Astrophys Space Sci (2011) 336:419–426
DOI 10.1007/s10509-011-0782-y

ORIGINAL PAPER

A class of well behaved charged superdense star models of embedding class one

Y.K. Gupta · Jitendra Kumar

Received: 8 April 2011 / Accepted: 21 June 2011 / Published online: 2 July 2011

Abstract A class of well behaved charged superdense star models of embedding class one is obtained by taking perfect fluid to be interior matter. In the process we come across the models for white dwarf, quark and neutron stars. Maximum mass of the star of this class is found to be $6.716998 M_\Theta$ with its radius is 18.92112 Km. In the absence of charge the models reduce to Schwarzschild's interior model with constant density.

Keywords Exact solutions · Charged fluids · Superdense stars · General relativity · Negative redshift

1 Introduction

1.1 Charged fluids

The presence of charge in a charged fluid has tendency to resist the gravitational collapse. This property persuades the research workers to work on the charged perfect fluid distributions. Many of the workers, charged the well known uncharged perfect fluid solutions e.g. Kuchowicz (1968) solutions by Nduka (1977), Adler (1974)–Wyman (1949) solutions by Nduka (1976) and Singh and Yadav (1978), Klein (1947)–Tolman (1939) solutions by Pant and Sah (1979) Tikekar (1984) and Cataldo and Mitskievic (1992). In addition to that Sharma et al. (2001), Gupta et al. (2011) and Gupta and Kumar (2005b, 2005c) charged the Vaidya–Tikekar type solutions (Mukherjee et al. 1997; Vaidya and Tikekar 1982) and Buchdahl's (1959) fluid spheres. In this context the work by Maharaj and Komathiraj (2007a, 2007b, 2008), Maharaj and Thirukkanesh (2006a, 2006b, 2009). Maharaj and Hensraj (2006), may also be visited. Recently Gupta and Maurya (2010a, 2010b) charged the Durgapal (1982), Durgapal and Fuloria (1985) Neeraj Pant et al. (2010) charged the Heintzmann (1969) solutions. Schwarzchild's solution has also been charged by Banerjee and Som (1981), Cohen and Cohen (1969), Grøn (1985), Gupta and Gupta (1986), Florides (1983), Guilfoyle (1999) and Gupta and Kumar (2005a). All the solutions mentioned above are reducible to their neutral counterpart in the absence of charge.

1.2 Models of astrophysical interest

In the present paper the charged fluid spheres of embedding class one are obtained and utilized to construct the models of superdense star models with surface density 2×10^{14} gm/cm^3. In this process we come across the various astrophysical objects like white dwarf, quark and neutron stars. The composition of the superdense matter in the core remains uncertain. One model describes the core as superfluid neutron-degenerate matter (mostly neutrons, with some protons and electrons). More exotic forms of matter are possible, including degenerate strange matter (containing strange quarks in addition to up and down quarks), matter containing high-energy pions and kaons in addition to neutrons or ultra-dense quark-degenerate matter. We are accustomed to longevity in astronomy. The Sun has burned for 4.5 billion years, orbited by planets of equal age. Many of the stars in our galaxy are over 10 billion years old. These stars will eventually burn out and grow cold, but they will change only slowly. But this is not the fate of all stars, for some stars are vulnerable to a catastrophic collapse. Gravitational collapse produce supernovae from blue giants and

Y.K. Gupta · J. Kumar (✉)
Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee, Uttrakhand 247667, India
e-mail: jitendark@gmail.com

degenerate dwarfs. Gravitational collapse sets a maximum mass for both the degenerate dwarf and the neutron star. Gravitational collapse is the creator of black hole. And gravitational collapse, whether of a blue giant, degenerate dwarf, or a neutron star, in all cases has the same principal cause. A typical neutron star has mass between 1.35 and 2.1 solar mass. About radius 1.9 solar mass neutron star can have radius 10.7, 11.1, 12.1 or 15.1 Km. In general, compact stars of less than 1.44 solar masses—the Chandrasekhar limit—are white dwarfs, and above 2 to 3 solar masses (the Tolman-Oppenheimer-Volkoff limit), a quark star might be created; however, this is uncertain. Gravitational collapse will usually occur on any compact star between 10 and 25 solar masses and produce a black hole. The equation of state for superdense stars is not very certain so far.

1.3 Embedding class

The idea that our space-time can be considered as a four-dimensional space embedded in a higher-dimensional flat space is an old and recursive one (Eddington 1924). Recently, due to a proposal by Randall and Sundram (1999) and discussions by Anchordoqui and Bergila (2000), this idea has again attracted much attention. It is well known that the manifold V_n can always be embedded in Pseudo-Euclidian space E_m of m dimensions, where $m = \frac{n(n+1)}{2}$. The minimum extra dimension p of the Pseudo-Euclidian space to embedded V_n in E_m, is called the class of the manifold V_n and must be less than or equal to the number $(m-n)$ i.e., $\frac{n(n-1)}{2}$. In case of relativistic space time V_4, the embedding class p turns out to be 6. In particular the class of spherical and plane symmetric space-time are 2 and 3 respectively. The famous Friedman-Robertson-Lemaitre space-time, (1933) is of class 1, while the Schwarzschild's exterior and interior solutions are of class 2 and class 1 respectively. Moreover the famous Kerr metric of class 5, (Kuzeev 1980). The postulates of general relativity do not provide any physical meaning to higher dimensional embedding space. However, it provides new characterizations of gravitational field, which hopefully, can be connected to physics. Some researchers are linked the group of motions of flat embedding space to the internal symmetries of elementary particle physics (Rayski 1976). Some have utilized the higher dimensions to study the singularity of the space-time. Recently, Pavsic and Tapia (2001) have published an article entitled "Resources letter on geometrical results for embedding and Brane" where many references regarding the applications of embedding to general relativity, extrinsic gravity, strings and membranes and new brane world are mentioned.

2 Charged fluids of embedding class one

Let us consider the metric to be

$$ds^2 = -a(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + c(r)dt^2 \tag{1}$$

which may represent space-time of embedding class one, if it satisfies the Karmarker condition (1948)

$$R_{1414} = \frac{R_{1212}R_{1414} - R_{1224}R_{1334}}{R_{2323}}$$

with $R_{2323} \neq 0$ (Pandey and Sharma 1981).

The above condition with reference to (1) yields the following differential equation

$$\frac{a'}{a(1-a)} = \frac{-2c''}{c'} + \frac{c'}{c} + \frac{a'}{a}, \quad a \neq 1 \tag{2}$$

if we set $c = y^2$, the solution of the differential equation gives rise

$$a = \left(1 + K y'^2\right) \tag{3}$$

In order to derive charged fluid solution $a(r)$ and $c(r)$ are to satisfy the Einstein-Maxwell equations

$$R^i_j - \frac{1}{2}R\delta^i_j = -\kappa\left[\left(c^2\rho + p\right)v^i v_j - p\delta^i_j + \frac{1}{4\pi}\left(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F_{mn}F^{mn}\right)\right] \tag{4}$$

where $\kappa = \frac{8\pi G}{c^4}$, ρ, p and v^i denote energy density, fluid pressure and flow vector of the fluid, respectively and F_{ik} being the skew-symmetric electromagnetic field tensor satisfying the Maxwell equations

$$F_{ik,j} + F_{kj,i} + F_{ji,k} = 0 \tag{5}$$

$$\frac{\partial}{\partial x^k}\left(\sqrt{-g}F^{ik}\right) = -4\pi\sqrt{-g}\, j^i \tag{6}$$

where $j^i = \sigma v^i$ represents the four-current vector of charged fluid while the charged density is denoted by σ.

The field equation (4) with respect to the metric equation (1) give (Dionysiou 1982)

$$-\frac{\nu'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = -\kappa p + \frac{q^2}{r^4} \tag{7}$$

$$-\left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right]e^{-\lambda} = -\kappa p - \frac{q^2}{r^4} \tag{8}$$

$$\frac{\lambda'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \kappa c^2\rho + \frac{q^2}{r^4} \tag{9}$$

where

$$q(r)=4\pi\int_0^r \sigma r^2 e^{\lambda/2}dr = r^2\sqrt{-F_{14}F^{14}} \\ = r^2 F^{41} e^{(\lambda+\nu)/2} \tag{10}$$

represents the total charge contained within the sphere of radius r. Equation (6) reduces to

$$\frac{\partial}{\partial r}\big(r^2 F^{41} e^{(\lambda+\nu)/2}\big) = -4\pi r^2 e^{(\lambda+\nu)/2} j^4 \tag{11}$$

here we have taken $a(r)=e^{\lambda(r)}$ and $c(r)=e^{\nu(r)}$.

At the pressure free interface $r=a$ the charged fluid sphere is expected to join with the Reissner-Nordstrom metric

$$ds^2 = -\left(1-\frac{2m}{r}+\frac{e^2}{r^2}\right)^{-1} dr^2 - r^2\big(d\theta^2+\sin^2\theta d\phi^2\big) \\ + \left(1-\frac{2m}{r}+\frac{e^2}{r^2}\right)dt^2 \tag{12}$$

where m is the gravitational mass of the distribution such that

$$m=\mu(a)+\varepsilon(a)$$

while

$$\mu(a)=\frac{\kappa}{2}\int_0^a \rho r^2 dr, \qquad \varepsilon(a)=\frac{\kappa}{2}\int_0^a r\sigma q e^{\lambda/2}dr, \tag{13}$$
$$e=q(a)$$

$\varepsilon(a)$ is the mass equivalence of the electromagnetic energy of distribution while $\mu(a)$ is the mass and e is the total charge inside the sphere (Florides 1983).

From (7)–(9) and (3), we get

$$\frac{y'}{r^2(1+Ky'^2)}\left(Ky'-\frac{2r}{y}\right)=\kappa p+\frac{q^2}{r^4} \tag{14}$$

$$\frac{1}{y(1+Ky'^2)}\left[\frac{Kyy'y''}{r(1+Ky'^2)}-\frac{y'}{r}-\frac{y''}{(1+Ky'^2)}\right] \\ =\kappa p-\frac{q^2}{r^4} \tag{15}$$

$$\frac{2Ky'y''}{r(1+Ky'^2)^2}+\frac{Ky'^2}{r^2(1+Ky'^2)}=\kappa c^2\rho+\frac{q^2}{r^4} \tag{16}$$

If we subtract (15) from (14), we get the expression for charge function as

$$\left(\frac{Kyy'}{r}-1\right)\left(\frac{y'}{ry(1+Ky'^2)}-\frac{y''}{y(1+Ky'^2)^2}\right)=\frac{2q^2}{r^4} \tag{17}$$

In the absence of charge either of the two factors on the left hand side of (17) has to be zero. It can be verified that the vanishing of the fist factor and then the consistency of (14) and (15) gives rise famous Kohler–Chao solution (1965)

$$ds^2=-\frac{(\alpha+2\beta r^2)}{(\alpha+\beta r^2)}dr^2-r^2\big(d\theta^2+\sin^2\theta d\phi^2\big) \\ +\big(\alpha+\beta r^2\big)dt^2 \tag{18}$$

with the expression for pressure and density given as

$$\kappa p=\frac{\beta}{\alpha+2\beta r^2} \tag{19}$$

$$\kappa c^2\rho=\beta\frac{(3\alpha+2\beta r^2)}{(\alpha+\beta r^2)^2} \tag{20}$$

Which cannot represent a compact star or sphere as is does not posses zero pressure for any finite radius. However the vanishing of the second factor gives rise ultimately the famous Schwarschild's interior solution

$$ds^2=-\left(1-\frac{r^2}{R^2}\right)^{-1}dr^2-r^2\big(d\theta^2+\sin^2\theta d\phi^2\big) \\ +\left(\alpha+\beta\sqrt{1-\frac{r^2}{R^2}}\right)^2 dt^2 \tag{21}$$

with its pressure and density to be

$$\kappa p=-\frac{(3\beta\sqrt{1-\frac{r^2}{R^2}})+\alpha}{R^2(\alpha+\beta\sqrt{1-\frac{r^2}{R^2}})} \tag{22}$$

$$\kappa c^2\rho=\frac{3}{R^2} \tag{23}$$

provided α and R are non zero and $\beta>0$.

Therefore it would be better to charge the Schwarzschild's interior solution.

In order to obtain the charge analogues of Schwarzschild's interior solution, let us consider

$$y=A+\sqrt{(B+Cr^2)}, \quad B>0,\ C,A\neq 0 \tag{24}$$

Now form (14)–(16) and (24), we get the expression of pressure, density and charge following as

$$\kappa p=\frac{C}{2(B+Cr^2+KC^2r^2)} \\ \times\left[\frac{(4B+3Cr^2)}{\sqrt{(B+Cr^2)}(A+\sqrt{(B+Cr^2)})}-KC \\ -\frac{KBC}{(B+Cr^2+KC^2r^2)} \\ \times\left[\frac{Cr^2}{\sqrt{(B+Cr^2)}(A+\sqrt{(B+Cr^2)})}+1\right]\right] \tag{25}$$

$$\kappa c^2\rho = C\left[\frac{KC}{(B+Cr^2+KC^2r^2)} + \frac{2KBC}{(B+Cr^2+KC^2r^2)^2} + \frac{Cr^2(1+KC)\sqrt{(B+Cr^2)}}{2(B+Cr^2+KC^2r^2)^2} \times \left\{\frac{1}{A+\sqrt{(B+Cr^2)}} - \frac{KC}{\sqrt{(B+Cr^2)}}\right\}\right] \tag{26}$$

$$q^2 = -\frac{C^2r^6(1+KC)\sqrt{(B+Cr^2)}}{2(B+Cr^2+KC^2r^2)^2} \times \left\{\frac{1}{A+\sqrt{(B+Cr^2)}} - \frac{KC}{\sqrt{(B+Cr^2)}}\right\} \tag{27}$$

The expressions for density and pressure gradient can be written as

$$\kappa c^2\frac{d\rho}{dr} = 2C^2r(M_1+M_2+N_3M_4+N_4M_3) \tag{28}$$

where

$$M_1 = -\frac{KC(1+KC)}{(B+Cr^2+KC^2r^2)^2},$$

$$M_2 = -\frac{4KBC(1+KC)}{(B+Cr^2+KC^2r^2)^3}$$

$$N_3 = \frac{Cr^2(1+KC)\sqrt{B+Cr^2}}{2(B+Cr^2+KC^2r^2)^2}$$

$$M_3 = \frac{(1+KC)}{2}\left[\frac{(B+Cr^2+KC^2r^2)(2B+3Cr^2)}{2\sqrt{B+Cr^2}(B+Cr^2+KC^2r^2)^3} - \frac{4Cr^2(1+KC)(B+Cr^2)}{2\sqrt{B+Cr^2}(B+Cr^2+KC^2r^2)^3}\right]$$

$$N_4 = \frac{1}{A+\sqrt{(B+Cr^2)}} - \frac{KC}{\sqrt{(B+Cr^2)}}$$

$$M_4 = \frac{KC}{2(B+Cr^2)^{3/2}} - \frac{1}{2\sqrt{(B+Cr^2)}(A+\sqrt{(B+Cr^2)})^2}$$

$$\kappa\frac{dp}{dr} = 2C^2r\big[P_1\big[\{Q_2-(P_3Q_4+P_4Q_3)\} \times (B+Cr^2) + (P_2-P_3P_4)\big] + Q_1\big[(P_2-P_3P_4)(B+Cr^2)-KC\big]\big] \tag{29}$$

where

$$P_1 = \frac{1}{2(B+Cr^2KC^2r^2)},$$

$$P_2 = \frac{(4B+3Cr^2)}{(B+Cr^2)^{3/2}(A+\sqrt{B+Cr^2})}$$

$$P_3 = \frac{KBC}{(B+Cr^2)(B+Cr^2+KC^2r^2)},$$

$$P_4 = \left(\frac{Cr^2}{\sqrt{B+Cr^2}(A+\sqrt{B+Cr^2})} + 1\right)$$

$$Q_1 = -\frac{(1+KC)}{2(B+Cr^2+KC^2r^2)^2}$$

$$Q_2 = \frac{3(B+Cr^2)^{3/2}(A+\sqrt{B+Cr^2})}{(B+Cr^2)^3(A+\sqrt{B+Cr^2})^2} - \frac{(4B+3Cr^2)\{\frac{3}{2}\sqrt{B+Cr^2}(A+\sqrt{B+Cr^2}) + \frac{(B+Cr^2)}{2}\}}{(B+Cr^2)^3(A+\sqrt{B+Cr^2})^2}$$

$$Q_3 = -\frac{KBC[(1+KC)(B+Cr^2)+(B+Cr^2+KC^2r^2)]}{(B+Cr^2)^2(B+Cr^2+KC^2r^2)^2}$$

$$Q_4 = \frac{2(B+Cr^2)(A+\sqrt{B+Cr^2})}{2(B+Cr^2)^{3/2}(A+\sqrt{B+Cr^2})^2} - \frac{Cr^2\{C+(A+\sqrt{B+Cr^2})\}}{2(B+Cr^2)^{3/2}(A+\sqrt{B+Cr^2})^2}$$

The expression for velocity of sound $\sqrt{\frac{dp}{d\rho}}$ can be had from (28) and (29) as

$$\frac{dp}{c^2d\rho} = \frac{dp/dr}{c^2d\rho/dr} \tag{30}$$

and the expression of mass read as

$$m(r) = \frac{r}{2}\left[1 - \frac{C^2r^4(1+KC)\sqrt{(B+Cr^2)}}{2(B+Cr^2+KC^2r^2)^2} \times \left\{\frac{1}{A+\sqrt{(B+Cr^2)}} - \frac{KC}{\sqrt{(B+Cr^2)}}\right\}\right] \tag{31}$$

such that

$$e^{-\lambda} = 1 - \frac{2m(r)}{r} + \frac{q^2(r)}{r^2}$$

3 Physical conditions to be satisfied

For the well behaved charged fluid sphere (CFS) depends upon the following conditions inside and on the sphere $r = a$ are required to be satisfied.

(i) $\rho > 0,\ 0 \le r \le a$,
(ii) $p > 0,\ r < a$,
(iii) $p = 0,\ r = a$,
(iv) $dp/dr < 0,\ d\rho/dr < 0,\ 0 < r < a$

(v) $c^2\rho \geq p$ weak energy condition (WEC) or $c^2\rho \geq 3p$ strong energy condition (SEC) $0 \leq r \leq a$.

(vi) The velocity of sound $(dp/d\rho)^{1/2}$ should be less than that of light throughout the CFS ($0 \leq r \leq a$).

(vii) $\frac{d}{dr}(\frac{p}{c^2\rho}) < 0$.

(viii) $\frac{d}{dr}(\frac{dp}{c^2 d\rho}) < 0$.

(ix) The adiabatic constant $\gamma = ((\frac{c^2\rho+p}{p})(\frac{dp}{c^2 d\rho})) > \frac{4}{3}$.

(x) Red shift read as $(e^{-\nu/2} - 1)$ or $\frac{1}{|y(r)|} - 1$ (in the present case).

Beside the above the smooth joining with the Reissner-Nordström metric, requires the continuity of $e^{\lambda}, e^{\upsilon}$ and q across the pressure free interface $r = a$ and we get,

$$\frac{(B + Ca^2)}{(B + Ca^2 + KC^2a^2)} = 1 - \frac{2m(a)}{a} + \frac{q^2(a)}{a^2} = n \quad \text{(say)} \tag{32}$$

$$y^2(a) = 1 - \frac{2m(a)}{a} + \frac{q^2(a)}{a^2} \tag{33}$$

$$q(a) = e \tag{34}$$

$$p_{(r=a)} = 0 \tag{35}$$

After solving (32)–(35), we get

$$A = \sqrt{n}\left[1 - \frac{a^2(1-n) - q^2}{2a^2(n-t)}\right]$$

$$B = \frac{t[a^2(1-n) - q^2]^2}{4a^4(n-t)^2}$$

$$C = \frac{[a^2(1-n) - q^2]^2}{4a^6(n-t)}$$

$$K = \frac{4a^6(1-n)}{[a^2(1-n) - q^2]^2}$$

where

$$n = 1 - \frac{2m(a)}{a} + \frac{q^2(a)}{a^2}, \qquad t = 1 - \frac{2q^2(a)}{m(a)a}$$

or

$$n = \left(1 - Y + \frac{SY}{2}\right), \qquad t = (1 - 2S)$$

where

$$S = \frac{q^2(a)}{ma} \quad \text{and} \quad Y = \frac{2m}{a}.$$

4 Numerical investigation of models

For numerical investigation of model let us assume that $S = q^2(a)/ma$, $Y = 2m/a$, and $x = r/a$.

Table 1 $Y = .01$

$S = 0.001$, $Y = 0.01$, Radius = 2.833771 Km, $M = 0.014169 M_{\Theta}$, $z_0 = 0.007572$, $z_a = 0.005035$, White dwarf star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2d\rho$	P/D	γ
0	0.000061	0.030045	0.029863	0	0.577882	0.00202	286.6403
0.2	0.000058	0.030041	0.029866	0.000051	0.577864	0.001939	298.5435
0.4	0.000051	0.030028	0.029875	0.000406	0.57781	0.001697	341.055
0.6	0.000039	0.030007	0.029891	0.001369	0.577721	0.001293	447.3339
0.8	0.000022	0.029978	0.029912	0.003245	0.577597	0.000728	794.513
1	0	0.02994	0.02994	0.006337	0.57744	0	Inf

Table 2 $0 < Y \leq 0.03$

$S = 0.0022$, $Y = 0.03$, Radius = 4.902337 Km, $M = 0.073535 M_{\Theta}$, $z_0 = 0.023171$, $z_a = 0.015329$, White dwarf star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2d\rho$	P/D	γ
0	0.000596	0.090298	0.088511	0	0.85954	0.006598	131.1369
0.2	0.000572	0.09027	0.088555	0.000225	0.859447	0.006334	136.5445
0.4	0.0005	0.090187	0.088687	0.001799	0.859173	0.005543	155.8576
0.6	0.00038	0.090048	0.088906	0.006074	0.858724	0.004224	204.1417
0.8	0.000214	0.089853	0.089213	0.014407	0.85811	0.002377	361.875
1	0	0.089604	0.089604	0.028162	0.857349	0	Inf

Table 3 $0.03 \leq Y \leq 0.07$

$S = 0.005$, $Y = 0.07$, Radius $= 7.467404$ Km, $M = 0.261359 M_{\Theta}$, $z_0 = 0.05632$, $z_a = 0.036854$, White dwarf star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.003413	0.211591	0.201352	0	0.928835	0.01613	58.51206
0.2	0.003274	0.211442	0.201618	0.000785	0.928556	0.015486	60.88806
0.4	0.00286	0.210995	0.202415	0.006285	0.927734	0.013554	69.37456
0.6	0.002172	0.210253	0.203737	0.021243	0.92641	0.010332	90.59444
0.8	0.001217	0.209221	0.205571	0.050454	0.924651	0.005815	159.9263
1	0	0.207904	0.207904	0.098784	0.922559	0	Inf

Table 4 $0.04 \leq Y \leq 0.12$

$S = 0.009$, $Y = 0.12$, Radius $= 9.737729$ Km, $M = 0.584264 M_{\Theta}$, $z_0 = 0.101865$, $z_a = 0.065677$, White dwarf star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.01058	0.364949	0.33321	0	0.933858	0.028989	33.14781
0.2	0.010145	0.364483	0.334049	0.001787	0.933272	0.027833	34.46451
0.4	0.008846	0.36309	0.336553	0.014321	0.931554	0.024363	39.16844
0.6	0.006701	0.360785	0.340681	0.048477	0.928826	0.018574	50.93457
0.8	0.00374	0.360785	0.34637	0.115348	0.9253	0.010459	89.39449
1	0	0.353539	0.353539	0.226284	0.921296	0	Inf

Table 5 $0.09 \leq Y \leq 0.24$

$S = 0.02$, $Y = 0.24$, Radius $= 13.61761$ Km, $M = 1.634113 M_{\Theta}$, $z_0 = 0.235042$, $z_a = 0.145272$, Neutron star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.048972	0.7425	0.595584	0	0.978143	0.065955	15.80851
0.2	0.04688	0.740359	0.59972	0.005145	0.976278	0.06332	16.3944
0.4	0.040675	0.733986	0.61196	0.041411	0.970863	0.055417	18.49009
0.6	0.030568	0.723529	0.631827	0.141035	0.962448	0.042248	23.74348
0.8	0.016878	0.709226	0.658593	0.337891	0.952024	0.023797	40.95795
1	0	0.691392	0.691392	0.667124	0.941178	0	Inf

Table 6 $0.2 \leq Y \leq 0.44$

$S = 0.05$, $Y = 0.44$, Radius $= 17.86688$ Km, $M = 3.930714 M_{\Theta}$, $z_0 = 0.579738$, $z_a = 0.323372$, Quark star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.219383	1.43	0.771852	0	0.976508	0.153414	7.341669
0.2	0.208849	1.419179	0.792632	0.013387	0.970367	0.147162	7.564246
0.4	0.178283	1.387388	0.85254	0.110086	0.952703	0.128503	8.366585
0.6	0.130551	1.336561	0.944908	0.384512	0.925796	0.097677	10.40395
0.8	0.069694	1.269638	1.060555	0.939981	0.893618	0.054893	17.17287
1	0	1.1902	1.1902	1.873894	0.862767	0	Inf

Table 7 $0.27 \leq Y \leq 0.52$

$S = 0.07$, $Y = 0.52$, Radius $= 19.00525$ Km, $M = 4.941364 M_{\Theta}$, $z_0 = 0.792823$, $z_a = 0.416766$, Quark star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.351432	1.750465	0.69617	0	0.955752	0.200765	5.716309
0.2	0.333142	1.73124	0.731814	0.017227	0.946962	0.19243	5.868038
0.4	0.280893	1.675319	0.83264	0.145407	0.921816	0.167665	6.419764
0.6	0.201736	1.587627	0.98242	0.52024	0.883875	0.127067	7.839833
0.8	0.105064	1.47535	1.160157	1.287547	0.838982	0.071213	12.62027
1	0	1.346696	1.346696	2.563948	0.796508	0	Inf

Table 8 $0.62 \leq Y \leq 0.71$

$S = 0.2$, $Y = 0.66$, Radius $= 18.24272$ Km, $M = 6.020098 M_{\Theta}$, $z_0 = 1.221319$, $z_a = 0.569412$, Quark star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.524151	2.97	1.397547	0	0.322634	0.176482	2.15078
0.2	0.482194	2.839613	1.393031	0.061965	0.320912	0.16981	2.210748
0.4	0.373594	2.498422	1.377639	0.46976	0.315413	0.149532	2.424743
0.6	0.23661	2.057543	1.347713	1.42933	0.305313	0.114996	2.960289
0.8	0.105925	1.619202	1.301427	2.90727	0.289759	0.065418	4.719095
1	0	1.2408	1.2408	4.68664	0.268484	0	Inf

Table 9 $0.36 \leq Y \leq 0.61$

$S = 0.1$, $Y = 0.61$, Radius $= 19.89809$ Km, $M = 6.068917 M_{\Theta}$, $z_0 = 1.125189$, $z_a = 0.542116$, Quark star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.580067	2.173125	0.432924	0	0.952213	0.266928	4.51952
0.2	0.545887	2.136982	0.49932	0.019744	0.939152	0.255448	4.615646
0.4	0.450692	2.033579	0.681503	0.18255	0.902198	0.221625	4.973029
0.6	0.313341	1.876545	0.936523	0.694896	0.847433	0.166978	5.922565
0.8	0.156715	1.684413	1.214269	1.758204	0.783773	0.093038	9.207976
1	0	1.4762	1.4762	3.475052	0.724092	0	Inf

Table 10 $Y = 0.71$

$S = 0.2455$, $Y = 0.71$, Radius $= 17.71275$ Km, $M = 6.288026 M_{\Theta}$, $z_0 = 1.425009$, $z_a = 0.628327$, Quark star

X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.597172	3.671007	1.879491	0	0.2461	0.162672	1.758959
0.2	0.540588	3.441099	1.819334	0.093954	0.246099	0.157098	1.812636
0.4	0.4012	2.873917	1.670316	0.670531	0.245138	0.139601	2.001131
0.6	0.239735	2.210508	1.491303	1.880987	0.240828	0.108452	2.461411
0.8	0.100823	1.622972	1.320502	3.512058	0.230466	0.062123	3.940309
1	0	1.169754	1.169754	5.229087	0.212103	0	Inf

Table 11 $Y = 0.71$

$S = 0.2$, $Y = 0.71$, Radius $= 18.92112$ Km, $M = 6.716998 M_\Theta$, $z_0 = 1.535807$, $z_a = 0.664357$, Quark star							
X	(P)	(D)	$(D-3P)$	(q)	$dp/c^2 d\rho$	P/D	γ
0	0.794469	3.195	0.811594	0	0.478238	0.24866	2.401498
0.2	0.728437	3.056106	0.870795	0.051457	0.472532	0.238355	2.455008
0.4	0.559147	2.691579	1.014139	0.42894	0.455841	0.207739	2.650136
0.6	0.349403	2.218242	1.170033	1.413314	0.429314	0.157513	3.154885
0.8	0.154082	1.745112	1.282865	3.029494	0.394659	0.088294	4.864497
1	0	1.3348	1.3348	5.041689	0.354508	0	Inf

Where

$$D = \frac{8\pi G}{c^2} a^2 \rho, \qquad P = \frac{8\pi G}{c^4} a^2 p,$$

$$c = 2.997 \times 10^{10} \text{ cm/s}, \qquad G = 6.673 \times 10^{-8} \text{ cm}^3/\text{gs}^2,$$

$$M_\Theta = 1.475 \text{ km}$$

also γ denotes the adiabatic constant and it is given by the expression $\gamma = ((\frac{c^2\rho + p}{p})(\frac{dp}{c^2 d\rho}))$. z_0 and z_a are red shift at the centre and surface $r = a$ respectively. $q(r)$ stands for charge.

5 Conclusion

Problem of charged fluid sphere of embedding class one is investigated fully. The charged fluids obtained satisfy the well behaved conditions and reduces to Schwarzchild's interior solution in the absence of charges. The maximum mass of charged fluid is computed to be $6.716998 M_\Theta$ with the corresponding radius 18.92112 Km. Adiabatic constant is found to be more than 4/3. Red shift of the model are found to be 1.535807 and 0.664357 at the centre and surface respectively. It can easily be observed from Tables 1–11 the class of models are representing the models for neutron, quark and white dwarf stars.

References

Adler, R.J.: J. Math. Phys. **15**, 729 (1974)
Anchordoqui, L., Bergila, S.P.: Phys. Rev. D **62**, 067502 (2000)
Banerjee, A., Som, M.M.: Int. J. Theor. Phys. **20**, 349 (1981)
Buchdahl, H.A.: Phys. Rev. **116**, 1027 (1959)
Cataldo, M., Mitskievic, N.V.: Class. Quantum Gravity **9**, 545 (1992)
Cohen, J.M., Cohen, M.D.: Nuovo Cimento **60**, 241 (1969)
Durgapal, M.C.: J. Phys. A **15**, 2637 (1982)
Durgapal, M.C., Fuloria, R.S.: Gen. Relativ. Gravit. **17**, 671 (1985)
Dionysiou, D.D.: Astrophys. Space Sci. **85**, 331 (1982)
Eddington, A.S.: The Mathematical Theory of Relativity. Cambridge University Press, Cambridge (1924)
Florides, P.S.: J. Phys. A, Math. Gen. **17**, 1419 (1983)
Grøn, Ø.: Phys. Rev. D **31**, 2129 (1985)
Guilfoyle, B.S.: Gen. Relativ. Gravit. **31**, 1645 (1999)
Gupta, Y.K., Gupta, R.S.: Acta Phys. Pol. B **17**, 855 (1986)
Gupta, Y.K., Jasim, M.K.: Astrophys. Space Sci. **283**, 337–346 (2003)
Gupta, Y.K., Kumar, M.: Gen. Relativ. Gravit. **37**(1), 575 (2005a)
Gupta, Y.K., Kumar, M.: Astrophys. Space Sci. **299**(1), 43–59 (2005b)
Gupta, Y.K., Kumar, M.: Gen. Relativ. Gravit. **37**(1), 233 (2005c)
Gupta, Y.K., Pratibha, Kumar Jitendra: Astrophys. Space Sci. **333**(1), 143–148 (2011)
Gupta, Y.K., Maurya, S.K.: Astrophys. Space Sci. (2010a). doi:10.1007/s10509-010-0445-4
Gupta, Y.K., Maurya, S.K.: Astrophys. Space Sci. (2010b). doi:10.1007/s10509-010-0503-y
Heintzmann, H.: Z. Phys. **228**, 489 (1969)
Ivanov, B.V.: Phys. Rev. D **65**, 104001 (2002)
Klein, O.: Ark. Mat. Astron. Fys. A **34**, N19 (1947)
Kohler, M., Chao, K.L.: Z. Naturforsch. Ser. A **20**, 1537 (1965)
Kuchowicz, B.: Acta Phys. Pol. **33**, 541 (1968)
Kuzeev, R.R.: Gravit. Teor. Otnosit. **16**, 93 (1980) (in Russian)
Maharaj, S.D., Hensraj, S.: Int. J. Mod. Phys. D **15**, 1311–1327 (2006)
Maharaj, S.D., Komathiraj, K.: Gen. Relativ. Gravit. **39**, 2079–2093 (2007a)
Maharaj, S.D., Komathiraj, K.: J. Math. Phys. **48**, 042501 (2007b)
Maharaj, S.D., Komathiraj, K.: arXiv:0808.1999v1 [gr-qc], 14 Aug. 2008
Maharaj, S.D., Thirukkanesh, S.: Class. Quantum Gravity **23**, 2697–2709 (2006a)
Maharaj, S.D., Thirukkanesh, S.: Math. Methods Appl. Sci. **29**, 1943–1952 (2006b)
Maharaj, S.D., Thirukkanesh, S.: arXiv:0904.0781v1 [gr-qc], 5 Apr. 2009
Mukherjee, S., Paul, B.C., Dadhich, N.K.: Class. Quantum Gravity **14**, 3475 (1997)
Nduka, A.: Acta Phys. Pol. B **8**, 75 (1977)
Nduka, A.: Gen. Relativ. Gravit. **7**, 493 (1976)
Pandey, S.N., Sharma, S.P.: Gene. Relativ. Gravit. **14** (1981)
Pant, D.N., Sah, A.: J. Math. Phys. **20**, 2537 (1979)
Pant, N., et al.: Astrophys. Space Sci. (2010). doi:10.1007/s10509-010-0383-1
Pavsic, M., Tapia, V.: arXiv:gr-qc/0010045 (2001)
Randall, L., Sundram, R.: Phys. Rev. Lett. **83**, 3370 (1999)
Rayski, J.: Preprint Dublin Institute for Advance Studies (1976)
Robertson, H.P.: Rev. Mod. Phys. **5**, 62 (1933)
Sharma, R., Mukherjee, S., Maharaj, S.D.: Gen. Relativ. Gravit. **33**, 999 (2001)
Singh, T., Yadav, R.B.S.: Acta Phys. Pol. B **9**, 475 (1978)
Tikekar, R.: J. Math. Phys. **25**, 1481 (1984)
Tolman, R.C.: Phys. Rev. **55**, 364 (1939)
Vaidya, P.C., Tikekar, R.: J. Astrophys. Astron. **3**, 325 (1982)
Wyman, M.: Phys. Rev. **75**, 1930 (1949)