Astrophys Space Sci (2011) 335:611–617
DOI 10.1007/s10509-011-0773-z

ORIGINAL ARTICLE

Statefinder diagnostic for variable modified Chaplygin gas in Bianchi type-V universe

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Received: 11 May 2011 / Accepted: 9 June 2011 / Published online: 28 June 2011

Abstract The Bianchi type-V cosmological model with variable modified Chaplygin gas having the equation of state $p = A\rho - B/\rho^{\alpha}$, where $0 \le \alpha \le 1$, A is a positive constant and B is a positive function of the average scale factor $a(t)$ of the universe [i.e. $B = B(a)$] has been studied. While studying its role in accelerated phase of the universe, it is observed that the equation of state of the variable modified Chaplygin gas interpolates from radiation dominated era to quintessence dominated era. The statefinder diagnostic pair $\{r, s\}$ is adopted to characterize different phases of the universe.

Keywords Bianchi type-V space-time · Variable modified Chaplygin gas · Statefinder parameters

1 Introduction

The direct evidence for "the expansion of the universe is accelerating" comes from the high redshift supernovae (Perlmutter et al. 1999; Riess et al. 1998) and the WMAP data (Bernardis et al. 2000; Hanany et al. 2000; Spergel et al. 2007). To explain these observations, two dark components are invoked: the pressureless cold dark matter (CDM) and the dark energy (DE) with negative pressure. The CDM contributes $\Omega_{DM} \sim 0.3$ and is mainly motivated by the theoretical interpretation of the galactic rotational curves and large scale structure formation. The DE is assumed to provide $\Omega_{DE} \sim 0.7$ and is responsible for the acceleration of the distant type Ia supernovae. The nature of dark energy as well as dark matter is unknown, and many radically different models have been proposed, such as, a tiny positive cosmological constant, quintessence (Caldwell et al. 1998; Liddle and Scherrer 1999; Steinhardt et al. 1999), DGP branes (Dvali et al. 2000; Deffayet 2001), the non-linear F(R) models (Capozziello et al. 2003; Carroll et al. 2003; Nojiri and Odintsov 2003), and dark energy in brane worlds, among many others (Townsend and Wohlfarth 2003; Gibbons 1985; Maldacena and Nuñez 2001; Ohta 2003a, 2003b, 2005; Wohlfarth 2003; Roy 2003; Webb et al. 2001; Cline and Vinet 2003; Chen et al. 2003; Bergshoeff 2004; Gong and Wang 2006; Neupane and Wiltshire 2005a, 2005b; Maeda and Ohta 2005; Neupane 2007; Gong et al. 2007; Pereira et al. 2006; Brandt et al. 2007) including the review articles (Copeland et al. 2006; Padmanabhan 2007). The existence of dark energy fluid comes from the observations of the accelerated expansion of the universe and the isotropic pressure cosmological models give the best fitting of the observations. Although some authors (Koivisto and Mota 2008) have suggested cosmological model with anisotropic and viscous dark energy in order to explain an anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles. The binary mixture of perfect fluid and dark energy has been studied for Bianchi type-I (Saha 2005) and for Bianchi type-V (Singh and Chaubey 2009). The anisotropic dark energy has been studied for Bianchi type-III (Akarsu and Kilinc 2010) and for Bianchi type-VI_0 (Adhav et al. 2011).

The scalar field known as Quintessence represents one type of the dark energy. There is another candidate for dark energy called as pure Chaplygin gas obeying the equation of state

$$p = -B/\rho, \tag{1.1}$$

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where p and ρ are respectively the pressure and energy density of dark energy, B is a positive constant.

There is no direct laboratory observational or experimental evidence of both CDM and DE. Therefore, it would be important if a unified dark matter—dark energy scenario could be found, in which these two components are different manifestations of a single fluid (Matos and Urena-Lopez 2000; Wetterich 2002; Padmanabhan and Choudhury 2002).

A candidate for such unification is the so-called generalized Chaplygin gas which is an exotic fluid with the equation of state

$$p = -B/\rho^{\alpha} \quad \text{with } 0 \le \alpha \le 1, \tag{1.2}$$

where $B\psi$ and $\alpha\psi$ are two parameters to be determined.

It was initially suggested by Kamenshchik et al. (2001a, 2001b) with $\alpha = 1$ and then generalized by Bento et al. (2002) for the case $\alpha \neq 1$.

Benaoum (2002) and Debnath et al. (2004) studied the models in which the isotropic pressure p of the cosmological fluid obeys a modified Chaplygin gas equation of state

$$p = A\rho - \frac{B}{\rho^{\alpha}}, \tag{1.3}$$

where $0 \le A \le 1$; $0 \le \alpha \le 1$, and B is a positive constant.

When $A = 1/3$ [i.e. when the scale factor $a(t)$ is vanishing small i.e. the co-moving volume of the universe is small i.e. $\rho \to \infty$], this (1.3) equation of state corresponds to a radiation dominated era at one extreme.

When the density is small [i.e. $\rho \to 0$ i.e. when the scale factor $a(t)$ is infinitely large], then this (1.3) equation of state corresponds to a cosmological fluid with negative pressure (the dark energy) i.e. it corresponds to ΛCDM model at the other extreme.

Generally, the modified Chaplygin gas equation of state corresponds to a mixture of ordinary matter and dark energy. For $\rho = (B/A)^{1/\alpha+1}$ the matter content is pure dust with $p = 0$.

Recently, Guo and Zhang (2005); Riess et al. (2004) have proposed the variable Chaplygin gas model with equation of state (1.1), where B is positive function of the cosmological scale factor 'a' i.e. $B = B(a)$. Bento et al. (2003) proved that this assumption is reasonable since $B(a)$ is related to the scale potential if we consider the Chaplygin gas as a Born-Infield scalar field.

An another form of equation of state for Chaplygin gas (Debnath 2007; Jamil and Rashid 2008) is considered which is given by

$$p = A\rho - B/\rho^{\alpha}, \tag{1.4}$$

where $0 \le \alpha \le 1$, A is a positive constant and B is a positive function of the average scale factor a of the universe [i.e. $B = B(a)$].

Sahni et al. (2003) proposed a cosmological diagnostic pair $\{r, s\}$ called statefinder, which is defined as

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{1.5}$$

to differentiate among different forms of dark energy. Here H is the Hubble parameter and q is the deceleration parameter.

The statefinder is a geometrical diagnostic which depends on the cosmic scalar factor $a(t)$. The statefinder pair $\{1, 0\}$ represents a cosmological constant with a fixed equation of state $w = -1$ and a fixed Newton's gravitational constant. The pair $\{1, 1\}$ represents the standard cold dark matter model containing no radiation. The Einstein static universe corresponds to the statefinder diagnostic pair $\{\infty, -\infty\}$ (Coles and Ellis 1994). The statefinder diagnostic pairs are analyzed for various dark energy candidates including holographic dark energy (Debnath 2008), agegraphic dark energy (Zhang 2005a), quintessence (Wei and Cai 2007), dilation dark energy (Zhang 2005b), Yang-Mills dark energy (Huang et al. 2008), viscous dark energy (Zhao 2008), interacting dark energy (Hu and Meng 2006), tachyon (Zimdahl and Pavon 2004), modified Chaplygin gas (Shao and Gui 2008), $f(R)$ gravity (Chakraborty and Debnath 2007) and so on.

In the present paper, the spatially homogeneous and anisotropic Bianchi type-V cosmological model with variable modified Chaplygin gas has been investigated. It is shown that the equation of state of this modified model is valid from the radiation era to the quintessence. The statefinder diagnostic pair i.e. $\{r, s\}$ parameter is adopted to characterize different phase of the universe. The geometrical and physical behavior of the model is also discussed.

2 Metric and field equations

The spatially homogeneous and anisotropic Bianchi type-V line element can be written as

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} dy^2 - a_3^2 e^{-2mx} dz^2, \tag{2.1}$$

where a_1, a_2 and a_3 are scale factors and are functions of cosmic time t only and m is constant.

Bianchi type-V universe is a generalization of the open universe in FRW cosmology and hence its study is important in the study of dark energy models of the universe with non-zero curvature.

The Einstein field equations are ($8\pi G = c = 1$)

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{3m^2}{a_1^2} = \rho \tag{2.2}$$

$$\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}-\frac{m^2}{a_1^2}=-p \tag{2.3}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{m^2}{a_1^2}=-p \tag{2.4}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{m^2}{a_1^2}=-p \tag{2.5}$$

$$2\frac{\dot{a}_1}{a_1}=\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}. \tag{2.6}$$

From (2.6), we get

$$a_1^2=a_2a_3. \tag{2.7}$$

The energy conservation equation is

$$\dot{\rho}+\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)(\rho+p)=0 \tag{2.8}$$

The spatial volume of the universe is defined by

$$V=a^3=a_1a_2a_3=a_1^3, \tag{2.9}$$

where a is an average scale factor of the universe.

We assume that the universe is filled with variable modified Chaplygin gas having equation of state

$$p=A\rho-B/\rho^{\alpha}, \tag{2.10}$$

where $0\le\alpha\le 1$, A is a positive constant and B is a positive function of the average scale factor of the universe $a(t)$ [i.e. $B=B(a)$].

At all stages it represents a mixture. There is also one stage, in between, when the pressure vanishes and the matter content is equivalent to a pure dust.

Now, for simplicity, assume $B(a)$ in the form

$$B(a)=B_0a^{-n}=B_0V^{-n/3}, \tag{2.11}$$

where $B_0>0$ and n are positive constants.

Such type of relations have been firstly considered by Berman (1983), Berman and Gomide (1988) for solving FRW models. Later on many authors have studied flat FRW and Bianchi models by using such law.

Using (2.8), (2.10) and (2.11), we obtain

$$\rho=\left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}}+\frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{1}{1+\alpha}}, \tag{2.12}$$

where $C>0$ is an arbitrary constant of integration.

For the requirement of expanding universe, n must be positive i.e. for positivity of first term, we need $3(1+\alpha)(1+A)>n$. Otherwise $(a\to\infty)\Rightarrow(\rho\to\infty)$ which is not the case of expanding universe.

In the variable modified Chaplygin gas scenario, the above equation (2.12) interpolates between a radiation dominated phase ($A=1/3$) and a quintessence dominated phase described by the equation of state $p=\gamma\rho$, where

$$\gamma=-1+\frac{n}{3(1+\alpha)}<-\frac{1}{3}.$$

Case (i) Now, for small values of the scale factors $a_1(t)$, $a_2(t)$ and $a_3(t)$, one may have

$$\rho\cong\frac{C^{\frac{1}{1+\alpha}}}{V^{(1+A)}}, \tag{2.13}$$

which is very large and corresponds to the universe dominated by an equation of state $p=A\rho$, i.e. we get, radiation dominated universe.

Subtracting (2.3) from (2.4), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)+\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=0. \tag{2.14}$$

Now, from (2.7) and (2.9), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)+\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V}=0. \tag{2.15}$$

Integrating the above equation, we get

$$\frac{a_1}{a_2}=d_1\exp\left(x_1\int\frac{dt}{V}\right),\quad d_1=\text{constant},\ x_1=\text{constant}. \tag{2.16}$$

Subtracting (2.5) from (2.3) and subtracting (2.4) from (2.5) and then by integration, we obtain

$$\frac{a_1}{a_3}=d_2\exp\left(x_2\int\frac{dt}{V}\right),\quad d_2=\text{constant},\ x_2=\text{constant}. \tag{2.17}$$

$$\frac{a_2}{a_3}=d_3\exp\left(x_3\int\frac{dt}{V}\right),\quad d_3=\text{constant},\ x_3=\text{constant}, \tag{2.18}$$

where d_1, d_2, d_3, x_1, x_2 and x_3 are integration constants.

In view of $V=a_1a_2a_3$, we find the following relation between the constants d_1, d_2, d_3, x_1, x_2 and x_3 as $d_2=d_1d_3$, $x_2=x_1+x_3$.

Using above results we write the scale factors $a_1(t)$, $a_2(t)$ and $a_3(t)$ in explicit form as

$$a_1(t)=D_1V^{1/3}\exp\left(X_1\int\frac{dt}{V}\right) \tag{2.19a}$$

$$a_2(t)=D_2V^{1/3}\exp\left(X_2\int\frac{dt}{V}\right) \tag{2.19b}$$

$$a_3(t)=D_3V^{1/3}\exp\left(X_3\int\frac{dt}{V}\right), \tag{2.19c}$$

where D_i $(i=1,2,3)$ and X_i $(i=1,2,3)$ satisfy the relations $D_1D_2D_3=1$ and $X_1+X_2+X_3=0$.

From (2.2)–(2.5), one can get

$$\frac{\ddot{V}}{V}-\frac{6m^2}{a_1^2}=\frac{3}{2}(\rho-p). \tag{2.20}$$

Using (2.13) and $p=A\rho$, (2.20) yields

$$\int\frac{dV}{\sqrt{3C^{\frac{1}{1+\alpha}}V^{(1-A)}+9m^2V^{4/3}+c_1}}=t+t_0, \tag{2.21}$$

where c_1 and t_0 are constants of integration.

For $A=1/3$, $c_1=0$ and $t_0=0$, (2.12) leads to

$$V=\left(m^2t^2-\frac{C^{1/1+\alpha}}{3m^2}\right)^{3/2}. \tag{2.22}$$

Using (2.22) in (2.19a)–(2.19c), we obtain the value of the scale factors as

$$a_1(t)=D_1\left[m^2t^2-\frac{C^{\frac{1}{1+\alpha}}}{3m^2}\right]^{1/2}\times\exp\left[\frac{-3X_1m}{C^{\frac{1}{1+\alpha}}}\sqrt{\frac{3m^4t^2}{3m^4t^2-C^{\frac{1}{1+\alpha}}}}\right]. \tag{2.23a}$$

$$a_2(t)=D_2\left[m^2t^2-\frac{C^{\frac{1}{1+\alpha}}}{3m^2}\right]^{1/2}\times\exp\left[\frac{-3X_2m}{C^{\frac{1}{1+\alpha}}}\sqrt{\frac{3m^4t^2}{3m^4t^2-C^{\frac{1}{1+\alpha}}}}\right]. \tag{2.23b}$$

$$a_3(t)=D_3\left[m^2t^2-\frac{C^{\frac{1}{1+\alpha}}}{3m^2}\right]^{1/2}\times\exp\left[\frac{-3X_3m}{C^{\frac{1}{1+\alpha}}}\sqrt{\frac{3m^4t^2}{3m^4t^2-C^{\frac{1}{1+\alpha}}}}\right]. \tag{2.23c}$$

From (2.22), the value of the pressure and the energy density of the universe is given by

$$p\cong\frac{1}{3}C^{\frac{1}{1+\alpha}}\left(m^2t^2-\frac{C^{\frac{1}{1+\alpha}}}{3m^2}\right)^{-2}\quad\text{and}$$
$$\rho\cong C^{\frac{1}{1+\alpha}}\left(m^2t^2-\frac{C^{\frac{1}{1+\alpha}}}{3m^2}\right)^{-2}. \tag{2.24}$$

The Hubble parameter H is found as

$$H=m^2t\left(m^2t^2-\frac{C^{\frac{1}{1+\alpha}}}{3m^2}\right)^{-1} \tag{2.25}$$

The physical quantities are as follows:

(i) The expansion scalar $\theta=3H$ is

$$\theta=3m^2t\left(m^2t^2-\frac{C^{\frac{1}{1+\alpha}}}{3m^2}\right)^{-1} \tag{2.26}$$

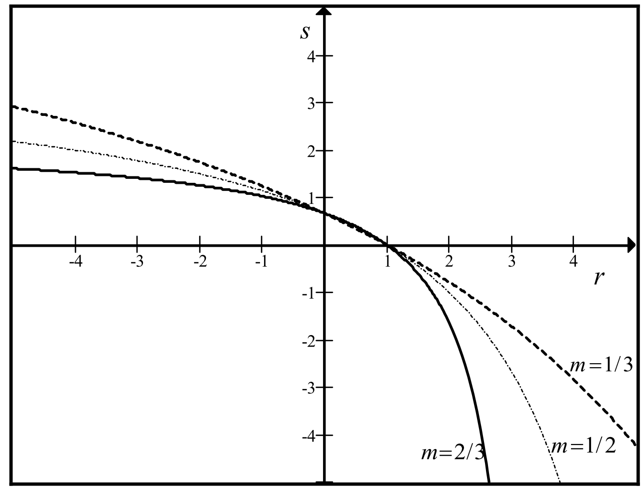

Fig. 1 Variation s against r for different values of $m(=1/3,1/2,2/3)$

(ii) The mean anisotropy parameter $\Delta=\frac{1}{3}\sum_{i=1}^{3}(\frac{H_i-H}{H})^2$ is found as

$$\Delta=\frac{X}{m^2t^2(3m^4t^2-C^{\frac{1}{1+\alpha}})}. \tag{2.27}$$

(iii) The shear scalar $\sigma^2=\frac{1}{2}(\sum_{i=1}^{3}H_i^2-3H^2)=\frac{3}{2}\Delta H^2$ is found as

$$\sigma^2=\frac{27m^6X}{2(3m^4t^2-C^{\frac{1}{1+\alpha}})^3}. \tag{2.28}$$

(iv) The deceleration parameter $q=\frac{d}{dt}(\frac{1}{H})-1$ is found as

$$q=\frac{C^{\frac{1}{1+\alpha}}}{3m^4t^2}, \tag{2.29}$$

where $X=X_1^2+X_2^2+X_3^2$.

From (1.2), the statefinder parameters are found as

$$r=\frac{C^{\frac{1}{1+\alpha}}}{m^6t^2}\quad\text{and}\quad s=\left(\frac{C^{\frac{1}{1+\alpha}}}{m^6t^2}-1\right)\left(\frac{C^{\frac{1}{1+\alpha}}}{m^4t^2}-\frac{3}{2}\right)^{-1}. \tag{2.30}$$

The relation between the statefinder parameters is

$$s=\frac{r-1}{rm^2-3/2}.$$

The variation of s with respect to r is shown in Fig. 1 above.

Case (ii) Now, for large values of the scale factors $a_1(t)$, $a_2(t)$ and $a_3(t)$, one may have

$$\rho=\left(\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\right)^{\frac{1}{1+\alpha}}V^{-n/3(1+\alpha)}. \tag{2.31}$$

and the pressure is given by

$$p = \left(-1 + \frac{n}{3(1+\alpha)}\right)\rho. \tag{2.32}$$

As per Guo and Zhang (2005) and Debnath (2008), this corresponds to Quintessence model (i.e. dark energy with constant equation of state).

One should note that when $n = 0$, this results corresponds to the original modified Chaplygin gas model discovered by Debnath et al. (2004) in which the modified Chaplygin gas behaves initially as radiation and later as a cosmological constant.

Hence, in order obtain the quintessence dominated era, one must have

$$\left(-1 + \frac{n}{3(1+\alpha)}\right) < -\frac{1}{3}.$$

Using (2.31) and (2.32) in (2.20), we get

$$\int \frac{dV}{\sqrt{kV^{\frac{6(1+\alpha)-n}{3(1+\alpha)}} + 9m^2V^{4/3} + c_2}} = t \tag{2.33}$$

where c_2 is constant of integration and $k = 3[\frac{3(1+\alpha)B_0}{3(1+\alpha)(1+A)-n}]^{\frac{1}{1+\alpha}}$.

For $n = 2(1+\alpha)$ and $c_2 = 0$, (2.33) leads to

$$V = \beta t^3, \quad \text{where } \beta = \left(\frac{k + 9m^2}{9}\right)^{3/2}. \tag{2.34}$$

Using (2.34), we obtain the values of the scale factors $a_1(t), a_2(t)$ and $a_3(t)$ as

$$a_1(t) = D_1\beta^{1/3} t \exp\left[\frac{-X_1}{2\beta t^2}\right] \tag{2.35a}$$

$$a_2(t) = D_2\beta^{1/3} t \exp\left[\frac{-X_2}{2\beta t^2}\right] \tag{2.35b}$$

$$a_3(t) = D_3\beta^{1/3} t \exp\left[\frac{-X_3}{2\beta t^2}\right]. \tag{2.35c}$$

From (2.34), the value of the energy density and the pressure of the universe is given by

$$\rho \cong \left[\frac{3(1+\alpha)B_0}{3(1+\alpha)(1+A)-n}\right]^{\frac{1}{1+\alpha}} \beta^{-n/3(1+\alpha)} t^{-n/(1+\alpha)} \quad \text{and}$$

$$p = \left(-1 + \frac{n}{3(1+\alpha)}\right) \times \left[\frac{3(1+\alpha)B_0}{3(1+\alpha)(1+A)-n}\right]^{\frac{1}{1+\alpha}} \beta^{-n/3(1+\alpha)} t^{-n/(1+\alpha)} \tag{2.36}$$

The Hubble parameter H is found as

$$H = \frac{1}{t}. \tag{2.37}$$

The physical quantities are as follows:

(i) The expansion scalar θ is

$$\theta = \frac{3}{t}, \tag{2.38}$$

(ii) The mean anisotropy parameter Δ is found as

$$\Delta = \frac{X}{3\beta^2 t^8}, \tag{2.39}$$

(iii) The shear scalar σ^2 is found as

$$\sigma^2 = \frac{X}{2\beta^2 t^{10}}, \tag{2.40}$$

(iv) The deceleration parameter q is found as

$$q = 0, \tag{2.41}$$

where $X = X_1^2 + X_2^2 + X_3^2$.

From (1.2), the statefinder parameters are found as

$$r = 0 \quad \text{and} \quad s = \frac{2}{3}. \tag{2.42}$$

Gorini et al. (2003, 2004) have shown that the simple flat Friedmann model with Chaplygin gas can equivalently be described in terms of a homogeneous minimally coupled scalar field ϕ and a self-interacting potential $v(\phi)$ with effective Lagrangian

$$L_\phi = \frac{1}{2}\dot{\phi}^2 - v(\phi). \tag{1.3}$$

Barrow (1988, 1990), Kamenshchik et al. (2001a, 2001b, 2004) have obtained homogeneous scalar field $\phi(t)$ and a potential $v(\phi)$ to describe Chaplygin cosmology.

Now, consider the energy density ρ_ϕ and pressure p_ϕ corresponding to a scalar field ϕ having a self-interacting potential $v(\phi)$. In view of variable modified Chaplygin gas model, the analogous energy density and the pressure of the scalar field are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + v(\phi) = \rho = \left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}} + \frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{1}{1+\alpha}} \tag{2.43}$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - v(\phi) = A\rho - \frac{B_0 V^{\frac{-n}{3}}}{\rho^\alpha}$$

$$= A\left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}} + \frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{1}{1+\alpha}} - B_0 V^{\frac{-n}{3}}\left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}} + \frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{-\alpha}{1+\alpha}} \tag{2.44}$$

From (2.43) and (2.44), we have

$$\dot{\phi}^2 = (1+A)\left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}} + \frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{1}{1+\alpha}} - B_0 V^{\frac{-n}{3}}\left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}} + \frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{-\alpha}{1+\alpha}} \tag{2.45}$$

and

$$v(\phi) = \frac{(1-A)}{2}\left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}} + \frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{1}{1+\alpha}} + \frac{B_0 V^{\frac{-n}{3}}}{2}\left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}}\frac{1}{V^{n/3}} + \frac{C}{V^{(1+\alpha)(1+A)}}\right]^{\frac{-\alpha}{1+\alpha}} \tag{2.46}$$

3 Conclusion

Here, a study has been carried on a spatially homogeneous and anisotropic Bianchi type-V model filled with variable modified Chaplygin gas. The equation of state of this cosmological model is valid from the radiation era to the quintessence model. In this model a detailed description of the universe has been given from radiation era ($A = 1/3$ & ρ is very large) to quintessence model (ρ is very small). As compared to Chaplygin gas models; this model describes the universe to a large extent. The physical and geometrical parameters are also discussed. The statefinder diagnostic pair i.e. $\{r, s\}$ parameter is adopted to differentiate among different forms of dark energy. It is interesting to note that this model reduces to Chaplygin gas model with $A = 0$ and $\alpha = 1$. Further it reduces to modified Chaplygin gas model with $n = 0$.

Acknowledgement The author is thankful to Hon'ble Referee and Editor (AP&SS) for valuable suggestions.

References

Adhav, K.S., et al.: Astrophys. Space Sci. **331**, 689 (2011)
Akarsu, O., Kilinc, C.B.: Gen. Relativ. Gravit. **42**(4), 763 (2010)
Barrow, J.D.: Nucl. Phys. B **310**, 743 (1988)
Barrow, J.D.: Phys. Lett. B **235**, 40 (1990)
Benaoum, H.B.: hep-th/0205140 (2002)
Bento, M.C., Bertolami, O., Sen, A.: Phys. Rev. D **66**, 043507 (2002)
Bento, M.C., Bertolami, O., Sen, A.A.: Phys. Lett. B **575**, 172 (2003)
Bergshoeff, E.: Class. Quantum Gravity **21**, 1947 (2004)
Berman, M.S.: Nuovo Cimento B **74**, 182 (1983)
Berman, M.S., Gomide, F.M.: Gen. Relativ. Gravit. **20**, 191 (1988)
Bernardis, P.D., et al.: Nature **404**, 995 (2000)
Brandt, C., et al.: Gen. Relativ. Gravit. **39**, 1675 (2007)
Caldwell, R.R., Dave, R., Steinhardt, P.J.: Phys. Rev. Lett. **80**, 1582 (1998)
Capozziello, S., Carloni, S., Troisi, A.: arXiv:astro-ph/0303041 (2003)
Carroll, S.M., et al.: Phys. Rev. D **70**, 043528 (2003)
Chakraborty, W., Debnath, U.: Mod. Phys. Lett. A **22**, 1805 (2007)
Chen, C.M., et al.: J. High Energy Phys. **10**, 058 (2003)
Cline, J.M., Vinet, J.: Phys. Rev. D **68**, 025015 (2003)
Coles, P., Ellis, G.F.R.: Nature **370**, 609 (1994)
Copeland, E.J., et al.: Int. J. Mod. Phys. D **15**, 1753 (2006)
Debnath, U.: Astrophys. Space Sci. **312**, 295 (2007)
Debnath, U.: Class. Quantum Gravity **25**, 205019 (2008)
Debnath, U., Banerjee, A., Chakraborty, S.: Class. Quantum Gravity **21**, 5609 (2004)
Deffayet, C.: Phys. Lett. B **502**, 199 (2001)
Dvali, G.R., Gabadadze, G., Porrati, M.: Phys. Lett. B **484**, 112 (2000)
Gibbons, G.W.: Aspects of supergravity theories. In: del Aguila, F., de Azcarraga, J.A., Ibanez, L.E. (eds.) Supersymmetry, Supergravity and Related Topics, pp. 346–351. World Scientific, Singapore (1985)
Gong, Y., Wang, A.: Class. Quantum Gravity **23**, 3419 (2006)
Gong, Y., Wang, A., Wu, Q.: arXiv:0711.1597 [gr-qc] (2007)
Gorini, V., et al.: Phys. Rev. D **67**, 063509 (2003)
Gorini, V., et al.: arXiv:gr-qc/0403062 (2004)
Guo, Z.K., Zhang, Y.Z.: astro-ph/0506091 (2005)
Hanany, S., et al.: Astrophys. J. **545**, L5 (2000)
Hu, M., Meng, X.H.: Phys. Lett. B **635**, 186 (2006)
Huang, J.Z., et al.: Astrophys. Space Sci. **315**, 175 (2008)
Jamil, M., Rashid, M.A.: Eur. Phys. J. C **58**, 111 (2008)
Kamenshchik, A., et al.: Phys. Lett. B **511**, 265 (2001a)
Kamenshchik, A., Moschella, U., Pasquier, V.: Phys. Lett. B **511**, 265 (2001b)
Kamenshchik, A., et al.: Phys. Rev. D, Part. Fields **69**, 123512 (2004)
Koivisto, T., Mota, D.F.: J. Cosmol. Astropart. Phys. **06**, 018 (2008)
Liddle, A.R., Scherrer, R.J.: Phys. Rev. D **59**, 023509 (1999)
Maeda, K., Ohta, N.: Phys. Rev. D **71**, 063520 (2005)
Maldacena, J.M., Nuñez, C.: Int. J. Mod. Phys. A **16**, 822 (2001)
Matos, T., Urena-Lopez, A.: Class. Quantum Gravity **17**, L75 (2000)
Neupane, I.P.: Phys. Rev. Lett. **98**, 061301 (2007)
Neupane, I.P., Wiltshire, D.L.: Phys. Lett. B **619**, 201 (2005a)
Neupane, I.P., Wiltshire, D.L.: Phys. Rev. D **72**, 083509 (2005b)
Nojiri, S., Odintsov, S.D.: Phys. Rev. D **68**, 123512 (2003)
Ohta, N.: Phys. Rev. Lett. **91**, 061303 (2003a)
Ohta, N.: Prog. Theor. Phys. **110**, 269 (2003b)
Ohta, N.: Int. J. Mod. Phys. A **20**, 1 (2005)
Padmanabhan, T.: arXiv:0705.2533 [gr-qc] (2007)
Padmanabhan, T., Choudhury, T.R.: Phys. Rev. D **66**, 081301 (2002)
Pereira, P.R.C.T., da Silva, M.F.A., Chan, R.: Int. J. Mod. Phys. D **15**, 991 (2006)
Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
Riess, A.G., et al.: Astrophys. J. **607**, 665 (2004)
Roy, S.: Phys. Lett. B **567**, 322 (2003)

Saha, Bijan: Chin. J. Phys. **43**, 1035 (2005)
Sahni, V., Saini, T.D., Starobinsky, A.A., Alam, U.: JETP Lett. **77**, 201 (2003)
Shao, Y., Gui, Y.: Mod. Phys. Lett. A **23**, 65 (2008)
Singh, T., Chaubey, R.: Astrophys. Space Sci. **319**(2–4), 149 (2009)
Spergel, D.N., et al.: Astron. Astrophys. Suppl. Ser. **170**, 377 (2007)
Steinhardt, P.J., Wang, L.M., Zlatev, I.: Phys. Rev. D **59**, 123504 (1999)
Townsend, P.K., Wohlfarth, N.R.: Phys. Rev. Lett. **91**, 061302 (2003)
Webb, J.K., et al.: Phys. Rev. Lett. **87**, 091301 (2001)
Wei, H., Cai, R.G.: Phys. Lett. B **655**, 1 (2007)
Wetterich, C.: Phys. Rev. D **65**, 123512 (2002)
Wohlfarth, N.R.: Phys. Lett. B **563**, 1 (2003)
Zhang, X.: Int. J. Mod. Phys. D **14**, 1597 (2005a)
Zhang, X.: Phys. Lett. B **611**, 1 (2005b)
Zhao, W.: Int. J. Mod. Phys. D **17**, 1245 (2008)
Zimdahl, W., Pavon, D.: Gen. Relativ. Gravit. **36**, 1483 (2004)