Astrophys Space Sci (2012) 337:217–222
DOI 10.1007/s10509-011-0816-5

ORIGINAL ARTICLE

Energy of electron acoustic solitons in plasmas with superthermal electron distribution

Hamid Reza Pakzad

Received: 26 April 2011 / Accepted: 25 July 2011 / Published online: 31 August 2011

Abstract The propagation of nonlinear waves in plasmas consisting of cold electron fluid and superthermal hot electrons and stationary ions is studied. The Korteweg-de Vries (KdV) equation is derived using the reductive perturbation theory. It is found that only the rarefractive solitons can be created. Moreover, the linear dispersion relation and energy of solitary waves in the presence of hot superthermal electrons are derived. Our investigation is of wide relevance to astronomers and space scientists working on interstellar space plasmas.

Keywords Electron-acoustic waves · Solitary waves · Superthermal electrons · Soliton energy

1 Introduction

The electron-acoustic wave, which is one of the basic wave processes in plasmas, is a high-frequency (in comparison with the ion plasma frequency) wave that occurs in plasmas having, in addition to positively charged ions, two electron components with widely disparate temperatures (Watanabe and Taniuti 1977; Tokar and Gary 1984; Gary and Tokar 1985). The electron acoustic solitary waves (EASWs) can also be generated by electron and laser beams (Gary and Tokar 1985; Montgomery et al. 2001). Recently, a great deal of interest has been shown in the studies on the propagation of EASWs. They have been observed in space and laboratory plasmas. On the other hand, they are investigated because of their importance in interpreting electrostatic component of the broadband electrostatic noise (BEN) observed in the cusp region of the terrestrial magnetosphere (Tokar and Gary 1984; Singh and Lakhina 2001), in the geomagnetic tail (Schriver and Ashour-Abdalla 1989), in the dayside auroral acceleration region (Dubouloz et al. 1977; Pottelette et al. 1999) and etc. The propagation of EASWs in a plasma system has been studied by several investigators in an unmagnetized two-electron plasma (Dubouloz et al. 1977; Chatterjee and Roychoudhury 1995; Berthomier et al. 2000; Mamun and Shukla 2002) as well as in magnetized plasmas (Mace and Hellberg 2001; Mamun et al. 2002; Berthomier et al. 2003; Shukla et al. 2004). On the other side, space plasma observations indicate clearly the presence of electron populations which are far away from their thermodynamic equilibrium (Gill et al. 2006; El-Shewy 2007b; Vasyliunas 1968; Leubner 1982; Younsi and Tribeche 2010; Pakzad and Tribeche 2010; Sahu 2010b; Armstrong et al. 1983). Numerous observations of space plasmas (Feldman et al. 1973; Formisano et al. 1973; Scudder et al. 1981; Marsch et al. 1982) clearly prove the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. Superthermal particles may arise due to the effect of external forces acting on the natural space environment plasmas or because of wave-particle interactions. Plasmas with an excess of superthermal (non-Maxwellian) electrons are generally characterized by a long tail in the high energy region. It has been found that generalized Lorentzian of k distribution can be modeled such space plasmas, better than the Maxwellian distribution (Hasegawa et al. 1985; Thorne and Summers 1991; Summers and Thorne 1991; Summers and Thorne 1994; Mace and Hellberg 1995b). Kappa distribution has been used by several authors (Hellberg and Mace 2002; Podesta

H.R. Pakzad (✉)
Department of Physics, Bojnourd Branch, Islamic Azad University, Bojnourd, Iran
e-mail: pakzad@bojnourdiau.ac

H.R. Pakzad (✉)
e-mail: ttaranomm83@yahoo.com

2005; Abbasi and Pajouh 2007; Baluku and Hellberg 2008; Hellberg et al. 2009; Sultana et al. 2010; Baluku et al. 2010) in studying the effect of Landau damping on various plasma modes. "Superthermal" plasma behavior was observed in various experimental plasma contexts, such as laser matter interactions or plasma turbulence (Magni et al. 2005). At very large values of the spectral index k, the velocity distribution function approaches a Maxwellian distribution, while for low values of k, they represent a "hard" spectrum with a strong non-Maxwellian tail having a power-law form at high speeds. The motivation of the presented paper is therefore to study the existence of EASWs in plasmas having stationary ions, cold inertial electrons and hot superthermal electrons.

2 Basic equations

We consider a homogeneous, unmagnetized plasma consisting of a cold electron fluid, hot electrons obeying a superthermal distribution and stationary ions. The nonlinear dynamics of the electron acoustic solitary waves is governed by the continuity and motion equations for cold electrons, and the Poisson's equation (Younsi and Tribeche 2010; Pakzad and Tribeche 2010; Sahu 2010b)

$$\begin{aligned} &\frac{\partial n_c}{\partial t}+\frac{\partial(n_c u_c)}{\partial x}=0\\ &\frac{\partial u_c}{\partial t}+u_c\frac{\partial u_c}{\partial x}-\alpha\frac{\partial\phi}{\partial x}=0\\ &\frac{\partial^2\phi}{\partial x^2}-\frac{1}{\alpha}n_c-n_h+\left(1+\frac{1}{\alpha}\right)=0 \end{aligned} \tag{1}$$

In the above equations, $n_c(n_h)$ is the cold (hot) electron number density normalized by its equilibrium value $n_{c0}(n_{h0})$, u_c is the cold electron fluid velocity normalized by $C_e=(k_B T_h/\alpha m_e)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_h/e$ while k_B is the Boltzmann's constant, e the electron charge, m_e electron mass and $\alpha = n_{h0}/n_{c0}$. The time and space variables are in units of the cold electron plasma period ω_{pc}^{-1} and the hot electron Debye radius λ_{Dh}, respectively.

n_h is the superthermal hot electron density and it is given by Pakzad and Tribeche (2010)

$$n_h=\left(1-\frac{\phi}{k-1/2}\right)^{-k-\frac{1}{2}} \tag{2}$$

The parameter κ shapes predominantly the superthermal tail of the distribution (Tribeche and Boubakour 2009) and the normalization is provided for any value of the spectral index $\kappa > 1/2$ (Boubakour et al. 2009). In the limit $\kappa\to\infty$, (2) reduces to the well known Maxwell-Boltzmann electron density. Low values of k represent distributions with a relatively large component of particles which their velocity are greater than the thermal speed ("superthermal particles") and an associated reduction in "thermal" particles, as one observes in a "hard" spectrum. Such a very hard spectrum, with an extreme accelerated superthermal component, may be found near very strong shocks associated with Fermi acceleration (Mace and Hellberg 1995a).

Now, we study the small but infinite amplitude waves in dust plasmas with superthermal electrons by using the reductive perturbation method. Firstly, we introduce the stretched coordinates as, $\xi=\varepsilon^{1/2}(x-\lambda t)$, $\tau=\varepsilon^{3/2}t$, where ε is a small dimensionless expansion parameter and λ is the wave speed normalized by C_e. Secondly, we expand dependent variables as follows,

$$\begin{cases} n_c=1+\varepsilon n_{1c}+\varepsilon^2 n_{2c}+\cdots\\ u_c=\varepsilon u_{1c}+\varepsilon^2 u_{2c}+\cdots\\ \phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\cdots \end{cases} \tag{3}$$

Substituting (3) into (1) and collecting the terms in different powers of ε the following equations can be obtained at the lower order of ε

$$n_{1c}=\frac{-\alpha\phi_1}{\lambda^2},\qquad u_{1c}=\frac{-\alpha\phi_1}{\lambda},\qquad \frac{1}{\lambda^2}=\frac{2k+1}{2k-1} \tag{4}$$

at the higher order of ε, we have

$$\begin{aligned} &\frac{\partial n_{1c}}{\partial\tau}-\lambda\frac{\partial n_{2c}}{\partial\xi}+\frac{\partial}{\partial\xi}(u_{2c}+n_{1c}u_{1c})=0\\ &\frac{\partial u_{1c}}{\partial\tau}-\lambda\frac{\partial u_{2c}}{\partial\xi}+u_{1c}\frac{\partial u_{1c}}{\partial\xi}-\alpha\frac{\partial\phi_2}{\partial\xi}=0\\ &\frac{\partial^2\phi_1}{\partial\xi^2}-\frac{n_{2c}}{\alpha}-\left[\frac{2k+1}{2k-1}\phi_2+\frac{(2k+1)(2k+3)}{(2k-1)^2}\phi_1^2\right]=0 \end{aligned} \tag{5}$$

Finally from (4) and (5) the KdV equation yields

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0 \tag{6}$$

where the coefficients are

$$A=-\left(\frac{3\alpha}{2\lambda}+\frac{2k+3}{2k-1}\lambda\right),\qquad B=\frac{\lambda^3}{2} \tag{7}$$

The above results can be compared with the results in Younsi and Tribeche (2010) and Pakzad and Tribeche (2010) for nonthermal and nonextensive hot electrons, respectively. In order to study a stationary solitary wave solution of (6), we assume that the stationary solution can be expressed as, $\phi_1=\phi_1(\chi)$, where $\chi=\xi-u\tau$. Substituting this expression into (6), we can obtained the stationary solitary wave solution

$$\phi_1=\phi_0\sec h^2\left(\frac{\chi}{w}\right) \tag{8}$$

where $\phi_0=3u/A$ is the soliton amplitude and $w=2\sqrt{B/u}$ is its width.

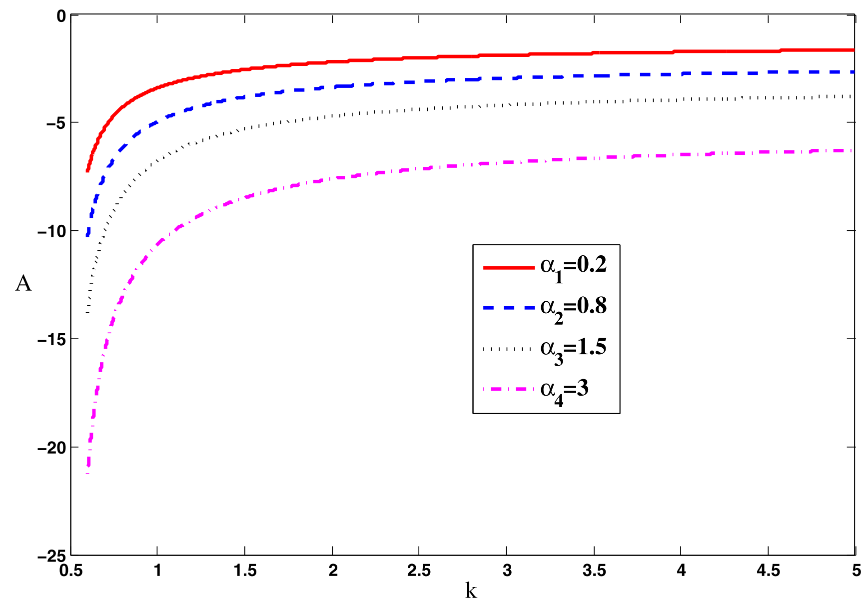

Fig. 1 The variation of nonlinear coefficient (A) respect to k-parameter for different values of α

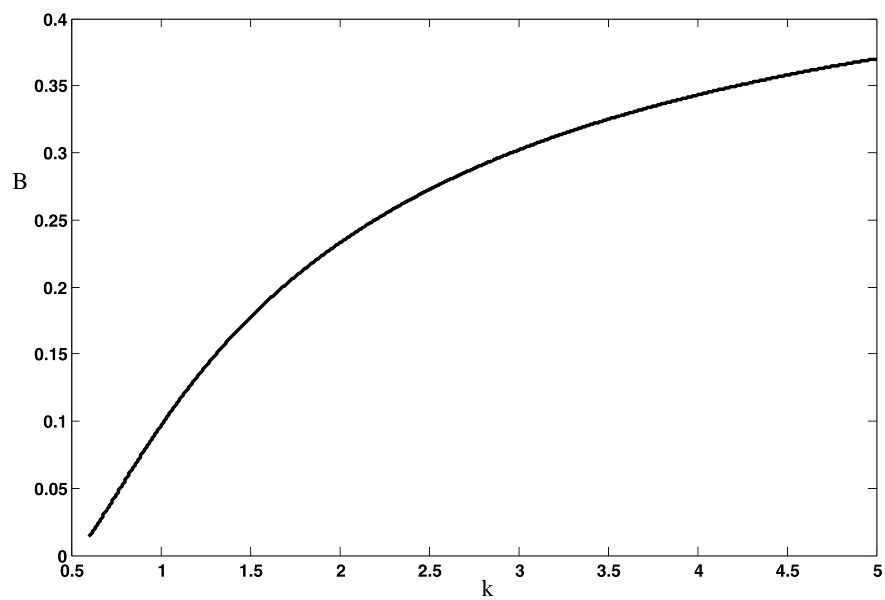

Fig. 2 The variation of dispersion coefficient (B) respect to k-parameter

3 Discussion

We can investigate the properties of solitary waves with numerical analysis of ϕ_1 using (8). In order to study electron acoustic solitary waves, we can use (8) in our plasma model. Equation (8) shows that the soliton amplitude depends on both α and k, but the soliton width depends on only k-parameter. It is seen that A is always negative and therefore only the rarefactive solitons can be created. This is congruent with the investigation of Sahu (2010a) which has been done using the Sagdeev's pseudopotential technique. Figure 1 shows the variation of nonlinear coefficient (A) as a function of superthermal parameter (k) for different values of α.

We can find from Figs. 1 and 2 and also (8) that the amplitude and width of solitons increases with an increasing in k. It is clear that the soliton amplitude decreases when α increases. In Fig. 3, the profile of solitons has been plotted with respect to χ for different values of k.

Because of using a weakly nonlinear analysis, the potential (ϕ_1) has to ranging from 0 to -1. Therefore the soliton amplitude ($|\phi_1|$) does not exceed 1 and we have to choose the adequate parameters to maintain the weakly nonlinear nature of our analysis. The variation of the soliton amplitude as a function of k can be studied by plotting the amplitude respect to k for the case of $u = 1$ and different values of α. Figure 4 shows that for $\alpha_1 = 0.4$ the amplitude becomes more than 1 for $k > k_{c1} = 1.47$ as the soliton struc-

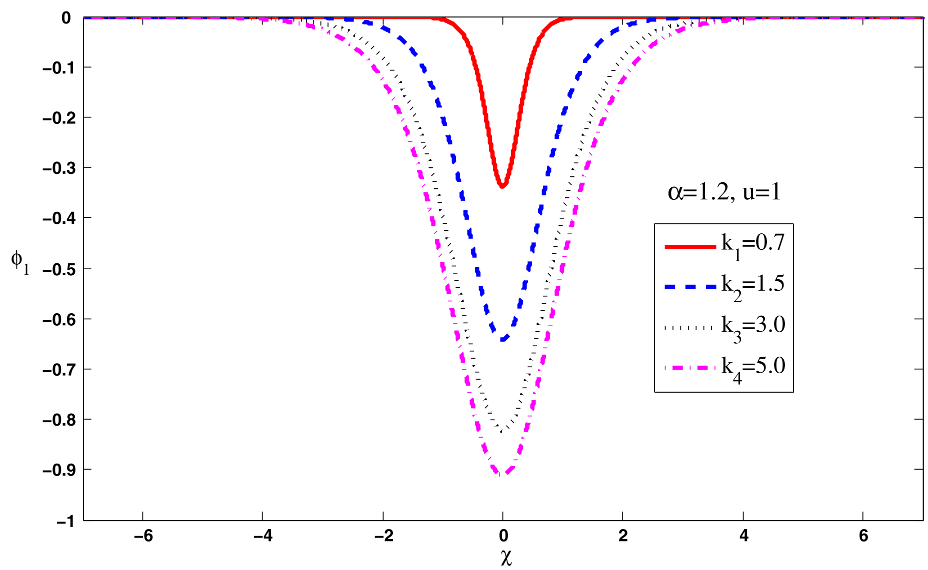

Fig. 3 The variation of profile of soliton (ϕ_1) respect to χ for different values of k-parameter

ture will be failed to exist, thus the values $k > 1.47$ are rule out from our model. Also our results which have been plotted in Figure 4, show that the admissible values of k are respectively, smaller than 2.26 and 4.66 for $\alpha_2 = 0.7$ and $\alpha_3 = 1$. The threshold superthermal parameters are shifted toward higher values as α increases. It is also obvious that for $\alpha_4 = 1.5$ there is no threshold value for superthermal parameter (k). Thus, there is no limitation for k-parameter with some values of α. It is also seen that the range of admissible values of k-parameter increases when hot electron density (α) increases. On the other hand, (8) shows that the amplitude depends on soliton velocity ($|\phi_0| = \frac{3u}{|A|}$). It is obvious that the threshold value of (k_c) decreases for higher values of "u".

We now proceed to obtain the linear dispersion relation for low frequency modes by using a variety of theoretical models. According to the standard normal-mode analysis, by linearization of dependent variables n_d, ϕ and u_d in term of their equilibrium and perturbed parts (Annou 1998), we have $n_d = 1 + n_{1d}$, $\phi = \phi_{1d}$ and $u_d = u_{1d}$. Assume that the wave perturbations behave like $e^{i(Kx-\omega t)}$ (K is the wave propagation constant in the direction of x-axis), from (1), we obtain the following equations

$$-i\omega n_1 + iKu_1 = 0$$

$$-i\omega u_1 - i\alpha K\phi_1 = 0$$

$$-\left(K^2 + \frac{2k+1}{2k-1}\right)\phi_1 = \frac{n_1}{\alpha}$$

Thus, we obtain the following dispersion relation for EAWs

$$\omega^2 = \frac{K^2}{K^2 + \frac{2k+1}{2k-1}} \tag{9}$$

For real values of ω, all of the variables oscillate harmonically. If at least one of the ω's has positive imaginary part, then the system is unstable since those normal modes will grow in time (Samanta et al. 2007). It is observed that the dispersion relation given by (9) depends upon the value of the superthermal parameter k. It is seen that ω increases as the value of K increases. Also one can show that ω is shifted towards higher values as $k \to \infty$, i.e., as the hot electron component evolves towards its Maxwell-Boltzmann thermodynamic equilibrium. Thus deviation from the Maxwellian distribution appears to decrease the energy of the wave.

The study of soliton energy is one of the ways to further recognize solitary waves in the plasmas. One of the main points of the paper is to study the effect of superthermal electrons on the energy of electron acoustic waves. Soliton energy is defined by the following integral (Singh and Honzawa 1993)

$$E = \int_{-\infty}^{+\infty} u_1^2(\chi)d\chi \tag{10}$$

where $\chi = \xi - u\tau$. Thus we can obtain (El-Shewy 2007a)

$$E = \frac{4}{3}u_m^2 w = \frac{24\alpha^2 u^2}{A^2}\left(\frac{2k+1}{2k-1}\right)\sqrt{\frac{B}{u}} \tag{11}$$

One can study the energy of the rarefactive solitary waves using Fig. 5. This figure demonstrates that the energy of solitons increases when superthermal parameter increases. Thus, the soliton energy decreases in the presence of superthermal electrons. We can also see that the energy of solitons increases with increasing values of α. Therefore the energy of solitons increases when density of hot electrons increases.

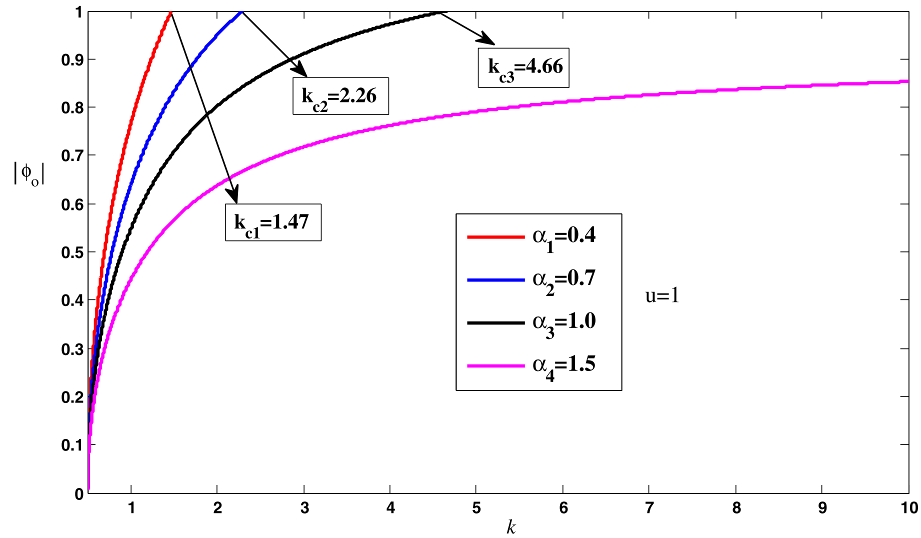

Fig. 4 The variation of the soliton amplitude with respect to k for $u = 1$ and different values of α

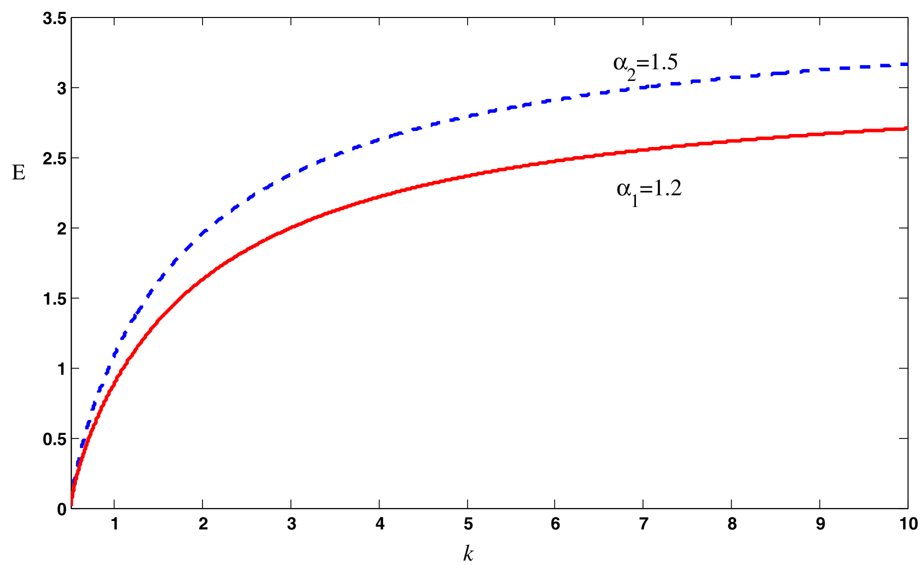

Fig. 5 Energy of soliton respect to k-parameter for $u = 1$ and different values of α

4 Conclusion

To conclude, we have addressed the problem of electron-acoustic oscillations in unmagnetized collisionless plasmas comprising cold fluid electrons, superthermal hot electrons and stationary ions. We have derived the linear dispersion relation and also found that ω is shifted towards higher values as the hot electron component evolves towards its Maxwell-Boltzmann thermodynamic equilibrium. We have also derived the KdV equation using the reductive perturbation method. Our results show that solitary waves appear in such plasmas and the effect of the superthermal hot electrons modifies the structure of the waves. In a weakly nonlinear analysis of solitary waves with superthermal hot electrons, there is threshold value for k (k_c), such that the acceptable value of k-parameter is smaller than k_c. In addition, the domain of allowable k-parameter shrinks as the relative number of hot electrons decreases. The energy of soliton and linear dispersion relation have been derived and discussed too. We have shown that increasing superthermal parameter results in the increase of the angular frequency. We have also found that the energy of compressive (rarefactive) solitons, increases (decreases) in the presence of superthermal electrons. It may be pointed out that the results of this investigation may be useful for understanding some nonlinear behavior of electrostatic waves about the strong spiky waveforms which have been observed in auroral electric fields (Ergun et al. 1998).

References

Abbasi, H., Pajouh, H.H.: Phys. Plasmas **14**, 012307 (2007)
Annou, R.: Phys. Plasmas **5**, 2813 (1998)

Armstrong, T.P., Paonessa, M.T., Bell, E.V., Krimigis, S.M.: J. Geophys. Res. **88**, 8893 (1983). doi:10.1029/JA088iA11p08893
Baluku, T.K., Hellberg, M.A.: Phys. Plasmas **15**, 123705 (2008)
Baluku, T.K., Hellberg, M.A., Kourakis, I., Saini, N.S.: Phys. Plasmas **17**, 053702 (2010)
Berthomier, M., Pottelette, R., Malingre, M., Khotyaintsev, Y.: Phys. Plasmas **7**, 2987 (2000)
Berthomier, M., Pottelette, R., Muschietti, L., Roth, I., Carlson, C.W.: Geophys. Res. Lett. **30**, 2148 (2003)
Boubakour, N., Tribeche, M., Aoutou, K.: Phys. Scr. **79**, 065503 (2009)
Chatterjee, P., Roychoudhury, R.K.: J. Plasma Phys. **53**, 25 (1995)
Dubouloz, N., Pottelette, R., Malingre, M., Treumann, R.A., Watanabe, K., Taniuti, T.: J. Phys. Soc. Jpn. **43**, 1819 (1977)
El-Shewy, E.K.: Chaos Solitons Fractals **31**, 1020 (2007a)
El-Shewy, E.K.: Chaos Solitons Fractals **34**, 628 (2007b)
Ergun, R., Carlson, B., McFadden, M.: Geophys. Res. Lett. **25**, 2025 (1998)
Feldman, W.C., Asbridge, J.R., Bame, S.J., Montgomery, M.D.: J. Geophys. Res. **78**, 2017 (1973)
Formisano, V., Moreno, G., Palmiotto, F.: J. Geophys. Res. **78**, 3714 (1973)
Gary, S.P., Tokar, R.L.: Phys. Fluids **28**, 2439 (1985)
Gill, T.S., Kaur, H., Saini, N.S.: Chaos Solitons Fractals **30**, 1020 (2006)
Hasegawa, A., Mima, K., Duong-Van, M.: Phys. Rev. Lett. **54**, 2608 (1985)
Hellberg, M.A., Mace, R.L.: Phys. Plasmas **9**, 1495 (2002)
Hellberg, M.A., Mace, R.L., Bakalu, T.K., Kourakis, I., Saini, N.S.: Phys. Plasmas **16**, 094701 (2009)
Leubner, M.P.: J. Geophys. Res. **87**, 6335 (1982). doi:10.1029/JA087iA08p06335
Mace, R.L., Hellberg, M.A.: Phys. Plasmas **2**, 2098 (1995a)
Mace, R.L., Hellberg, M.A.: Phys. Plasmas **2**, 2098 (1995b)
Mace, R.L., Hellberg, M.A.: Phys. Plasmas **8**, 2649 (2001)
Magni, S., Roman, H.E., Barni, R., Riccardi, C., Pierre, Th., Guyomarc'h, D.: Phys. Rev. E **72**, 026403 (2005)
Mamun, A.A., Shukla, P.K.: J. Geophys. Res. **107**, 1135 (2002)
Mamun, A.A., Shukla, P.K., Stenflo, L.: Phys. Plasmas **9**, 1474 (2002)
Marsch, E., Muhlhauser, K.H., Schwenn, R., Rosenbauer, H., Pilipp, W., Neubauer, F.M.: J. Geophys. Res. **87**, 52 (1982)
Montgomery, D.S., Focia, R.J., Rose, H.A., Russell, D.A., Cobble, J.A., Fernandez, J.C., Johnson, R.P.: Phys. Rev. Lett. **87**, 155001 (2001)
Pakzad, H.R., Tribeche, M.: Astrophys. Space Sci. Trans. **330**, 95–99 (2010)
Podesta, J.J.: Phys. Plasmas **12**, 052101 (2005)
Pottelette, R., Ergun, R.E., Treumann, R.A., Berthomier, M., Carlson, C.W., McFadden, J.P., Roth, I.: Geophys. Res. Lett. **26**, 2629 (1999)
Sahu, B.: Phys. Plasmas **17**, 122305 (2010a)
Sahu, B.: Phys. Plasmas **17**, 122305 (2010b)
Samanta, S., Misra, A.P., Chowdhury, A.P.: Planet. Space Sci. **55**, 1380 (2007)
Schriver, D., Ashour-Abdalla, M.: Geophys. Res. Lett. **16**, 899 (1989)
Scudder, J.D., Sittler, E.C., Bridge, H.S.: J. Geophys. Res. **86**, 8157 (1981)
Shukla, P.K., Mamun, A.A., Eliasson, B.: Geophys. Res. Lett. **31**, L07803 (2004)
Singh, S., Honzawa, T.: Phys. Fluids B **5**, 2093 (1993)
Singh, S.V., Lakhina, G.S.: Planet. Space Sci. **49**, 107 (2001)
Sultana, S., Kourakis, I., Saini, N.S., Hellberg, M.A.: Phys. Plasmas **17**, 032310 (2010)
Summers, D., Thorne, R.M.: Phys. Fluids B **3**, 1835 (1991)
Summers, D., Thorne, R.M.: Phys. Plasmas **1**, 2012 (1994)
Thorne, R.M., Summers, D.: Phys. Fluids B **3**, 2117 (1991)
Tokar, R.L., Gary, S.P.: Geophys. Res. Lett. **11**, 1180 (1984)
Tribeche, M., Boubakour, N.: Phys. Plasmas **16**, 084502 (2009)
Vasyliunas, V.M.: J. Geophys. Res. **73**, 2839 (1968). doi:10.1029/JA073i009p02839
Watanabe, K., Taniuti, T.: J. Phys. Soc. Jpn. **43**, 1819 (1977)
Younsi, S., Tribeche, M.: Astrophys. Space Sci. Trans. **330**, 295–300 (2010)