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ORIGINAL ARTICLE

Early universe cosmology with particle creation: kinematics tests

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Abstract We study some properties of the early evolution of the universe with particle creation in the framework of the flat Friedmann-Robertson-Walker line element. The field equations are solved by using "gamma-law" equation of state $p = (\gamma - 1)\rho$, where the parameter γ varies with cosmological time. A unified description of the early evolution of the universe is presented in which an inflationary phase is followed by a radiation-dominated phase. Exact expressions for the lookback time, proper distance, luminosity distance and angular diameter distance versus redshift are derived and their meaning discussed in detail. It is found that the negative pressure due to the particle creation may play the role of an accelerating universe.

Keywords Cosmology · Exact solutions · Thermodynamics · Particle creation

1 Introduction

The creation of particle remains one of the most unsolved problem in cosmology. Many authors have tried to understand the particle creation phenomena and its effects on the evolution of the universe. Bondi and Gold (1948), Hoyle (1948), and Hoyle and Narlikar (1962) studied the creation of particle during the expansion of the universe. The approach of Bondi and Gold was based on the Perfect Cosmological Principle whereas Hoyle and Narlikar introduced a scalar field and gave the action for their theory based on the creation field.

Many attempts to treat the particle creation process at a phenomenological macroscopic level have also long been considered in literature based on bulk viscosity. The basic idea is that bulk viscosity (particle creation) contributes at the level of the Einstein field equations (EFE's) as a negative pressure term. Specifically, Barrow (1986, 1988) introduced this idea in the framework of new inflationary scenario.

However, Prigogine et al. (1989) pointed out that the bulk viscosity and particle creation are not only two independent processes but, in general, lead to different histories of the evolution of the universe. They presented a new type of cosmological history that includes large-scale entropy production. The phenomenological macroscopic approach of Prigogine et al. allows for both particle creation and entropy production. They argued that, at the expense of the gravitational field, particle creation can occur only as an irreversible process constrained by the usual requirements of non-equilibrium thermodynamics. The basic idea of this formulation is to modify the usual energy momentum conservation law in 'open' system in the framework of cosmology, which adds a balance equation for the number density of the created particles to the dynamic equations of the universe. In this framework, the thermodynamic second law leads naturally to a reinterpretation of the energy momentum tensor corresponding to an additional stress term (creation pressure), which in turn depends on the particle creation rate, and may considerably alter several predictions of the standard big bang cosmology.

A detailed study of the thermodynamics of the particle creation with changing specific entropy have been discussed

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by Lima et al. (1991), Calvão et al. (1992), Lima and Germano (1992) and Lima et al. (1996). Zimdahl et al. (1996) have studied the back reaction of particle creation process on the cosmological dynamics. They found that the adiabatic creation of massive particles implies power-law inflation. Sudharsan and Johri (1994), Johri and Desikan (1996), Desikan (1997), Singh and Beesham (1999), and Singh et al. (2002) have studied cosmological models with particle creation using Prigogine et al.'s hypothesis in general relativity and some of its modified theories. After the discovery of the accelerating universe, this model was reconsidered to explain it and got some unexpected results. The particle creation pressure, which is negative, might play the role of dark energy component. Lima and Alcaniz (1999), and Alcaniz and Lima (1999) tested the models through kinematics tests. It was shown that the models are consistent with the observational data. Zimdahl et al. (2001) tested the particle creation with SNe Ia data and got the result of accelerating universe. Recently, Qiang et al. (2007) have studied the universe with adiabatic particle creation and showed that the model is consistent with SNe Ia data.

In the standard model, the history of the universe begins with the radiation phase and then evolves to the present matter-dominated era. In order to overcome some of the difficulties met by standard model, Guth (1981) proposed an inflationary phase and this would happen prior to the radiation-dominated phase. In general, the field equations are solved separately for the different epochs. However, Some authors have tried to solve the field equations in a unified manner. Madsen and Ellis (1988) presented the evolution of the universe for inflationary, radiation and matter-dominated phases in a unified manner by assuming gamma (γ) of "gamma-law" equation of state $p=(\gamma-1)\rho$ as a function of scale factor of the FRW metric. Later on, Israelit and Rosen (1989, 1993) used a different equation of state to describe the transition from pre-matter to radiation and then radiation to matter-dominated phase in a unified manner.

In a similar way, Carvalho (1996) studied flat Friedmann-Robertson-Walker (FRW) model in general relativity by using the "gamma-law" equation of state where γ varies with cosmic time to describe the early phases (inflation and radiation) of the evolution of the universe in a unified manner. Therefore, it is not realistic to assume γ as a constant throughout the history of the universe. We can obtain a reasonably realistic model if we assume the universe evolves through the epoches each of which γ is constant. Singh et al. (2007) studied flat viscous FRW model with variable gravitational and cosmological constant with varying γ, and the possibility that the present acceleration of the universe is driven by viscous fluid, is studied. These works motivates us to consider for further work in some of the modified theories or modified energy momentum tensor.

The aim of this paper is to extend Carvalho's work to include the theory of particle creation. We study a flat FRW model with perfect fluid and particle creation pressure in Einstein' theory of gravitation. We discuss the evolution of the universe as it goes from an inflationary phase to a radiation-dominated phase. The similarities and differences among solutions with particle creation have been analyzed from both formal and observational points of view. Exact expressions for the lookback-time, age of the universe, proper distance, luminosity distance and angular diameter distance versus redshift are derived and their meaning are discussed in detail.

2 Thermodynamics of particle creation

In the standard model, the universe is considered as a 'closed' system and the corresponding laws of thermodynamics have the form

$$d(\rho V)=dQ-pdV, \tag{1}$$

and

$$TdS=d(\rho V)+pdV, \tag{2}$$

where ρ is the energy density, p the thermodynamic pressure, V the volume containing N particles, Q is the heat during the cosmic time t, T the temperature and S is the entropy of the system.

From (1) and (2), we see that the entropy production is given by

$$TdS=dQ. \tag{3}$$

Consequently, for a closed adiabatic system ($dQ=0$), the entropy remains constant. If one treats, following Prigogine et al. (1989) that, the expansion is described by an 'open' thermodynamic system, allowing for irreversible particle creation from the energy of gravitational field, one can account for entropy production right from the beginning and the second law of thermodynamics is also incorporated into the evolutionary equations in a more meaningful manner. In this case the number of particles N in a given volume V is not to be a constant.

Thus allowing for particle creation, the modification of (1), into account of variation of particle number, leads to

$$d(\rho V)=dQ-pdV+\frac{h}{N}dN, \tag{4}$$

where $h=(\rho+p)V$ is the total enthalpy of the system. For open system, adiabatic transformations ($dQ=0$) reduce (4) to

$$d(\rho V)+pdV-\frac{h}{N}dN=0. \tag{5}$$

Equation (5) can be rewritten as

$$d(\rho V) + (p + p_c)dV = 0, \tag{6}$$

where

$$p_c = -\frac{(\rho + p)V}{N}\frac{dN}{dV}. \tag{7}$$

Thus, the creation of particle corresponds to a supplementary pressure p_c. It is noted that (6) is equivalent to the conservation equation with additional pressure due to the particle creation and therefore this supplementary pressure must be considered as a part of the cosmological pressure entering into the Einstein field equations. p_c is negative or zero depending on the presence or absence of particle creation. In such a transformation the thermal energy received by the system is entirely due to the change of the number of particles. This change is due to the transfer of energy from the gravitational field of matter. Hence, the creation of particle acts as a source of internal energy.

Now, the entropy change dS, given in (2) in an 'open' system becomes

$$TdS = d(\rho V) + pdV - \mu dN, \tag{8}$$

where $\mu = \frac{h-TS}{N} \geqslant 0$ is the chemical potential associated to the non conservation of the particle number. Therefore, from (5) and (8), we get

$$\frac{dS}{S} = \frac{dN}{N}. \tag{9}$$

The second law of thermodynamics requires that $dS \geqslant 0$. We regard the second law of thermodynamics as one of the most fundamental laws of physics and it should hold whether creation of particle takes place or not. Therefore, from (9), we must have

$$dN \geqslant 0. \tag{10}$$

This inequality implies that the space-time can produce particle.

3 Model and field equations

We start with the homogeneous and isotropic flat Friedman-Robertson-Walker (FRW) line element ($c = 1$)

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{11}$$

where r, θ, and ϕ are dimensionless comoving coordinates and R is the scale factor. The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi G T_{ij}, \tag{12}$$

where T_{ij} is the effective energy momentum tensor of the cosmic fluid in the presence of the particle creation, and is given by

$$T_{ij} = (\rho + p + p_c)u_i u_j - (p + p_c)g_{ij}, \tag{13}$$

where p_c is the particle creation pressure given by (7).

In context of 'open' system with adiabatic creation, the non-trivial Einstein's field equations for a fluid endowed with matter creation can be written as

$$3\frac{\dot{R}^2}{R^2} = 8\pi G\rho, \tag{14}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = -8\pi G(p + p_c). \tag{15}$$

In models with adiabatic creation, the balance equation for the particle number density (see, Calvão et al. 1992; Lima and Germano 1992) is given by

$$\frac{\dot{n}}{n} + 3\frac{\dot{R}}{R} = \frac{\psi(t)}{n} = \frac{\dot{N}}{N}, \tag{16}$$

where $n = \frac{N}{V}$ is the particle number density and $\psi(t)$ is the matter creation rate ($\psi > 0$ corresponds to particle creation while $\psi < 0$ to particle decay). The creation pressure p_c depends on the particle creation rate. A dot denotes derivative with respect to cosmic time t. Equations (14) and (15) lead to the continuity equation

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = -3p_c\frac{\dot{R}}{R}, \tag{17}$$

which is same as described in (6). Now, We suppose that the pressure p and energy density ρ are related through the "gamma-law" equation of state

$$p = (\gamma - 1)\rho. \tag{18}$$

Substituting this into (15) and using (14) and (7), we finally obtain

$$2\dot{H} + 3\gamma H^2 = \gamma H\frac{\dot{N}}{N}, \tag{19}$$

where $H = \dot{R}/R$ is the Hubble parameter. Equation (19) shows that the rate of expansion is related to the rate of creation of particles in the universe. To proceed further it is necessary to assume a physically reasonable expression to the particle creation rate. We need to know the exact form of the function ψ the one which is determined from a more fundamental theory than involves quantum processes.

In this work we take the simple phenomenological expression of the particle creation rate (Lima and Alcaniz 1999)

$$\psi(t) = 3\beta n H. \tag{20}$$

Thus, we get

$$\frac{\dot{N}}{N}=3\beta H, \tag{21}$$

where the parameter β is a dimensionless constant, defined on the interval [0, 1]. The most interesting situations emerge during phase in which $\beta \approx 1$, i.e., of order of unity.

Using (20) into (19), we get

$$\dot{H}+\frac{3}{2}(1-\beta)\gamma H^2=0. \tag{22}$$

In general, the value of γ is taken to be constant and lying in the interval $0 \le \gamma \le 2$. But our aim in this paper is to let the parameter γ depends on scale factor R to describe the early phases, inflationary and radiation-dominated evolution of the universe in a unified manner. We assume, following Carvalho (1996) that, the functional form of γ as

$$\gamma(R)=\frac{4}{3}\frac{A(R/R_*)^2+(a/2)(R/R_*)^a}{A(R/R_*)^2+(R/R_*)^a}, \tag{23}$$

where A is a constant and 'a' is free parameter related to the power of the cosmic time t during the inflationary phase. Here, R_* is a certain reference value of R. The function $\gamma(R)$ is defined in such a manner that when the scale factor R is less than R_*, i.e., when $R \ll R_*$, an inflationary phase ($\gamma \le 2a/3$) can be obtained and for $R \gg R_*$ we have a radiation-dominated phase ($\gamma = 4/3$). The expression of $\gamma(R)$ in (23) is an increasing function of R. In the limit $R \to 0$, $\gamma(R) = 2a/3$. Thus, 1 is the maximum value of 'a' for an inflation epoch to exist. As 'a' approaches to zero we have an exponential inflation ($\gamma = 0$). Therefore, a must lie in the interval $0 \le a < 1$.

To solve (22), we rewrite it in the form

$$H'+\frac{3}{2}(1-\beta)\gamma(R)\frac{H}{R}=0, \tag{24}$$

where $H' = dH/dR$.

4 Solution of the field equations

Substituting (23) into (24) and integrating, we get

$$H=\frac{C}{[A(R/R_*)^2+(R/R_*)^a]^{(1-\beta)}}, \tag{25}$$

where C is a constant of integration. If $H = H_*$ for $R = R_*$, we have a relation between constants A and C as

$$C=H_*(1+A)^{(1-\beta)}. \tag{26}$$

An important observational quantity is the deceleration parameter $q=-\frac{R\ddot{R}}{\dot{R}^2}$. A unified expression of q for both inflationary and radiation-dominated phases can be expressed as a function of scale factor R as

$$q=\frac{(1-2\beta)A(R/R_*)^2+(a(1-\beta)-1)(R/R_*)^a}{A(R/R_*)^2+(R/R_*)^a}. \tag{27}$$

Integrating (25) for $H=\dot{R}/R$, an expression for t in terms of the scale factor R, in case of ($a \neq 0$), is given by

$$Ct=\int\frac{[A(R/R_*)^2+(R/R_*)^a]^{(1-\beta)}}{R}dR. \tag{28}$$

It is difficult to integrate (28) for a general power of $(1-\beta)$. In the following subsections, we solve (28) for two early phases of the evolution of the universe according as $R \ll R_*$ or $R \gg R_*$.

4.1 Inflationary phase

For inflationary phase ($R << R_*$), the second term of the integral in (28) on right hand side dominates which gives the power-law solution of the scale factor ($a \neq 0$) as

$$R=R_*[a(1-\beta)Ct]^{\frac{1}{a(1-\beta)}}, \tag{29}$$

which shows that the dimension of the universe increases according to power-law inflation $R \propto t^{\frac{1}{a(1-\beta)}}$, where $0 \le \beta < 1$. For $\beta = 0$, the above (29) reduces to the standard FRW model (see, Carvalho 1996).

The Hubble parameter in terms of t is given by

$$H=\frac{1}{a(1-\beta)}\frac{1}{t}. \tag{30}$$

Using (29) we may obtain energy density, the particle creation pressure and the particle number density as functions of the scale factor R. For $a > 0$, inserting (29) into (14), we get

$$\rho=\rho_{0i}\left(\frac{R}{R_*}\right)^{-2a(1-\beta)}, \tag{31}$$

where $\rho_{0i}=\frac{3C^2}{8\pi G}$ is the present observed value during inflationary phase. It is found that the explicit dependence of the energy density on the scale factor is slightly modified in comparison with the standard case. We observe that as $R \to 0$, $\rho \to \infty$. The expanding model has the singularity at $t = 0$.

From (31) and (7), the pressure due to particle creation can be written as

$$p_c=-\frac{2a}{3}\beta\rho_{0i}\left(\frac{R}{R_*}\right)^{-2a(1-\beta)}. \tag{32}$$

The particle number density is given by

$$n=n_{0i}\left(\frac{R}{R_*}\right)^{-3(1-\beta)}. \tag{33}$$

We note from (33) that the effect of particle creation is measured by the parameter β. The particle density $N = nR^3$ is given by

$$N = N_{0i}\left(\frac{R}{R_*}\right)^{3\beta}. \tag{34}$$

In the above expressions the subscript '$0i$' refers to the present observed values of the parameters during inflationary phase. It follows from (34) that N increases as a power of R. For $\beta = 0$, N would remain constant throughout the evolution of the universe and we would recover the standard FRW model of the universe. In this case the cosmic expansion is due to the big bang impulse and there is no creation of particle. The solutions identically satisfy the conservation equation (17).

Let us consider the entropy behavior during the inflationary phase. From (9) and (34), one may write the entropy as

$$S = S_{0i}\left(\frac{R}{R_*}\right)^{3\beta}. \tag{35}$$

The entropy increases with the increase of the rate of creation of particle. It is readily seen from (27) that the deceleration parameter q in inflationary phase is given by

$$q = a(1-\beta) - 1. \tag{36}$$

Therefore, for $0 \le a < 1$ and $0 \le \beta < 1$, the deceleration parameter with matter creation is always negative.

4.2 Radiation-dominated phase

For radiation-dominated phase ($R \gg R_*$), the first term of the integrand in (28) on right hand side dominates over first term, which gives the power-law solution of the scale factor as

$$R = R_*\left[\frac{2(1-\beta)}{A^{(1-\beta)}}Ct\right]^{\frac{1}{2(1-\beta)}}, \tag{37}$$

which shows that the dimension of the universe increases according to the law $R \propto t^{\frac{1}{2(1-\beta)}}$, where $0 \le \beta < 1$. For $\beta = 0$ (no particle creation), the above (37) reduces to the standard FRW model. The Hubble parameter in terms of t is given by

$$H = \frac{1}{2(1-\beta)}\frac{1}{t}. \tag{38}$$

It is straight forward to obtain the energy density, creation pressure, particle number density, particle number and entropy production as functions of the scale factor R and of the β parameter. These quantities are, respectively, given by

$$\rho = \rho_{0r}\left(\frac{R}{R_*}\right)^{-4(1-\beta)}, \tag{39}$$

$$p_c = -4\beta\rho_{0r}\left(\frac{R}{R_*}\right)^{-4(1-\beta)}, \tag{40}$$

$$n = n_{0r}\left(\frac{R}{R_*}\right)^{-3(1-\beta)}, \tag{41}$$

$$N = N_{0r}\left(\frac{R}{R_*}\right)^{3\beta}, \tag{42}$$

$$S = S_{0r}\left(\frac{R}{R_*}\right)^{3\beta}, \tag{43}$$

where $\rho_{0r} = \frac{3C^2}{8\pi G A^{2(1-\beta)}}$ is the present observed value of energy density during radiation-dominated phase.

The deceleration parameter is given by

$$q = 1 - 2\beta. \tag{44}$$

Therefore, for a given value of β, q with particle creation is always smaller than the corresponding one of standard flat FRW model. The critical case ($\beta = 1/2$, $q = 0$), describes a "coasting cosmology", i.e., marginal inflation. For $0 \le \beta < 1/2$, we have $q > 0$ whereas for $\beta \ge 1/2$, one get $q \le 0$. The later result is in line of the recent measurements of q using Type Ia supernovae (see, Perlmutter et al. 1998; Riess et al. 1998; Garnavich et al. 1998). Such observations indicate that the universe may be accelerating. In the present framework, this is due to the negative creation pressure that provides the additional acceleration measured by negative q and not an exotic equation of state.

The energy density is decreasing function of R. As $R \to 0$, $\rho \to \infty$, thus the model has singularity at $t = 0$. Equations (31) and (39) show that the densities in inflation and radiation, respectively, as $\rho \propto R^{-2a(1-\beta)}$ and $\rho \propto R^{-4(1-\beta)}$. The solutions identically satisfy the conservation equation (17). Hence, in a model with inflation and radiation, the transition from inflation to a radiation-dominated phase, in course of expansion, happens exactly as in the standard model. We also observe that the same expressions describe the evolution of the particle number density, number of particle and entropy production either for inflationary phase or radiation-dominated phase

5 Particular case

We now study the solution in the limit $a \to 0$. In this case, (25) becomes

$$H = \frac{H_i}{[A(R/R_*)^2 + 1]^{(1-\beta)}}, \tag{45}$$

where H_i is the initial value of H at $R = 0$. A unified expression for deceleration parameter in terms of scale factor has the form

$$q = \frac{(1-2\beta)A(R/R_*)^2 - 1}{A(R/R_*)^2 + 1}. \tag{46}$$

Integrating (45), we get

$$H_i t = \int \frac{[A(R/R_*)^2 + 1]^{(1-\beta)}}{R} dR. \tag{47}$$

Again, in the limit of very small R ($R \ll R_*$), we get

$$R = R_* \exp(H_i t), \tag{48}$$

which corresponds to de Sitter expansion driven by creation of particles. As $t \to -\infty$, $R \to 0$, which shows that the universe is infinitely old. There is no physical singularity as the energy density assume a finite value. These solutions help to resolve several cosmological problems (flatness, horizon, monopole etc.) associated with the standard model. In this case there exists an event horizon, which is given by

$$R_E = R(t_0) \int_{t_0}^{\infty} \frac{dt}{R(t)} = \frac{1}{H_i}. \tag{49}$$

This implies that no observer beyond this proper distance at $t = t_0$ can communicate with another observer.

The energy density is constant during inflationary phase. The particle creation pressure p_c is zero. The exponential inflation is due to the vacuum energy density ($p = -\rho$). Thus the cosmological constant may be considered to give rise the exponential expansion in the absence of negative creation pressure. Also the particle number density, number of particles and entropy are given by

$$n = n_0 \exp[-3(1-\beta) H_i t], \tag{50}$$

$$N = N_0 \exp(3\beta H_i t), \tag{51}$$

$$S = S_0 \exp(3\beta H_i t). \tag{52}$$

Hence, particle number density decreases whereas the number of particles and entropy increase exponentially in this phase. As such this model represents a steady-state inflationary universe. It is noteworthy that this model is analogous to the steady-state model of the universe [3].

In the limit of very large R ($R \gg R_*$), when the universe inters to radiation-dominated phase, we obtain

$$R = R_* \left[\frac{2(1-\beta)}{A^{(1-\beta)}} H_i t\right]^{\frac{1}{2(1-\beta)}}. \tag{53}$$

The other results for various parameters are the same as obtained in Sect. 4.2. We see from (46) that the deceleration parameter varies from $q = -1$ at $R = 0$ to $q = (1 - 2\beta)$ for radiation-dominated phase.

6 Kinematics tests

Now we derive some observable tests of the models in radiation-dominated phase as proposed in the preceding section.

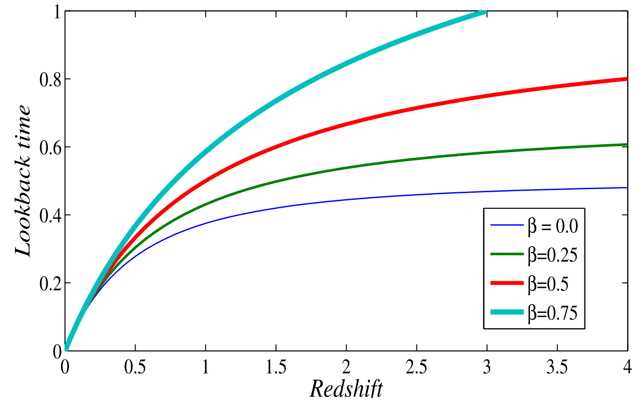

Fig. 1 Lookback time *vs.* redshift for selected values of β in unit of H_0^{-1}

6.1 Lookback time-redshift

The lookback time, $\Delta = t_0 - t(z)$, is the difference between the age of the universe at the present time ($z = 0$) and the age of the universe when a particular light ray at redshift z was emitted. For a given redshift z, the expansion scale factor of the universe $R(t_z)$ is related to R_0 by $1 + z = R_0/R$, where R_0 is the present scale factor. Therefore, from (37), we get

$$1 + z = \left(\frac{t_0}{t}\right)^{\frac{1}{2(1-\beta)}}, \tag{54}$$

which can be rewritten as

$$t_0 - t(z) = \frac{H_0^{-1}}{2(1-\beta)}[1 - (1+z)^{-2(1-\beta)}], \tag{55}$$

where H_0 is the Hubble constant at present in $\mathrm{km\,s^{-1}\,M\,pc^{-1}}$ and its value is believed to be some where between 50 and 100. However, H_0 is dimensionally similar to the reciprocal of time t. The reciprocal of Hubble constant is called the Hubble time T_H: $T_H = H_0^{-1}$, where T_H is expressed in s and H_0 in s^{-1}. If H_0 is expressed in $\mathrm{km\,s^{-1}\,M\,pc^{-1}}$ and T_H in gigayears, then $T_H = 977.8/H_0$. In Fig. 1 we plot the lookback time as a function of the redshift for selected values of β in unit of H_0^{-1}. We observe that the lookback time increases for higher values of β. For lower redshift, all models coincide since they follow the same behavior. Therefore, the models with larger matter creation rate are older.

For small z, (55) gives

$$(t_0 - t(z))H_0 = z - \frac{(1-2\beta)}{2} z^2 + \cdots. \tag{56}$$

The solution of age problem depends on the parameter β. Taking limit $z \to \infty$ in (55), the present age of the universe is

$$H_0 t_0 = \frac{1}{2(1-\beta)} = \frac{1}{(1+q)}. \tag{57}$$

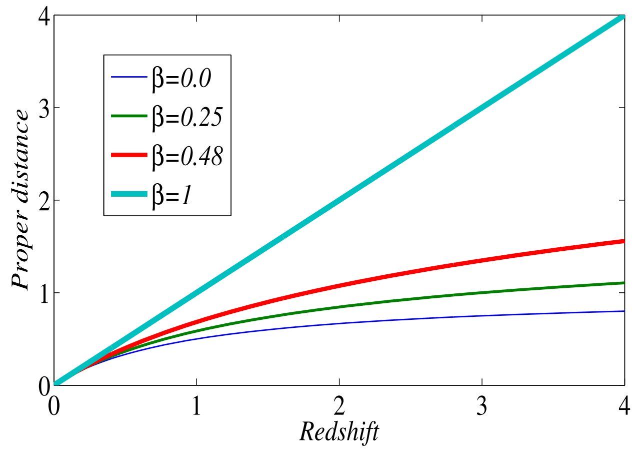

Fig. 2 Proper distance *vs.* redshift for selected values of β in unit of H_0^{-1}

It is seen from (57) that for a given H_0 the age t_0 is always larger than $H_0^{-1}/2$. It is exactly H_0^{-1} for $\beta = 1/2$, i.e., coasting cosmology. In standard flat model ($\beta = 0$), one would obtain exactly $t_0 = H_0^{-1}/2$. We may conclude that the particle creation changes the predictions of the standard cosmology. For $\beta = 1/4$, (55) becomes

$$t_0 - t(z) = \frac{2H_0^{-1}}{3}[1 - (1+z)^{-3/2}], \tag{58}$$

which can be rewritten as

$$t_0 - t(z) = \frac{2}{3}T_H[1 - (1+z)^{-3/2}], \tag{59}$$

which gives the well-known lookback time in Einstein-de Sitter universe. In the limit as $z \to \infty$, we obtain the present age of the universe as $t = (2/3)T_H$.

6.2 Proper distance-redshift

The radial coordinate distance $r(z)$ of the object at light emission as a function of redshift is given by

$$r(z) = \int_t^{t_0} \frac{dt}{R} = \frac{H_0^{-1}R_0^{-1}}{(1-2\beta)}[1-(1+z)^{(2\beta-1)}], \quad (\beta \neq 1/2). \tag{60}$$

Therefore, the proper distance $d(z)$ between the source and the observer as a function of redshift is given by

$$d(z) = r(z)R_0 = \frac{H_0^{-1}}{(1-2\beta)}[1 - (1+z)^{(2\beta-1)}]. \tag{61}$$

In Fig. 2 the proper distance as a function of redshift and for different values of the parameter β is shown. For small z, expanding (61) we get

$$H_0 d(z) = z - (1-\beta)z^2 + \cdots, \tag{62}$$

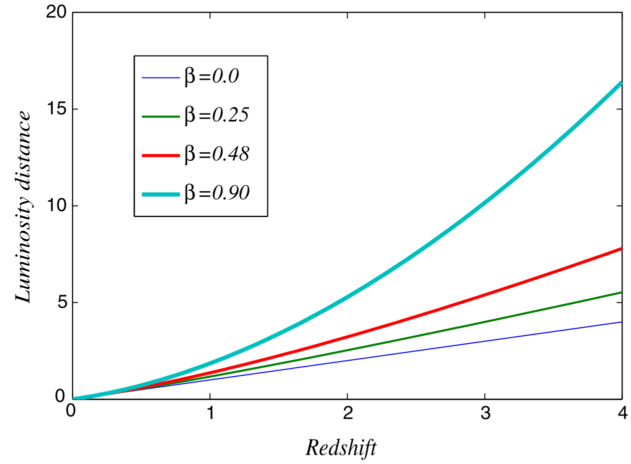

Fig. 3 Luminosity distance *vs.* redshift for selected values of β in unit of H_0^{-1}

which depends on the particle creation β. Taking $z \to \infty$, we obtain

$$d(z = \infty) = \frac{H_0^{-1}}{(1-2\beta)}. \tag{63}$$

6.3 Luminosity distance-redshift

The luminosity distance of a light source is defined as the ratio of the detected energy flux l and the apparent luminosity L, i.e., $d_l^2 = l/4\pi L$. In the standard FRW model, it takes the form

$$d_l = R_0 r(z)(1+z) = d(z)(1+z). \tag{64}$$

From (61) and (64), we get

$$d_l = \frac{H_0^{-1}(1+z)}{(1-2\beta)}[1 - (1+z)^{(2\beta-1)}], \quad (\beta \neq 1/2). \tag{65}$$

For small z after some algebra, (62) gives

$$H_0 d_l = z + \frac{1}{2}(1-q)z^2 + \cdots. \tag{66}$$

The luminosity distance depends on the particle creation β parameter. However, (66) shows that for small z, it depends on the effective deceleration parameter q. The luminosity distance as a function of the redshift is shown in Fig. 3. As expected, we find the same behavior for different models at $z \ll 1$ and the possible difference in behavior for different models come at large redshift ($z \gg 1$). In Fig. 3 we observe that all curves start off with the linear Hubble law ($z = d_l H_0$) for small z, but then, only the curve for $q = 1$, i.e., $\beta = 0$ stays linear all the way. We also note that for the small redshift the luminosity distance is larger for lower values of q. Thus, for $q = 1$, we have

$$d_l = H_0^{-1} z, \tag{67}$$

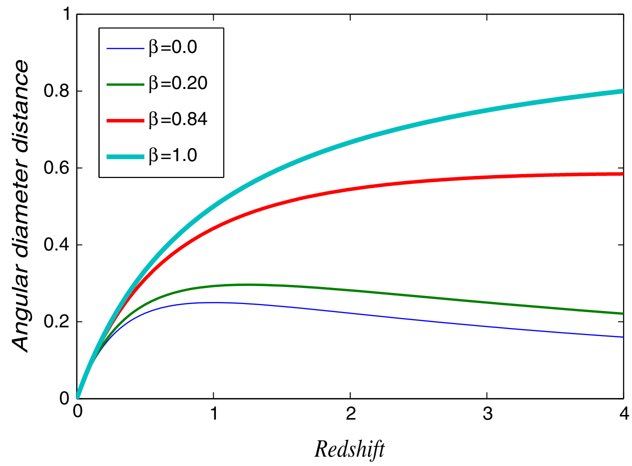

Fig. 4 Angular diameter distance *vs.* redshift for selected values of β in unit of H_0^{-1}

and for $q=0$, i.e., $\beta=1/2$ we get

$$d_l = H_0^{-1}\left(z+\frac{1}{2}z^2\right). \tag{68}$$

6.4 Angular diameter distance-redshift

The angular diameter d_A of a light source of proper distance $d(z)$ is defined as

$$d_A = d(z)(1+z)^{-1} = d_l(1+z)^{-2}. \tag{69}$$

Using (61) and (65), we get

$$d_A = \frac{H_0^{-1}}{(1-2\beta)}\left[\frac{1-(1+z)^{(2\beta-1)}}{(1+z)}\right], \quad (\beta \neq 1/2). \tag{70}$$

In Fig. 4 we plot the angular diameter distance versus redshift for some selected values of β in unit of H_0^{-1}. The angular diameter distance initially decreases with increasing z and eventually begins to increase for higher values of β.

The limit on the parameter β for all kinematics tests are in the range $0.50 \le \beta \le 1$, i.e., $-1 < q \le 0$, which is acceptable range observed by SNe Ia data (Qiang et al. 2007). The kinematics tests for the inflationary phase may be derived in a similar way as discussed above.

7 Conclusion

In this paper we have discussed the flat FRW cosmological models described by "open" thermodynamics systems, i.e., including particle creation at the expense of gravitational field. We have solved the field equations by using "gamma-law" equation state, in which the adiabatic parameter γ is the function of scale factor R. A unified description of early phases of the evolution of the universe has been presented with the creation of particle in which an inflationary phase is followed by an radiation-dominated phase. We have obtained power-law and exponential expansion in inflationary phase for $0 \le a < 1$ and $0 \le \beta < 1$. In case of power-law expansion in both phases (inflation and radiation), the model has singularity at $t=0$. The dimension of scale factor vary as $R \propto t^{1/a(1-\beta)}$ in inflationary phase and $R \propto t^{1/2(1-\beta)}$ in radiation-dominated phase which are the simple generalization of Carvalho's work.

In the case of exponential expansion ($a=0$), the model is non-singular as the energy density is finite at $t=0$ and at $t \to -\infty$, $R \to 0$ which shows that the universe is infinitely old. We find that the exponential expansion of the universe is due to the negative pressure caused by vacuum energy density ($p=-\rho$) without negative pressure due to particle creation. The possibility that $q<0$ has come in this case, which indicates that the universe is accelerating and the models studied here are alternatives to universe dominated by a cosmological constant. Thus the inflationary model with de Sitter expansion belongs to particle creation domain with $q=-1$. We may expect that the process of particle creation is also an ingredient accounting for this unexpected observational result. The changes introduced by the particle creation process, which is quantified by the parameter β, provides a reasonable observational results. The new fact justifying the present work is that we have considered the thermodynamics approach for which particle creation is at the expense of the gravitational field. A general expression relating the energy densities and particle number density as function of scale factor have been established. One may find that the particle creation changes the predictions of standard cosmology, thereby alleviating the problem of reconciling observations with the inflationary scenario. We have derived the lookback time, proper distance, luminosity distance and angular diameter distance versus redshift with particle creation. We have observed that the age of the universe with particle creation is always greater than the corresponding FRW model without particle creation. For further work one may expect that negative creation pressure and cosmological constant may jointly generate the accelerating universe.

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