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ORIGINAL ARTICLE

Anisotropic universe with cosmic strings and bulk viscosity in a scalar–tensor theory of gravitation

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Abstract A spatially homogeneous and anisotropic Bianchi type-I cosmological model is obtained in a scalar–tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A 113:467, 1986) when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings. Some physical and kinematical properties of the model are discussed. It is observed that the bulk viscosity has a greater role in getting an accelerated expansion of the universe in this theory.

Keywords Viscosity · Anisotropic universe · Cosmic strings · Scalar tensor theory

1 Introduction

In recent years there has been a considerable interest in cosmological models with bulk viscosity, since bulk viscosity leads to the accelerated expansion phase if the early universe, popularly known as the inflationary phase. The possibility of bulk viscosity leading to inflationary-like solutions in general relativistic FRW models has been discussed by several authors (Barrow 1986; Padmanabhan and Chitre 1987; Pavon et al. 1991; Martens 1995; Lima et al. 1993). It is well known that the bulk viscosity contributes a negative pressure term giving rise to an effective total negative pressure leading to an repulsive gravity. This overcomes the attractive gravity of the matter and gives an impetus for rapid expansion of the universe. Roy and Tiwari (1983), Mohanty and Pattanaik (1991), Mohanty and Pradhan (1992), Singh and Shi Ram (1996), Singh (2005) are some of the authors who have investigated cosmological models with bulk viscosity in general relativity.

Of late, string cosmological models are attracting more and more attentions of research workers. In a gauze theory, spontaneous symmetry breaking in elementary particle physics has given rise to topological defects known as cosmic strings. The gravitational effects of such objects are of particular interest since they are considered as possible 'seeds' for galaxy for motion and gravitational lenses. Latelier (1983), Krori et al. (1990), Mahanta and Mukherjee (2001), Battacharjee and Baruah (2001) have studied several aspects of string cosmological models in general relativity. Recently, Wang (2004, 2005, 2006), Bali and Dave (2002), Bali and Pradhan (2007), Tripathy et al. (2009, 2010) have studied the Bianchi type cosmological models in the presence of cosmic strings and bulk viscosity.

Scalar–Tensor theories of gravitation are considered to be essential to describe the gravitational interactions near the plank scale: string theory, extended inflations and many higher order theories imply scalar field. Brans and Dicke (1961) scalar–tensor theory of gravitation introduces an additional scalar field ϕ besides the metric tensor g_{ij} and a dimensionless coupling constant ω. This theory goes to general relativity for large values of the coupling constant $\omega > 500$. Saez and Ballester (1986) formulated a scalar–tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non-flat FRW cosmologies.

Johri and Sudharsan (1989), Pimental (1994), Banerjee and Beesham (1996), Sing et al. (1997) have discussed bulk

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viscous cosmological models in Brans-Dicke theory of gravitation. Reddy (2003a, 2003b), Reddy and Naidu (2007a, 2007b) and Rao et al. (2007, 2008a, 2008b) have studied several aspects of string cosmological models in scalar–tensor theory of gravitation.

In this paper we study spatially homogeneous and anisotropic locally rotational symmetric (LRS) Bianchi type-I universe I the presence of bulk viscous fluid containing massive strings in the frame work of Saez and Ballester (1986) scalar–tensor theory of gravitation. The advantages of these anisotropic models in the scalar–tensor theory are that they describe the evolution of the early phase of the universe.

2 Metric and field equations

We consider the LRS Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \tag{2.1}$$

where A and B are the Eulerian functions of time t only.

The field equations given by Saez and Ballester (1986) for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{2.2}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{,i}{}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2.3}$$

Here ω and n are constants, T_{ij} is the energy momentum tensor of the matter and comma and semicolon denotes partial and covariant differentiation respectively.

Also

$$T^{ij}{}_{;j} = 0 \tag{2.4}$$

is a consequence of the field equations (2.2) and (2.3).

We consider the energy momentum tensor for a bulk viscous fluid containing one dimensional string as

$$T_{ij} = (\rho + \vec{p})u_iu_j + \vec{p}g_{ij} - \lambda x_ix_j \tag{2.5}$$

where ρ is the rest energy density of the system, λ is tension in the string and

$$\vec{p} = p - 3\xi H \tag{2.6}$$

is the total pressure which includes the proper pressure, $\xi(t)$ is the coefficient of bulk viscosity, $3\,\xi H$ is usually known as bulk viscous pressure, H is the Hubble parameter, $u^i = \delta^i_4$ is the four velocity vector and x^i is a space-like vector which represents the anisotropic directions of the string.

Here u^i and x^i satisfy the equations

$$\begin{aligned} &g_{ij}u^iu^j = -1 \\ &g_{ij}x^ix^j = 1 \\ &u^ix_i = 0 \end{aligned} \tag{2.7}$$

we assume the string to be lying along the x-axis. The one dimensional strings are assumed to be loaded with particles and the particle energy density is $\rho_p = \rho - \lambda$.

We also consider ρ, λ, $\vec{p}$ and ϕ are functions of time t only.

By adopting comoving coordinates the field equation (2.2) and (2.3) take the form

$$2\dot{H}_2 + 3H_2^2 - \frac{\omega}{2}\phi^n\dot{\phi}^2 = \lambda - \vec{p}, \tag{2.8}$$

$$\dot{H}_1 + H_1^2 + \dot{H}_2 + H_2^2 + H_1H_2 - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -\vec{p}, \tag{2.9}$$

$$2H_1H_2 + H_2^2 + \frac{\omega}{2}\phi^n\dot{\phi}^2 = \rho, \tag{2.10}$$

$$\ddot{\phi} + (H_1 + H_2)\dot{\phi} + \frac{n\dot{\phi}^2}{2\phi} = 0 \tag{2.11}$$

It may be noted that (2.4) being a consequence of (2.2) and (2.3) we consider (2.8) to (2.11) only where $H_1 = \frac{\dot{A}}{A}$ and $H_2 = \frac{\dot{B}}{B}$ are the directional Hubble parameters, so that $H = \frac{(H_1+2H_2)}{3}$, and the overhead dots denote ordinary time derivatives.

The expansion scalar θ and the shear scalar σ for the metric (2.1) are defined as

$$\theta = u^i_{ji} = H_1 + 2H_2, \tag{2.12}$$

$$\sigma^2 = \frac{1}{2}\left(H_1^2 + 2H_2^2 - \frac{1}{3}\theta^2\right) = \frac{1}{3}(H_1 - H_2)^2 \tag{2.13}$$

3 Solutions of the field equations and the models

The field equations (2.8) to (2.11) are highly non-linear in nature and therefore we require the following plausible physical conditions:

(1) The shear scalar σ be proportional to scalar expansion θ, so that we can take a linear relationship between the Hubble parameters H_1 and H_2, i.e.,

$$H_1 = kH_2 \tag{3.1}$$

where k is an arbitrary constant which takes positive values only and it takes care of the anisotropic nature of the model.

(2) A more general relationship between the proper rest energy density ρ and string tension density λ is taken to be

$$\rho = r\lambda \tag{3.2}$$

where r is an arbitrary constant which can take both positive and negative values. The negative value of r leads to the absence of strings in the universe and the positive value shows the presence of one dimensional strings in the cosmic fluid. The energy density of the particles attached to the strings is

$$\rho_p = \rho - \lambda = (r-1)\lambda \tag{3.3}$$

(3) For a bartropic fluid, the combined effect of the proper pressure and the barotropic bulk viscous pressure can be expressed as

$$\vec{p} = p - 3\xi H = (\epsilon\rho) \tag{3.4}$$

where

$$\epsilon = \epsilon_0 - \zeta \quad \text{and} \quad p = \epsilon_0\rho \quad (0 \le \epsilon_0 \le 1) \tag{3.5}$$

With the assumption (3.1), the field equations (2.8)–(2.11) reduce to

$$2\dot{H}_2 + 3H_2^2 - \frac{\omega}{2}\phi^n\dot{\phi}^2 = \lambda - \vec{p}, \tag{3.6}$$

$$(k+1)\dot{H}_2 + (k^2+k+1)H_2^2 - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -\vec{p}, \tag{3.7}$$

$$(2k+1)H_2^2 + \frac{\omega}{2}\phi^n\dot{\phi}^2 = \rho, \tag{3.8}$$

$$\ddot{\phi} + (k+1)H_2\dot{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{3.9}$$

Addition of (3.6) and (3.8) and then using (3.2) and (3.4), we obtain

$$2\dot{H}_2 + (2k+4)H_2^2 = \left(\frac{1}{r} - \epsilon + 1\right)\rho \tag{3.10}$$

Addition of (3.7) and (3.8) yields

$$(k+1)\dot{H}_2 + (k^2+3k+2)H_2^2 = (1-\epsilon)\rho \tag{3.11}$$

Equations (3.10) and (3.11), immediately, gives us

$$-\frac{\dot{H}_2}{H_2^2} = m \tag{3.12}$$

where

$$m = \frac{2r(\epsilon+1) - k[1 - r(\epsilon+1)]}{k[r(\epsilon+1)-1] - r(\epsilon-1) - 1} \tag{3.13}$$

Integrating (3.12), we get

$$H_2 = \frac{\dot{B}}{B} = \frac{1}{mt + m_1} \tag{3.14}$$

which on further integration yields

$$B = B_0(mt + m_1)^{\frac{1}{m}} \tag{3.15}$$

and consequently, we obtain

$$A = A_0(mt + m_1)^{\frac{k}{m}} \tag{3.16}$$

where m_1, A_0 and B_0 are constants of integration.

With a suitable choice of constants, the metric (2.1) with the help of (3.15) and (3.16) can be written as

$$ds^2 = -dt^2 + (mt)^{\frac{2k}{m}}dx^2 + (mt)^{\frac{2}{m}}(dy^2 + dz^2) \tag{3.17}$$

which represents bulk viscous string cosmological model in Saez-Ballester theory.

4 Some physical properties

The model (3.17) represents an exact bulk viscous string cosmological model in Saez-Ballester scalar–tensor theory of gravitation. The scalar field of the theory in the model can be obtained from (3.9) using (3.14) as

$$\phi = \left[\frac{(n+1)m\phi_0 t^{\frac{1}{m}}}{2}\right]^{\frac{2}{n+2}} \tag{4.1}$$

where ϕ_0 is a constant of integration.

The physical quantities that are important in cosmology are:

The string tension density

$$\lambda = \frac{1}{r}\left[\left(\frac{2k+1}{m^2}\right)\frac{1}{t^2} + \frac{\omega\phi_0^2}{2}\frac{1}{t^{\frac{2(k+1)}{m}}}\right] \tag{4.2}$$

Energy density

$$\rho = \left[\left(\frac{2k+1}{m^2}\right)\frac{1}{t^2} + \frac{\omega\phi_0^2}{2}\frac{1}{t^{\frac{2(k+1)}{m}}}\right] \tag{4.3}$$

Proper pressure

$$p = \epsilon_0\left[\left(\frac{2k+1}{m^2}\right)\frac{1}{t^2} + \frac{\omega\phi_0^2}{2}\frac{1}{t^{\frac{2(k+1)}{m}}}\right] \tag{4.4}$$

Particle energy density

$$\rho_p = \left(\frac{r-1}{r}\right)\left[\left(\frac{2k+1}{m^2}\right)\frac{1}{t^2} + \frac{\omega\phi_0^2}{2}\frac{1}{t^{\frac{2(k+1)}{m}}}\right] \tag{4.5}$$

Coefficient of bulk viscosity

$$\xi = \left(\frac{m\xi}{k+2}\right)\left[\left(\frac{2k+1}{m^2}\right)\frac{1}{t} + \frac{\omega\phi_0^2}{2}\frac{1}{t^{\frac{2(k+1)}{m}}}\right] \tag{4.6}$$

The expansion scalar θ and the shear scalar σ^2 for the model (3.17) are given by

$$\theta = (k+2)\frac{1}{mt}, \tag{4.7}$$

$$\sigma^2 = \frac{(k-1)^2}{3m^2t^2} \tag{4.8}$$

The scale factor of the model can be expressed as

$$R = AB^2 = mt\frac{(k+2)}{m} \tag{4.9}$$

The deceleration parameter for such a volume scale factor is given by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \frac{m}{(k+2)} - 1 \tag{4.10}$$

It can be observed that the model (3.17) has no initial singularity.

The spatial volume in the model increases as t increases while the scalar of expansion θ and the shear scalar σ^2 decreases. At the initial epoch, physical quantities $\theta, \sigma^2, \lambda, \rho, \xi$ and p diverge when $t \to \infty, \lambda, p, \rho, \theta, \xi$ and σ vanish. Also, since

$$\lim_{t\to\infty}\left(\frac{\sigma^2}{\theta^2}\right) = \frac{1}{6} \neq 0$$

The model does not approach isotropy for large values of t.

It may be noted here that for accelerated expansion of the model the deceleration parameter q should be less than zero (i.e., $q < 0$). Hence in order to get an accelerated expansion model, we should have $\frac{m}{k+2} < 1$.

5 Conclusions

Here, the field equations for spatially homogeneous and anisotropic LRS Bianchi type-I metric are solved in the frame work of Saez-Ballester(1986) scalar–tensor theory of gravitation when the source of the energy-momentum tensor is a viscous fluid containing one dimensional strings. It is well known that scalar field and bulk viscosity play a vital role in getting an accelerated universe. In other words, bulk viscosity has a greater role in obtaining inflationary model. The model obtained represents a bulk viscous inflationary cosmological model in Saez-Ballester scalar–tensor theory of gravitation.

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