tag{2.18}$$

and

$$\frac{\phi_{44}}{\phi_4}+\frac{2A_4}{A}+\frac{B_4}{B}=0 \tag{2.19}$$

integrating (2.19), we get

$$\phi_4\cdot A^2B=C_1,\quad \text{where } C_1 \text{ is integral constant}$$

$$\Rightarrow\quad \phi_4=\frac{C_1}{A^2B} \tag{2.20}$$

To solve the non linear field equations (2.16) to (2.18), we take a relation between the metric coefficients

$$A=B^n \tag{2.21}$$

Using (2.20) and (2.21), the field equations (2.16) to (2.18) admits the exact solution given by

$$A=\left(\frac{4n-1}{3n}\right)^{\frac{3n}{4n-1}}T^{\frac{3n}{4n-1}} \tag{2.22}$$

where $T=C_2^{1/3}t+C_3$, C_2, C_3 are integration constants.

$$B=\left(\frac{4n-1}{3n}\right)^{\frac{3}{4n-1}}T^{\frac{3}{4n-1}} \tag{2.23}$$

$$\phi = -\frac{c_1(3n)^{\frac{6n+3}{4n-1}}}{(2n+4)(4n-1)^{\frac{2n+4}{4n-1}}} T^{-\frac{2n+4}{4n-1}} \tag{2.24}$$

$$\rho = 3p = \frac{3c_1c_2^2}{16\pi}\left(\frac{3n}{4n-1}\right)^{\frac{10n+2}{4n-1}} \times \frac{1}{T^{\frac{10n+2}{4n-1}}} \tag{2.25}$$

The corresponding string model of the solution can be written, through a proper choice of constants of integration and coordinates as

$$ds^2 = dT^2 - \left(\frac{4n-1}{3n}T\right)^{\frac{3n}{4n-1}}[d\chi^2 + f^2(\chi)d\phi^2] - \left(\frac{4n-1}{3n}T\right)^{\frac{3}{4n-1}} dz^2 \tag{2.26}$$

3 Some physical properties of the model

Equation (2.26) represents axially symmetric radiating cosmological model in the frame work of second self-creation theory of gravitation proposed by Barber (1982) in the presence of a perfect fluid source. We observe that the model has no initial singularity for $n \neq \frac{1}{4}$.

For the model (2.26) the physical and kinematical parameters which are important in the discussion of cosmology are

- Energy density

$$\rho = 3p = \frac{3}{16\pi}\left(\frac{3n}{4n-1}\right)^{\frac{10n+2}{4n-1}} T^{\frac{-(10n+2)}{4n-1}} \tag{3.1}$$

- Spatial volume

$$\gamma^3 = \sqrt{-g} = A^2B = \frac{(2n+4)(4n-1)^{\frac{2n+4}{4n-1}}}{(3n)^{\frac{6n+3}{4n-1}}} - T^{\frac{2n+4}{4n-1}} \tag{3.2}$$

- Expansion Scalar

$$\theta = \frac{1}{3}U^i_{;i} = \frac{n+1}{4n-1}\cdot\frac{1}{T} \tag{3.3}$$

- Shear Scalar

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\theta^2 = \frac{1}{6}\left[\frac{n+1}{4n-1}\cdot\frac{1}{T}\right]^2 \tag{3.4}$$

- Deceleration parameter

$$q = -3\theta^{-2}\left[\theta_{;\alpha}u^\alpha + \frac{1}{3}\theta^2\right] = -\frac{(4-11n)}{n+1} \tag{3.5}$$

Equation (3.2) shows the anisotropic expansion of the universe (2.26) with $4n-1>0$. The energy density ρ and the isotropic pressure p tend to the zero as time increases indefinitely. For this model the expansion scalar θ shear scalar σ tend to zero as $T\to\infty$. Since $n>0$ we have a positive value of the deceleration parameter q which shows that the model decelerates in the standard way. However the model does not admit rotation and acceleration. Hence the model (2.26) represents expanding, shearing, non-rotating and non-singular universe which decelerate in the standard way. However the scalar field diverges as $T\to 0$. Also, since $\mathrm{lt}_{T\to\infty}\frac{\sigma^2}{\theta^2}\neq 0$, the model does not approach isotropy for large values of T.

4 Conclusions

It is well known that it is still a challenging problem to unravel the secrets of the early stages of evolution of the universe. The radiating model and the presence of long range scalar fields help to understand the evolution of early stages of the universe.

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