Astrophys Space Sci (2012) 337:147–150
DOI 10.1007/s10509-011-0809-4

ORIGINAL ARTICLE

New class of well behaved exact Solutions for static charged Neutron-star with *perfect fluid*

Neeraj Pant

Received: 27 June 2011 / Accepted: 20 July 2011 / Published online: 1 September 2011

Abstract The paper presents a class of interior solutions of Einstein–Maxwell field equations in general relativity for a static spherically symmetric distribution of a charged fluid. This class of solutions describes well behaved charged fluid balls. This solution gives us wide range of parameter K ($0.53 \leq K \leq 0.95$), for which the solution is well behaved hence, suitable for modeling of super dense star. For this solution the mass of a star is maximized with all degree of suitability and by assuming the surface density $\rho_b = 2 \times 10^{14}$ g/cm^3. Corresponding to $K = 0.95$ with $X = -0.15$, the maximum mass of the star comes out to be $M = 1.56\ M_\Theta$ with radius $r_b \approx 9.22$ km and the surface red shift $Z_b \approx 0.124207$. It has been observed that under well behaved conditions this class of solutions gives us the mass of super dense object within the range of neutron star. However, its neutral counter part is not well behaved.

Keywords Charge fluid · Reissener–Nordstrom · General relativity · Exact solution

1 Introduction

Ever since the formulation of Einstein's field equations, the relativists have been proposing different models of immense gravity astrophysical objects by considering the distinct nature of matter or radiation(energy-momentum tensor) present in them. Such models successfully explain the characteristics of massive objects like quasar, neutron star, pulsar, quark star, black-hole or other super-dense object. These stars are specified in terms of their masses as white dwarfs (Mass <1.44 solar mass), Quark star (2 solar mass–3 solar mass) and Neutron star (1.35 solar mass–2.1 solar mass).

Exact solutions with well behaved nature of Einstein–Maxwell field equations are of vital importance in relativistic astrophysics. Such solutions may be used to make a suitable model of super dense object with charge matter. Eventually, these exact solutions of the Einstein-Maxwell field equations joining smoothly to the Nordstrom solution at the pressure free interface. It is interesting to observe that, in the presence of charge, the gravitational collapse of a spherical symmetric distribution of the matter to a point singularity may be avoided. In this scenario gravitational attraction is counter balanced by the Colombian repulsive force together with the pressure gradient. On account of the nonlinearity of Einstein–Maxwell field equations, not many realistic well behaved, analytic solutions are known for the description of relativistic charge fluid spheres. For well behaved model of relativistic star with charged and perfect fluid matter, following conditions should be satisfied (Pant et al. 2011a):

(i) The solution should be free from physical and geometrical singularities i.e. finite and positive values of central pressure, central density and non zero positive values of e^{λ} and e^{ν}.
(ii) The solution should have positive and monotonically decreasing expressions for pressure and density (p and ρ) with the increase of r. The solution should have positive value of ratio of pressure-density and less than 1 (weak energy condition) and less than 1/3 (strong energy condition) throughout within the star.
(iii) The solution should have positive and monotonically decreasing expression for fluid parameter $\frac{p}{\rho c^2}$ with the increase of r.
(iv) The solution should have positive and monotonically decreasing expression for velocity of sound ($\frac{dp}{d\rho}$) with

N. Pant (✉)
Department of Mathematics, National Defence Academy
Khadakwasla, Pune 411023, India
e-mail: neeraj.pant@yahoo.com

the increase of r and causality condition should be obeyed at the centre i.e. $\frac{dp}{c^2 d\rho} < 1$.

(v) The red shift Z should be positive, finite and monotonically decreasing in nature with the increase of r.

(vi) Electric intensity E is positive and monotonically increasing from centre to boundary and at the centre the Electric intensity is zero.

Under these well behaved conditions, one has to assume the gravitational potential and electric field intensity in such a way that the field equation can be integrated and solution should be well behaved. Keeping in view of this aspect, several authors obtained the parametric class of exact solutions Pant et al. (2011a, 2011b), Gupta and Maurya (2011), Maurya and Gupta (2011a, 2011b), Pant (2011a, 2011b), Bijalwan (2011) etc. These coupled solutions are well behaved with some positive values of charge parameter $\boldsymbol{K}$ and completely describe interior of the super-dense astrophysical object with charge matter. Further, the mass of the such modeled super dense object can be maximized by assuming surface density is $\rho_b = 2\times 10^{14}$ g/cm^3. In the present paper we have obtained yet another new parametric class of well behaved exact solutions of Einstein–Maxwell field equations, which is compatible within the range of Neutron star.

2 Einstein–Maxwell equation for charged fluid distribution

Let us consider a spherical symmetric metric in curvature coordinates

$$ds^2 = -e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu} dt^2 \tag{1}$$

where the functions $\lambda(r)$ and $\nu(r)$ satisfy the Einstein–Maxwell equations

$$\begin{aligned} -\frac{8\pi G}{c^4} T^i_j &= R^i_j - \frac{1}{2} R\delta^i_j \\ &= -\frac{8\pi G}{c^4}\Big[(c^2\rho + p)v^i v_j - p\delta^i_j \\ &\quad + \frac{1}{4\pi}\Big(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F_{mn}F^{mn}\Big)\Big] \end{aligned} \tag{2}$$

where ρ, p, v^i, F_{ij} denote energy density, fluid pressure, velocity vector and skew-symmetric electromagnetic field tensor respectively.

In view of the metric (1), the field equation (2) gives,

$$\frac{\nu'}{r} e^{-\lambda} - \frac{(1-e^{-\lambda})}{r^2} = \frac{8\pi G}{c^4} p - \frac{q^2}{r^4}, \tag{3a}$$

$$\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu'-\lambda'}{2r}\right) e^{-\lambda} = \frac{8\pi G}{c^4} p + \frac{q^2}{r^4}, \tag{3b}$$

$$\frac{\lambda'}{r} e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \frac{8\pi G}{c^2}\rho + \frac{q^2}{r^4} \tag{3c}$$

where, prime ($'$) denotes the differentiation with respect to r and $q(r)$ represents the total charge contained within the sphere of radius r.

By using the transformation

$$e^{\nu} = B(1+x)^{-1/3}, \quad \text{and} \quad e^{-\lambda} = Y. \tag{4}$$

where B being positive constants. Now putting (4) into (3a)–(3c), we have

$$\frac{-2Y}{3(1+x)} - \frac{(1-Y)}{x} + \frac{c_1 q^2}{x^2} = \frac{1}{c_1}\frac{8\pi G}{c^4} p, \tag{5}$$

$$\frac{(1-Y)}{x} - 2\frac{dY}{dx} - \frac{c_1 q^2}{x^2} = \frac{1}{c_1}\frac{8\pi G}{c^2}\rho \tag{6}$$

and Y satisfying the equation

$$\frac{dY}{dx} + \frac{-2x^2 - 18x - 9}{3x(1+x)(3+2x)} Y = \frac{3(1+x)}{x(3+2x)}\left(\frac{2c_1 q^2}{x} - 1\right), \tag{7}$$

$$\frac{E^2}{c_1} = \frac{c_1 q^2}{x^2} = \frac{9Kx}{2(3+2x)^2} \tag{8}$$

In view of (8) differential equation (7) yields the following solution

$$\begin{aligned} Y &= e^{-\lambda} \\ &= -81K.\frac{x(1+x)^2}{(3+2x)^3} \\ &\quad + \frac{27}{(3+2x)^3}\left[Ax(1+x)^{\frac{7}{3}} + \frac{(3+4x)(1+x)^2}{3}\right] \end{aligned} \tag{9}$$

where A is an arbitrary constant of integration.

3 Properties of the new class of solutions

Using (4), (8), (9) into (5) and (6), we get the following expressions for pressure and energy density

$$\begin{aligned} \frac{1}{c_1}\frac{8\pi G}{c^4} p &= 9K\frac{(-4x^2 - 21x - 18)}{2(2+3x)^3} \\ &\quad + \frac{1}{(3+2x)^3}\big[9A(1+x)^{\frac{4}{3}}(x+3) \\ &\quad + (18 + 21x + 4x^2)\big] \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{1}{c_1}\frac{8\pi G}{c^2}\rho &= 9K.\frac{(32x^3 + 294x^2 + 423x + 162)}{(3+2x)^4 2} \\ &\quad - \frac{1}{2(3+2x)^4}\big[18A(27 + 51x + 10x^2)(1+x)^{\frac{4}{3}} \\ &\quad + (-648 - 1458x - 924x^2 - 112x^3)\big] \end{aligned} \tag{11}$$

Differentiating (10) and (11) w.r.t. x., we get;

$$\frac{1}{c_1}\cdot\frac{8\pi G}{c^4}\frac{dp}{dx}=\frac{9K(8x^2+60x+45)+6A(-9-21x-4x^2)(1+x)^{\frac{1}{3}}-90-120x-16x^2}{(3+2x)^4} \tag{12}$$

$$\frac{1}{c_1}\cdot\frac{8\pi G}{c^2}\frac{d\rho}{dx}=\frac{9K(-64x^3-888x^2-774x-27)}{2(3+2x)^5}+\frac{224x^3+2688x^2+3204x+810-6A(1+x)^{\frac{1}{3}}\{-40x^3-330x^2+99x+135)}{2(3+2x)^5} \tag{13}$$

The expression for gravitational red-shift (z) is given by

$$z=\frac{(1+x)^{1/6}}{\sqrt{B}}-1 \tag{14}$$

The central value of gravitational red shift to be non zero positive finite and monotonically decreasing with the increase of radial coordinate r, we have

$$1>\sqrt{B}>0 \tag{15}$$

$$x<0 \quad \text{or} \quad c_1<0 \tag{16}$$

4 Boundary conditions

The solutions so obtained are to be matched over the boundary with Reissner–Nordstrom metric;

$$ds^2=-\left(1-\frac{2GM}{c^2r_b}+\frac{e^2}{r_b^2}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2)+\left(1-\frac{2GM}{c^2r_b}+\frac{e^2}{r_b^2}\right)dt^2 \tag{17}$$

which requires the continuity of e^{λ}, e^{ν} and q across the boundary $r=r_b$

$$e^{-\lambda(r_b)}=1-\frac{2GM}{c^2r_b}+\frac{e^2}{r_b^2}, \tag{18}$$

$$e^{\nu(r_b)}==1-\frac{2GM}{c^2r_b}+\frac{e^2}{r_b^2}, \tag{19}$$

$$q(r_b)=e, \tag{20}$$

$$p(r_b)=0 \tag{21}$$

On setting $x_{r=r_b}=X=c_1r_b^2$, (r_b being the radius of the charged sphere)

Pressure at $p_{(r=r_b)}=0$ gives

$$A=-\frac{9K(4X^2+21X+18)-(8X^2+42X+36)}{18(X+3)(1+X)^{\frac{3}{2}}} \tag{22}$$

In view of (18) and (19) we get,

$$B=\left\{-81K\cdot\frac{X(1+X)^2}{(3+2X)^3}+\frac{27}{(3+2X)^3}\times\left[AX(1+X)^{\frac{7}{3}}+\frac{(3+4X)(1+X)^2}{3}\right]\right\}(1+X)^{\frac{1}{3}} \tag{23}$$

The expression for mass can be written as

$$\frac{GM}{c^2}=\frac{r_b}{2}\left[1+-81K\cdot\frac{X(1+X)^2}{(3+2X)^3}+\frac{27}{(3+2X)^3}\times\left[AX(1+X)^{\frac{7}{3}}+\frac{(3+4X)(1+X)^2}{3}\right]\right] \tag{24}$$

The surface density is given by

$$\frac{8\pi G}{c^2}\rho_b r_b^2=X\left\{\begin{array}{l}9K\cdot\frac{(32X^3+294X^2+423X+162)}{(3+2X)^42}+\\ -\frac{1}{2(3+2X)^4}\left[18A(27+51X+10X^2)(1+X)^{\frac{4}{3}}+(-648-1458X-924X^2-112X^3)\right]\end{array}\right\} \tag{25}$$

Centre red shift is given by

$$z_0=B^{-1/2}-1 \tag{26}$$

5 Discussion

In view of and Table 1 it has been observed that all the physical parameters (p, ρ, $\frac{p}{\rho c^2}$, $\frac{dp}{d\rho}$, z and E) are positive at the centre and within the limit of realistic equation of state and well behaved conditions for all values of K satisfying the inequalities $0.53\le K\le 0.95$. However, corresponding to any value of $K<0.53$, there exist no value of X for which causality principle is obeyed at the centre and for $K>0.95$, the central pressure is negative for all values of X.

It has been observed that under well behaved conditions this class of solutions gives us the mass of super dense object

Table 1 The march of pressure, density, pressure–density ratio, square of adiabatic sound speed, gravitational red shift and electric field intensity within the ball corresponding to $K = 0.95$ with $X = -0.15$. By assuming surface density; $\rho_b = 2 \times 10^{14}\ \mathrm{g/cm^3}$. The resulting well behaved model has the mass $M = 1.56\ M_\Theta$ with radius $r_b \approx 9.22$ km

r/r_b	$\frac{8\pi G}{c^4} p r_b^2$	$\frac{8\pi G}{c^2} \rho r_b^2$	$(\frac{p}{\rho c^2})$	$\frac{1}{c^2}(\frac{dp}{d\rho})$	Z	$E \times r_b$
0	0.027864	0.383593	0.07264	0.373806	0.170824	0
0.1	0.027632	0.382972	0.072152	0.364936	0.170385	0.217926
0.2	0.026931	0.381102	0.070666	0.357237	0.169064	0.274879
0.3	0.025746	0.377959	0.068117	0.350794	0.166852	0.315251
0.4	0.02405	0.373507	0.06439	0.345747	0.163735	0.3479
0.5	0.021808	0.367694	0.059309	0.33922	0.15969	0.376051
0.6	0.018967	0.360452	0.05262	0.320727	0.154687	0.401304
0.7	0.015462	0.351706	0.043963	0.315442	0.14869	0.424596
0.8	0.011208	0.341367	0.032834	0.305003	0.141652	0.446537
0.9	0.006099	0.329346	0.018519	0.295222	0.133514	0.467559
1.0	0	0.315554	0	0.283806	0.124207	0.487987

within the range of neutron star. When $K = 0$, we are left with a solution which is not well behaved.

We now present here a model of super dense star based on the particular solution discussed above corresponding to $K = 0.95$ with $X = -0.15$, by assuming surface density; $\rho_b = 2 \times 10^{14}\ \mathrm{g/cm^3}$. The resulting well behaved model has the mass $M = 1.56\ M_\Theta$ with radius $r_b \approx 9.22$ km and the surface red shift $Z_b \approx 0.124207$.

Acknowledgements Neeraj Pant acknowledges his gratitude to Lt. Gen. Jatinder Singh AVSM, SM, Comdt, NDA his motivation and encouragement.

N. Pant is grateful to the referee for pointing out the errors in original manuscript and making constructive suggestions.

References

Bijalwan, N.: Astrophys. Space Sci. (2011). doi:10.1007/s10509-011-0691-0
Gupta, Y.K., Maurya, S.K.: Astrophys. Space Sci. **332**, 155 (2011)
Maurya, S.K., Gupta, Y.K.: Astrophys. Space Sci. **332**, 481 (2011a)
Maurya, S.K., Gupta, Y.K.: Astrophys. Space Sci. (2011b). doi:10.1007/s10509-011-0616-y
Pant, N.: Astrophys. Space Sci. **332**, 403 (2011a)
Pant, N.: Astrophys. Space Sci. (2011b). doi:10.1007/s10509-011-0720-z
Pant, N., et al.: Astrophys. Space Sci. **332**, 473 (2011a)
Pant, N., et al.: Astrophys. Space Sci. **333**, 161 (2011b)