Astrophys Space Sci (2011) 335:565–575
DOI 10.1007/s10509-011-0745-3

ORIGINAL ARTICLE

# Some anisotropic dark energy models in Bianchi type-V space-time

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Received: 22 January 2011 / Accepted: 12 May 2011 / Published online: 28 May 2011


**Abstract** The paper deals with Bianchi type V Universe, which has dynamical energy density. We consider Bianchi type V space-time, introducing three different skewness parameters along spatial directions to quantify the deviation of pressure from isotropy. To study the anisotropic nature of the dynamical dark energy, we assume that the skewness parameters are time dependent. It is found that the Universe achieves flatness in quintessence model. The physical behavior of the Universe has been discussed in detail.

**Keywords** Bianchi-V space-time · Hubble's parameter · Deceleration parameter · Dark energy

## 1 Introduction

Recent observations have revolutionized our understanding of cosmology. Analysis of type Ia supernovae (SN Ia) (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 2004), cosmic microwave background (CMB) anisotropy (Caldwell 2002; Huang et al. 2006), and large scale structure (Daniel et al. 2008) strongly indicate that dark energy (DE) dominates the present Universe, causing cosmic acceleration. This acceleration is realized with negative pressure and positive energy density that violate the strong energy condition. This violation gives a reverse gravitational effect. Due to this effect, the Universe gets a jerk and the transition from the earlier deceleration phase to the recent acceleration phase take place (Caldwell et al. 2006). The cause of this sudden transition and the source of accelerated expansion is still unknown. In physical cosmology and astronomy, the simplest candidate for the DE is the cosmological constant ($\Lambda$), but it needs to be extremely fine-tuned to satisfy the current value of the DE density, which is a serious problem. Alternatively, to explain the decay of the density, the different forms of dynamically changing DE with an effective equation of state (EoS), $\omega = p^{(de)}/\rho^{(de)} < -1/3$, were proposed instead of the constant vacuum energy density. Other possible forms of DE include quintessence ($\omega > -1$) (Steinhardt et al. 1999), phantom ($\omega < -1$) (Caldwell 2002) etc. While the the possibility $\omega \ll -1$ is ruled out by current cosmological data from SN Ia (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) (Riess et al. 2004; Astier et al. 2006), CMBR (WMAP, BOOMERANG) (Eisentein et al. 2005; MacTavish et al. 2006) and large scale structure (Sloan Digital Sky Survey) (Komatsu et al. 2009) data, the dynamically evolving DE crossing the phantom divide line (PDL) ($\omega = -1$) is mildly favored.

The anisotropy of the DE within the framework of Bianchi type space-times is found to be useful in generating arbitrary ellipsoidality to the Universe, and to fine tune the observed CMBR anisotropies. Koivisto and Mota (2008a, 2008b) have investigated cosmological model with anisotropic EoS. They have proposed a different approach to resolve CMB anisotropy problem; even if the CMB formed isotropically at early time, it could be distorted by the direction dependent acceleration of the future Universe in such a way that it appears to us anomalous at the largest scales. They have investigated a cosmological model containing a DE component which has a non dynamical anisotropic EoS and interacts with the perfect fluid component. They have also suggested that cosmological models with anisotropic EoS can explain the quadrupole problem and can be tested by SN Ia data. Kumar and Singh (2011) have studied Bianchi type I cosmological models

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with constant deceleration parameter (DP) in the presence of anisotropic DE and perfect fluid. They have considered phenomenological parametrization of minimally interacting DE in terms of its EoS and time-dependent skewness parameters $(\delta(t), \gamma(t), \eta(t))$. Leon and Sarikadis (2010) have investigated that anisotropic geometries in modified gravitational frameworks present radically difference cosmological behaviors comparing to the simple isotropic scenarios. Akarsu and Kilinc (2010) have investigated Bianchi-I anisotropic DE model with constant DP. Yadav and Yadav (2011), Yadav et al. (2011) have studied anisotropic DE models with variable EoS parameter. They have suggested that the dynamics of EoS parameter describe the present acceleration of Universe i.e. from earlier deceleration phase to recent acceleration phase. Recently Pradhan and Amirhashchi (2011), Amirhashchi et al. (2011) have studied anisotropic DE models in different physical contexts. They have found that in the earlier stage EoS parameter was positive and it evolves with negative sign at present epoch.

Bianchi type-V Universe is generalization of the open Universe in FRW cosmology and hence it's study is important in the study of DE models in Universe with nonzero curvature (1994). A number of authors such as Collins (1974), Coles and Ellis (1994), Maartens and Nel (1978), Wainwright et al. (1979), Camci et al. (2001), Pradhan et al. (2004), Singh et al. (2008), Yadav (2009) have studied Bianchi type-V model in different physical contexts. Recently Kumar and Yadav (2011) have studied isotropic DE model with variable EoS parameter in Bianchi type V space-time and found that the Universe is dominated by DE at present epoch and after dominance of DE, Universe achieves flatness. Following Eriksen (2004), it is found that some large-angle anomalies appear in CMB radiations which violate the statistical isotropy of the Universe. This motivates the researcher to consider the model of Universe with anisotropic DE.

In this paper, we have studied some physically realistic and totally anisotropic Bianchi-V models with anisotropic DE and perfect fluid. To study the anisotropic nature of DE, we have assumed the time dependent skewness parameter, which modify EoS. The time dependent forms of the skewness parameter provide exact solutions of Einstein's field equation together with the special law of variation of Hubble's parameter. The paper is organized as follows. In Sect. 2, the models and field equations have been presented. The Sect. 3 deals with the exact solutions of the field equations and physical behavior of the models. Finally, the results are discussed in Sect. 4.

## 2 Model and field equations

The spatially homogeneous and anisotropic Bianca-V space-time is described by the line element

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha x}(B^2dy^2 + C^2dz^2), \tag{1}$$

where $A$, $B$ and $C$ are the metric functions of cosmic time $t$ and $\alpha$ is a constant.

We define $a = (ABC)^{\frac{1}{3}}$ as the average scale factor of the space-time (1) so that the average Hubble's parameter reads as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{2}$$

where $a = (ABC)^{\frac{1}{3}}$ is the average scale factor and an over dot denotes derivative with respect to the cosmic time $t$.

The directional Hubble parameters along $x$, $y$ and $z$ coordinate axes, respectively, may be defined as

$$H_x = \frac{\dot{A}}{A}, \qquad H_y = \frac{\dot{B}}{B}, \qquad H_z = \frac{\dot{C}}{C}. \tag{3}$$

The Einstein's field equations (in gravitational units $8\pi G = c = 1$) read as

$$R^i_j - \frac{1}{2}g^i_jR = -T^{(m)i}_j - T^{(de)i}_j, \tag{4}$$

where $T^{(m)i}_j$ and $T^{(de)i}_j$ are the energy momentum tensors of perfect fluid and DE, respectively. These are given by

$$T^{(m)i}_j = \mathrm{diag}[-\rho^{(m)}, p^{(m)}, p^{(m)}, p^{(m)}] \tag{5}$$

and

$$\begin{aligned} T^{(de)i}_j &= \mathrm{diag}[-\rho^{(de)}, p^{(de)}_x, p^{(de)}_y, p^{(de)}_z] \\ &= \mathrm{diag}[-1, \omega_x, \omega_y, \omega_z]\rho^{(de)} \\ &= \mathrm{diag}[-1, w+\delta, w+\gamma, w+\eta]\rho^{(de)} \end{aligned} \tag{6}$$

where $\rho^{(m)}$ and $p^{(m)}$ are, respectively the energy density and pressure of the perfect fluid component; $\rho^{(de)}$ is the energy density of the DE component; $\delta(t)$, $\gamma(t)$ and $\eta(t)$ are skewness parameters, which modify EoS (hence pressure) of the DE component and are functions of the cosmic time $t$; $\omega$ is the EoS parameter of DE; $\omega_x$, $\omega_y$ and $\omega_z$ are the directional EoS parameters along $x$, $y$ and $z$ coordinate axes, respectively and we assume the four velocity vector $u^i = (1, 0, 0, 0)$ satisfying $u^iu_i = -1$.

In a co moving coordinate system ($u^i = \delta^i_0$), the field equations (4), for the anisotropic Bianchi type-I space-time (1), in case of (5) and (6), read as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -p^{(m)} - (\omega+\delta)\rho^{(de)}, \tag{7}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}-\frac{\alpha^2}{A^2}=-p^{(m)}-(\omega+\gamma)\rho^{(de)}, \quad (8)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-p^{(m)}-(\omega+\eta)\rho^{(de)}, \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3\alpha^2}{A^2}=\rho^{(m)}+\rho^{(de)}. \quad (10)$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \quad (11)$$

We assume that the perfect fluid and DE components interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately.

The energy conservation equation $T^{(m)j}_{;j}=0$, of the perfect fluid leads to

$$\dot{\rho}^{(m)}+3(\rho^{(m)}+p^{(m)})H=0, \quad (12)$$

whereas the energy conservation equation $T^{(de)j}_{;j}=0$, of the DE component yields

$$\dot{\rho}^{(de)}+3\rho^{(de)}(\omega+1)H+\rho^{(de)}(\delta H_x+\gamma H_y+\eta H_z)=0, \quad (13)$$

where we have used the equation of state $p^{(de)}=\omega\rho^{(de)}$.

Equations (7)–(10) can be written in terms of $H$, $\sigma$ and $q$ as

$$p^{(de)}+\frac{1}{3}(3\omega+\delta+\gamma+\eta)\rho^{(de)} = H^2(2q-1)-\sigma^2+\frac{\alpha^2}{A^2} \quad (14)$$

$$\rho^m+\rho^{de}=3H^2-\sigma^2-\frac{3\alpha^2}{A^2} \quad (15)$$

Where $q$ and $\sigma$ are deceleration parameter and shear scalar respectively.

## 3 Solution of field equations

Berman (1983), Berman and Gomide (1988) obtained some FRW cosmological models with constant DP and showed that the constant DP models stand adequately for our present view of different phases of the evolution of Universe. In this paper, we show how the constant deceleration parameter models with metric (1) behave in the presence of anisotropic DE. According to the law, the variation of the average Hubble parameter is given by Singh et al. (2008)

$$H=Da^{-n}, \quad (16)$$

where $D>0$ and $n\geq 0$ are constants.

Following, Akarsu and Kilinc (2010), we split the conservation of energy momentum tensor of the DE into two parts, One corresponds to deviations of EoS parameter and other is the deviation-free part of $T^{(de)ij}_{;j}=0$:

$$\dot{\rho}+3\rho^{(de)}(\omega+1)H=0 \quad (17)$$

and

$$\rho^{(de)}(\delta H_x+\gamma H_y+\eta H_z)=0, \quad (18)$$

According to (17) and (18) the behaviour of $\rho^{(de)}$ is controlled by the deviation-free part of EoS parameter of DE but deviations will affect $\rho^{(de)}$ indirectly, since, as can be seen later, they affect the value of EoS parameter. Of course, the choice of skewness parameters are quite arbitrary but, since we are looking for a physically viable models of Universe consistent with observations. We consider the skewness parameters $\delta$, $\gamma$ and $\eta$ be function of cosmic time and we constrained $\delta$, $\gamma$ and $\eta$ by assuming a special dynamics which is consistent with (18). The dynamics of skewness parameters on $x$-axis, $y$-axis and $z$-axis are assumed to be

$$\delta(t)=\alpha(H_y+H_z)\frac{1}{\rho^{(de)}}, \quad (19)$$

$$\gamma(t)=-\alpha H_x\frac{1}{\rho^{(de)}}, \quad (20)$$

$$\eta(t)=-\alpha H_x\frac{1}{\rho^{(de)}}, \quad (21)$$

where $\alpha$ is an arbitrary constant, which parameterizes the anisotropy of the DE. In literature, many authors have considered totally anisotropic Bianchi-I, Bianchi-III and Bianchi-V space-times with only two skewness parameters of DE (Akarsu and Kilinc 2010; Yadav and Yadav 2011; Yadav et al. 2011; Pradhan and Amirhashchi 2011; Amirhashchi et al. 2011).

Finally, we assume that $\omega=$ const. so that we can study different models related to the DE by choosing different values of $\omega$, viz. phantom ($\omega<-1$), cosmological constant ($\omega=-1$) and quintessence ($\omega>-1$).

In view of the assumptions (19)–(21) and $\omega=$ const., equation (11) can be integrated to obtain

$$\rho^{(de)}(t)=\rho_0 a^{-3(\omega+1)}, \quad (22)$$

where $\rho_0$ is a positive constant of integration.

Integrating (11) and absorbing the constant of integration in $B$ or $C$, without loss of generality, we obtain

$$A^2=BC. \quad (23)$$

Subtracting (7) from (8), (7) from (9), (8) from (10) and taking second integral of each, we get the following three relations respectively:

$$\frac{A}{B} = d_1 \exp\left[x_1 \int a^{-3} dt - \frac{\alpha\rho_0}{\omega} \int a^{-3(\omega+1)} dt\right], \tag{24}$$

$$\frac{A}{C} = d_2 \exp\left[x_2 \int a^{-3} dt - \frac{\alpha\rho_0}{\omega} \int a^{-3(\omega+1)} dt\right], \tag{25}$$

$$\frac{B}{C} = d_3 \exp\left(x_3 \int a^{-3} dt\right), \tag{26}$$

where $d_1$, $x_1$, $d_2$, $x_2$, $d_3$ and $x_3$ are constants of integration. From (24)–(26) and (23), the metric functions can be explicitly written as

$$A(t) = a \exp\left[-\frac{2\alpha\rho_0}{3\omega} \int a^{-3(\omega+1)} dt\right], \tag{27}$$

$$B(t) = ma \exp\left[l \int a^{-3} dt + \frac{\alpha\rho_0}{3\omega} \int a^{-3(\omega+1)} dt\right], \tag{28}$$

$$C(t) = m^{-1} a \exp\left[-l \int a^{-3} dt + \frac{\alpha\rho_0^{(de)}}{3\omega} \int a^{-3(\omega+1)} dt\right], \tag{29}$$

where

$$m = \sqrt[3]{d_2 d_3}, \qquad l = \frac{(x_2 + x_3)}{3} \tag{30}$$

with

$$d_2 = d_1^{-1}, \qquad x_2 = -x_1. \tag{31}$$

In the following subsections, we discuss the cosmologies for $n \neq 0$ and $n = 0$, respectively.

### 3.1 DE cosmology for $n \neq 0$

In this case, integration of (16) leads to

$$a(t) = (nDt)^{\frac{1}{n}}, \tag{32}$$

where the constant of integration has been omitted by assuming that $a = 0$ at $t = 0$.

Using (32) into (27)–(29), we get the following expressions for scale factors:

$$A(t) = (nDt)^{\frac{1}{n}} \exp\left[-\frac{2\alpha\rho_0}{3\omega D(n-3\omega-3)} (nDt)^{\frac{n-3\omega-3}{n}}\right], \tag{33}$$

$$B(t) = m(nDt)^{\frac{1}{n}} \exp\left[\frac{l}{D(n-3)} (nDt)^{\frac{n-3}{n}} + \frac{\alpha\rho_0}{3\omega D(n-3\omega-3)} (nDt)^{\frac{n-3\omega-3}{n}}\right], \tag{34}$$

$$C(t) = m^{-1}(nDt)^{\frac{1}{n}} \exp\left[\frac{-l}{D(n-3)} (nDt)^{\frac{n-3}{n}} + \frac{\alpha\rho_0}{3\omega D(n-3\omega-3)} (nDt)^{\frac{n-3\omega-3}{n}}\right], \tag{35}$$

where $n \neq 3$.

The physical parameters such as directional Hubble parameters $(H_x, H_y, H_z)$, average Hubble parameter $(H)$, anisotropy parameter $(\bar{A})$, expansion scalar $(\theta)$ and spatial volume $(V)$ are, respectively, given by

$$H_x = (nt)^{-1} - \frac{2\alpha\rho_0}{3\omega} (nDt)^{\frac{-3(\omega+1)}{n}}, \tag{36}$$

$$H_y = (nt)^{-1} + l(nDt)^{\frac{-3}{n}} + \frac{\alpha\rho_0}{3\omega} (nDt)^{\frac{-3(\omega+1)}{n}}, \tag{37}$$

$$H_z = (nt)^{-1} - l(nDt)^{\frac{-3}{n}} + \frac{\alpha\rho_0}{3\omega} (nDt)^{\frac{-3(\omega+1)}{n}}, \tag{38}$$

$$H = (nt)^{-1}, \tag{39}$$

$$\bar{A} = \frac{1}{3}\left[\left(\frac{H_x - H}{H}\right)^2 + \left(\frac{H_y - H}{H}\right)^2 + \left(\frac{H_z - H}{H}\right)^2\right] = \frac{2}{3D^2}\left[l^2 (nDt)^{\frac{2(n-3)}{n}} + \frac{\alpha^2 {\rho_0}^2}{3\omega^2} (nDt)^{\frac{2(n-3\omega-3)}{n}}\right], \tag{40}$$

$$\theta = u^i_{;i} = \frac{3\dot{a}}{a} = 3(nt)^{-1}, \tag{41}$$

$$V = (nDt)^{\frac{3}{n}} \exp(2\alpha x), \tag{42}$$

Shear scalar of the model reads as

$$\sigma^2 = l^2 (nDt)^{\frac{-6}{n}} + \frac{\alpha^2 {\rho_0}^2}{3\omega^2} (nDt)^{\frac{-6(\omega+1)}{n}}. \tag{43}$$

The value of DP $(q)$ is found to be

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = n - 1, \tag{44}$$

which is a constant. The sign of $q$ indicates whether the model inflates or not. A positive sign of $q$, i.e., $n > 1$ corresponds to the standard decelerating model whereas the negative sign of $q$, i.e., $0 < n < 1$ indicates acceleration. The expansion of the Universe at a constant rate corresponds to $n = 1$, i.e., $q = 0$. Also, recent observations of SN Ia, reveal that the present Universe is accelerating and value of DP lies somewhere in the range $-1 < q < 0$. It follows that in the derived model, one can choose the values of DP consistent with the observations.

The skewness parameters of DE are as follows:

$$\delta(t) = \frac{\alpha}{\rho_0}\left[2D(Dnt)^{-\frac{3(\omega+1)-n}{n}} + \frac{2\alpha\rho_0}{3\omega}\right], \tag{45}$$

$$\gamma(t) = \eta(t) = -\frac{\alpha}{\rho_0}\left[2D(Dnt)^{-\frac{3(\omega+1)-n}{n}} - \frac{2\alpha\rho_0}{3\omega}\right]. \tag{46}$$

In view of (5), the directional EoS parameters of DE are given by

$$\omega_x = \omega + \frac{\alpha}{\rho_0}\left[2D(Dnt)^{-\frac{3(\omega+1)-n}{n}} + \frac{2\alpha\rho_0}{3\omega}\right], \tag{47}$$

$$\omega_y = \omega_z = \omega - \frac{\alpha}{\rho_0}\left[2D(Dnt)^{-\frac{3(\omega+1)-n}{n}} - \frac{2\alpha\rho_0}{3\omega}\right]. \tag{48}$$

The energy density and pressure of the DE components are obtained as

$$\rho^{(de)} = \rho_0(nDt)^{\frac{-3(\omega+1)}{n}}, \tag{49}$$

$$p^{(de)} = \omega\rho_0(nDt)^{\frac{-3(\omega+1)}{n}}. \tag{50}$$

From (14) and (15), the pressure and energy density of the perfect fluid are obtained as

$$\begin{aligned} p^{(m)} &= (2n-3)(nt)^{-2} - l^2(nDt)^{\frac{-6}{n}} \\ &\quad - \frac{(2\omega+1)\alpha^2{\rho_0}^2}{3\omega^2}(nDt)^{\frac{-6(\omega+1)}{n}} \\ &\quad - \omega\rho_0(nDt)^{\frac{-3(\omega+1)}{n}} \\ &\quad + \frac{\alpha^2}{(nDt)^{\frac{2}{n}}\exp[-\frac{4\alpha\rho_0}{3\omega D(n-3\omega-3)}(nDt)^{\frac{n-3\omega-3}{n}}]}, \end{aligned} \tag{51}$$

$$\begin{aligned} \rho^{(m)} &= 3(nt)^{-2} - l^2(nDt)^{\frac{-6}{n}} - \frac{\alpha^2{\rho_0}^2}{3\omega^2}(nDt)^{\frac{-6(\omega+1)}{n}} \\ &\quad - \frac{3\alpha^2}{(nDt)^{\frac{2}{n}}\exp[-\frac{4\alpha\rho_0}{3\omega D(n-3\omega-3)}(nDt)^{\frac{n-3\omega-3}{n}}]} \\ &\quad - \rho_0(nDt)^{\frac{-3(\omega+1)}{n}}. \end{aligned} \tag{52}$$

The perfect fluid density parameter ($\Omega^{(m)}$) and DE density parameter ($\Omega^{(de)}$) are given by

$$\begin{aligned} \Omega^{(m)} &= 1 - \frac{l^2(nDt)^{\frac{-6}{n}}}{3(nt)^{-2}} - \frac{\alpha^2{\rho_0}^2}{9\omega^2(nt^{-2})}(nDt)^{\frac{-6(\omega+1)}{n}} \\ &\quad - \frac{1}{(nt)^{-2}} \\ &\quad \times \left(\frac{3\alpha^2}{(nDt)^{\frac{2}{n}}\exp[-\frac{4\alpha\rho_0}{3\omega D(n-3\omega-3)}(nDt)^{\frac{n-3\omega-3}{n}}]} \right. \\ &\quad \left. + \rho_0(nDt)^{\frac{-3(\omega+1)}{n}}\right). \end{aligned} \tag{53}$$

$$\Omega^{(de)} = \frac{\rho_0 D^{-3(\omega+1)}}{n}(nt)^{\frac{2n-3(\omega+1)}{n}} \tag{54}$$

Thus the overall density parameter ($\Omega$) is obtained as

$$\begin{aligned} \Omega &= \Omega^{(m)} + \Omega^{(de)} \\ &= 1 - \frac{1}{3(nt)^{-2}}\left[l^2(nDt)^{\frac{-6}{n}} - \frac{\alpha^2{\rho_0}^2}{3\omega^2}(nDt)^{\frac{-6(\omega+1)}{n}}\right. \end{aligned}$$

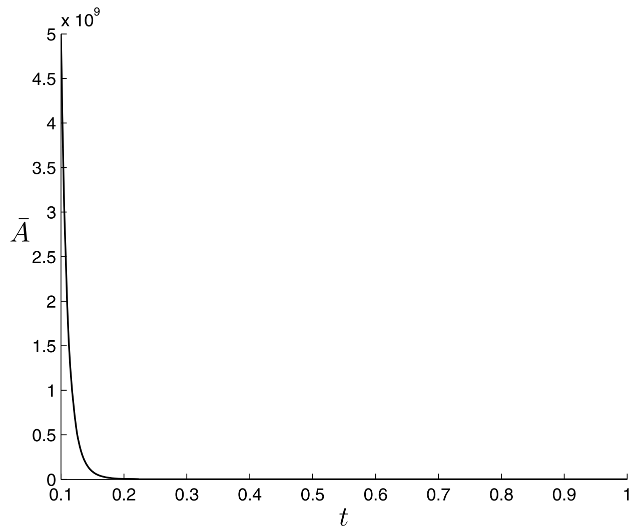



**Fig. 1** Plot of anisotropic parameter ($\bar{A}$) versus time ($t$)

$$\left. - \frac{3\alpha^2}{(nDt)^{\frac{2}{n}}\exp[-\frac{4\alpha\rho_0}{3\omega D(n-3\omega-3)}(nDt)^{\frac{n-3\omega-3}{n}}]}\right]. \tag{55}$$

It is observed that at $t = 0$, the spatial volume vanishes while all other parameters diverge. Thus the derived model starts expanding with big bang singularity at $t = 0$. This singularity is point type because the directional scale factors $A(t)$, $(B(t))$ and $C(t)$ vanish at initial moment. The solutions for the scale factors have a combination of a power-law term and exponential term in the product form. The DE term appears in exponential form and thus affects their evolution significantly. For $\alpha < 0$ and $n > 3\omega + 3$, the DE contributes to the expansion of $A(t)$ while opposing to the expansion of $B(t)$ and $C(t)$. Likewise, for $\alpha > 0$ and $n > 3\omega + 3$, the anisotropic DE opposes the expansion of $A(t)$ while contributing to the expansion of $B(t)$ and $C(t)$.

The difference between the directional EoS parameters and hence the pressures of the DE, along $x$-axis and $y$-axis (or $z$-axis) is $3\alpha(nt)^{-1}$, which decreases as $t$ increases. Therefore, the anisotropy of the DE decreases as $t$ increases and finally drops to zero at late time. The variation of mean anisotropic parameter ($\bar{A}$) versus has been graphed in Fig. 1 by choosing $D = 2$, $\omega = -1.1$, $n = 0.5$ and other constant as unity. Since the current observations strongly recommend that the present Universe is accelerating (i.e. $q < 0$ ). We consider $n = 0.5$ i.e. $q = -0.5$ in the remaining discussion of the model.

Figure 2 depicts the variation of directional EoS parameter (i.e. $\omega_x$, $\omega_y$, $\omega_z$) versus cosmic time. We observe that directional EoS parameter along $x$-axis (i.e. $\omega_x$) is decreasing function of time while directional EoS parameters along $y$-axis (or $z$-axis) are increasing function of time. At the later stage of evolution, all the directional EoS parameters approaches to $-1$ as expected. The same is predicted by current observations.

**Fig. 2** Plot of directional EoS parameters versus time ($t$)

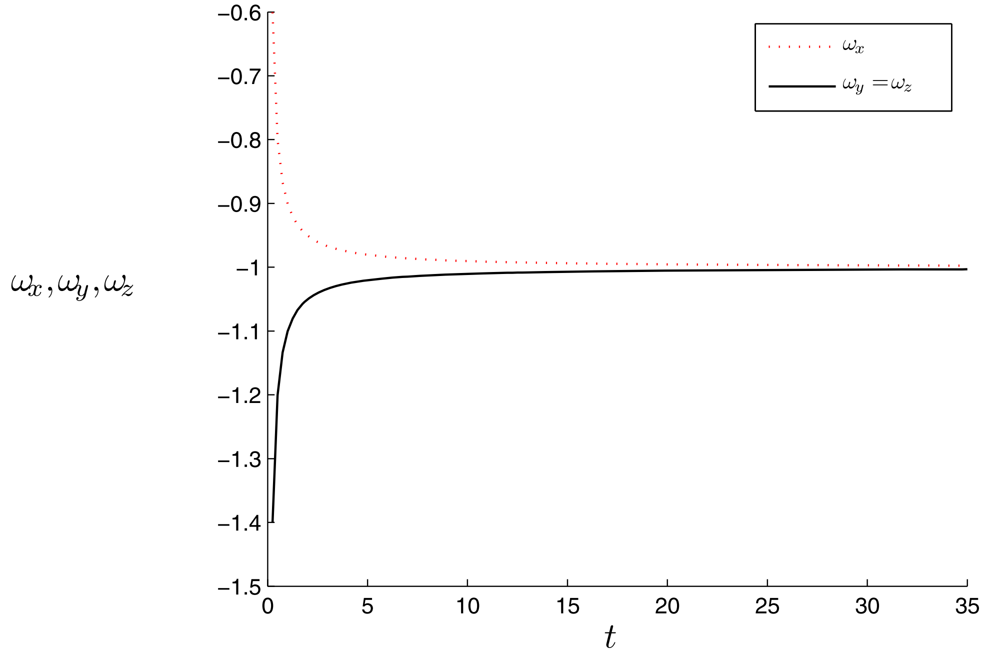
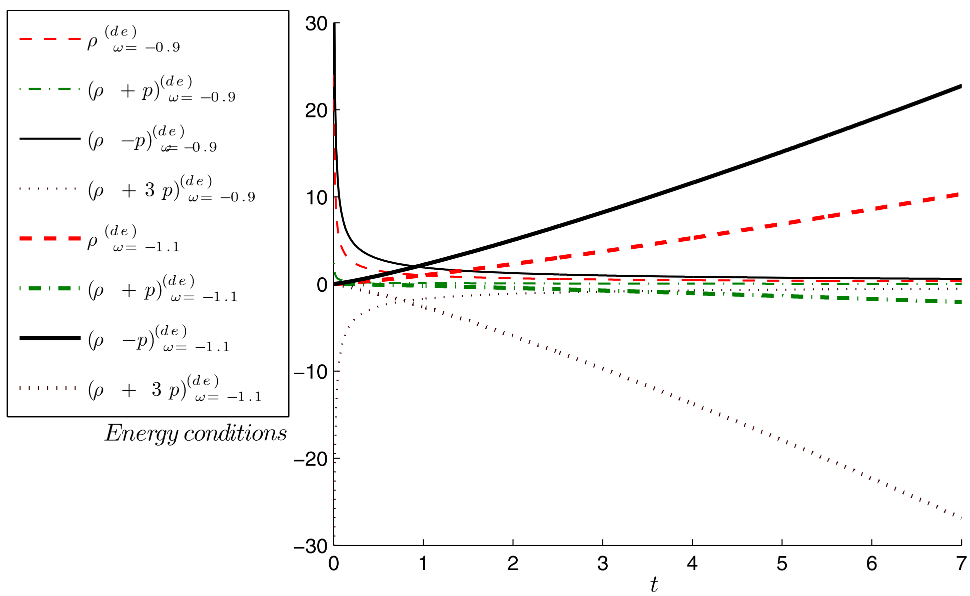



**Fig. 3** Single plot of energy conditions




Following, Caldwell (2002), Srivastava (2005) and recently Yadav (2011) have investigated phantom model with $\omega < -1$. They have suggested that at late time, phantom energy has appeared as a potential DE candidate which violates the weak as well as strong energy condition. The left hand side of energy conditions have graphed in Fig. 3.

From Fig. 3, for $\omega = -0.9$ (i.e. quintessence model), we observe that

(i) $\rho^{(de)} \geq 0$
(ii) $\rho^{(de)} + p^{(de)} \geq 0$
(iii) $\rho^{(de)} - p^{(de)} \geq 0$
(iv) $\rho^{(de)} + 3p^{(de)} < 0$

Thus the derived quintessence model violates the strong energy conditions, as expected.

Further, for $\omega = -1.1$ (i.e. phantom model), it is observed that

(i) $\rho^{(de)} > 0$
(ii) $\rho^{(de)} - p^{(de)} > 0$
(iii) $\rho^{(de)} + p^{(de)} < 0$
(iv) $\rho^{(de)} + 3p^{(de)} < 0$

Thus the derived phantom model violates the weak as well as strong energy conditions. The same is predicted by current astronomical observations.

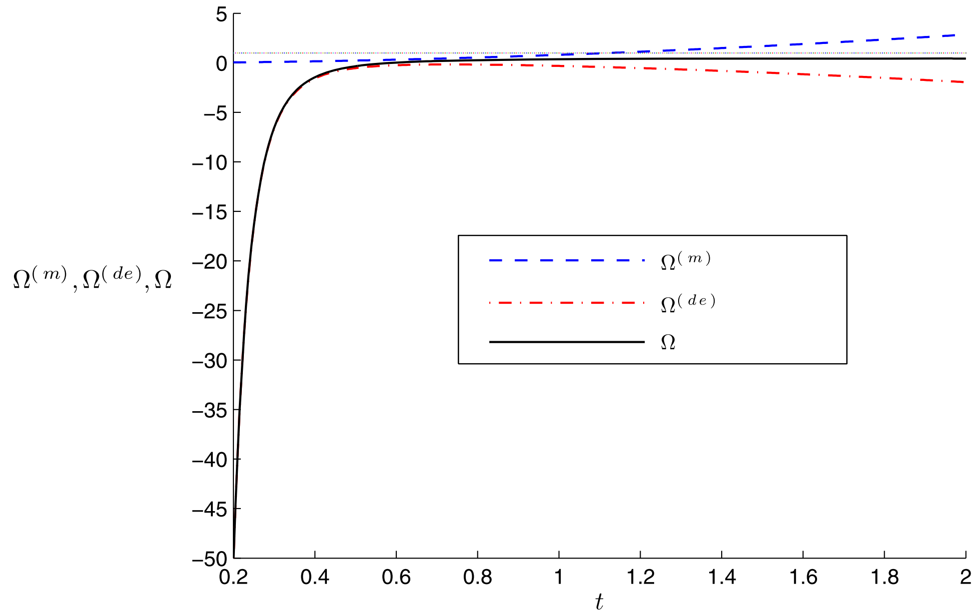



**Fig. 4** Plot of density parameters versus time ($t$)

From (55), it is observed that for $0 < n < 1$ and $\omega > -1$, the overall density parameter ($\Omega$) approaches to 1 for sufficiently large times. Thus, the derived model predicts a flat Universe in quintessence model for sufficiently large times. Figure 4, depicts the variation of density parameters versus cosmic time during the evolution of Universe.

### 3.2 DE cosmology for $n = 0$

In this case, integration of (16) yields

$$a(t) = c_1 e^{Dt}, \tag{56}$$

where $c_1$ is a positive constant of integration.

The metric functions, therefore, read as

$$A(t) = c_1 \exp\left[Dt + \frac{2\alpha\rho_0}{9\omega D(\omega+1)c_1^{3(\omega+1)}} e^{-3D(\omega+1)t}\right], \tag{57}$$

$$B(t) = mc_1 \exp\left[Dt - \frac{l}{3Dc_1^3} e^{-3Dt} - \frac{\alpha\rho_0}{9\omega D(\omega+1)c_1^{3(\omega+1)}} e^{-3D(\omega+1)t}\right], \tag{58}$$

$$C(t) = m^{-1}c_1 \exp\left[Dt + \frac{l}{3Dc_1^3} e^{-3Dt} - \frac{\alpha\rho_0}{9\omega D(\omega+1)c_1^{3(\omega+1)}} e^{-3D(\omega+1)t}\right]. \tag{59}$$

provided $\omega \neq -1$. For $\omega = -1$, we have

$$A(t) = c_1 \exp\left[Dt + \frac{2\alpha\rho_0}{3} t\right], \tag{60}$$

$$B(t) = mc_1 \exp\left[Dt - \frac{l}{3Dc_1^3} e^{-3Dt} - \frac{\alpha\rho_0}{3} t\right], \tag{61}$$

$$C(t) = m^{-1}c_1 \exp\left[Dt + \frac{l}{3Dc_1^3} e^{-3Dt} - \frac{\alpha\rho_0}{3} t\right]. \tag{62}$$

The other cosmological parameters of the model have the following expressions:

$$H_x = D - \frac{2\alpha\rho_0}{3\omega c_1^{3(\omega+1)}} e^{-3D(\omega+1)t}, \tag{63}$$

$$H_y = D + \frac{\alpha\rho_0}{3\omega c_1^{3(\omega+1)}} e^{-3D(\omega+1)t}, \tag{64}$$

$$H_z = D + \frac{\alpha\rho_0}{3\omega c_1^{3(\omega+1)}} e^{-3D(\omega+1)t}, \tag{65}$$

$$H = D, \tag{66}$$

$$\bar{A} = \frac{2\alpha^2{\rho_0}^2}{3D^2\omega^2 c_1^{6(\omega+1)}} e^{-6D(\omega+1)t}, \tag{67}$$

$$\theta = 3D, \tag{68}$$

$$V = c_1^3 e^{3Dt}, \tag{69}$$

$$\sigma^2 = \frac{\alpha^2{\rho_0}^2}{3\omega^2 c_1^{6(\omega+1)}} e^{-6D(\omega+1)t}. \tag{70}$$

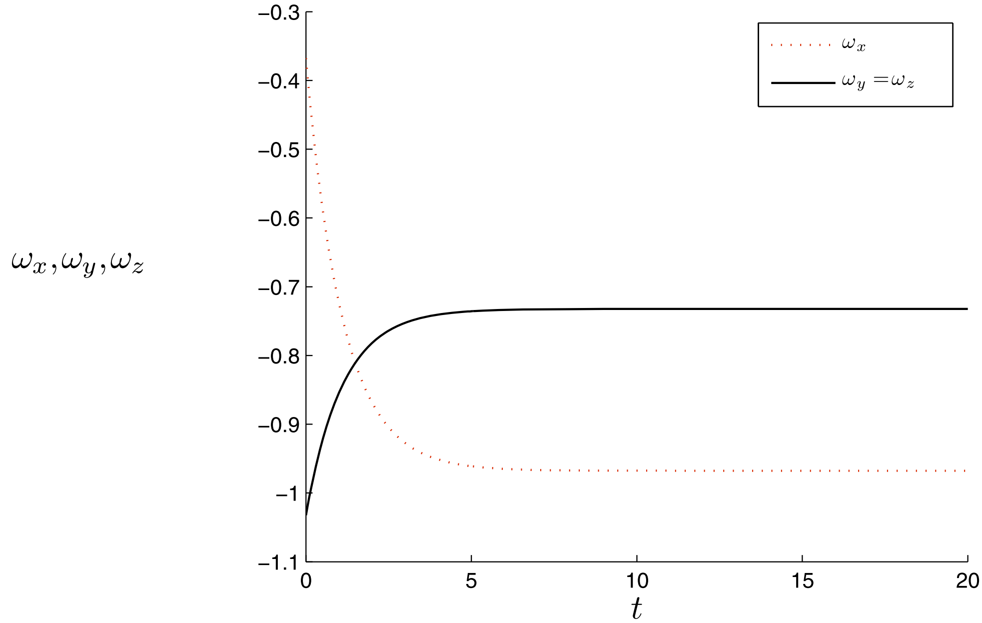



**Fig. 5** Plot of directional EoS parameters versus time ($t$)

The skewness parameters and the directional EoS parameters of DE, respectively, are given by

$$\delta(t) = \frac{\alpha c_1^{3(\omega+1)}}{\rho_0}\left[2De^{3D(\omega+1)t} + \frac{2\alpha\rho_0}{3\omega c_1^{3(\omega+1)}}\right], \tag{71}$$

$$\gamma(t) = \eta(t) = -\frac{\alpha c_1^{3(\omega+1)}}{\rho_0}\left[De^{3D(\omega+1)t} - \frac{2\alpha\rho_0}{3\omega c_1^{3(\omega+1)}}\right], \tag{72}$$

$$\omega_x = \omega + \frac{\alpha c_1^{3(\omega+1)}}{\rho_0}\left[2De^{3D(\omega+1)t} + \frac{2\alpha\rho_0}{3\omega c_1^{3(\omega+1)}}\right], \tag{73}$$

$$\omega_y = \omega_z = \omega - \frac{\alpha c_1^{3(\omega+1)}}{\rho_0}\left[De^{3D(\omega+1)t} - \frac{2\alpha\rho_0}{3\omega c_1^{3(\omega+1)}}\right], \tag{74}$$

The energy density and pressure of the DE components are given by

$$\rho^{(de)} = \rho_0 c_1^{-3(\omega+1)} e^{-3D(\omega+1)t}, \tag{75}$$

$$p^{(de)} = \omega\rho_0 c_1^{-3(\omega+1)} e^{-3D(\omega+1)t}. \tag{76}$$

From (14) and (15), the pressure and energy density of the perfect fluid components are obtained as

$$p^{(m)} = -3D^2 - \frac{l^2}{c_1^6}e^{-6Dt} - \frac{(2\omega+1)\alpha^2\rho_0{}^2}{3\omega^2 c_1^{6(\omega+1)}}e^{-6D(\omega+1)t} - \frac{\omega\rho_0}{c_1^{3(\omega+1)}}e^{-3D(\omega+1)t}, \tag{77}$$

$$\rho^{(m)} = 3D^2 - \frac{l^2}{c_1^6}e^{-6Dt} - \frac{\alpha^2\rho_0{}^2}{3\omega^2 c_1^{6(\omega+1)}}e^{-6D(\omega+1)t} - \frac{\rho_0}{c_1^{3(\omega+1)}}e^{-3D(\omega+1)t}. \tag{78}$$

The perfect fluid density parameter ($\Omega^{(m)}$) and DE density parameter ($\Omega^{(de)}$) are given by

$$\Omega^{(m)} = 1 - \frac{1}{3D^2}\left[\frac{l^2}{c_1^6}e^{-6Dt} + \frac{\alpha^2\rho_0{}^2}{3\omega^2 c_1^{6(\omega+1)}}e^{-6D(\omega+1)t} + \frac{\rho_0}{c_1^{3(\omega+1)}}e^{-3D(\omega+1)t}\right] \tag{79}$$

$$\Omega^{(de)} = \frac{\rho_0 c_1^{-3(\omega+1)}}{3D^2}e^{-3D(\omega+1)t} \tag{80}$$

Thus the overall density parameter ($\Omega$) is obtained as

$$\Omega = 1 - \frac{1}{3D^2}\left[\frac{l^2}{c_1^6}e^{-6Dt} + \frac{\alpha^2\rho_0{}^2}{3\omega^2 c_1^{6(\omega+1)}}e^{-6D(\omega+1)t}\right] \tag{81}$$

We observe that the model has no initial singularity. The directional scale factors and all other physical quantities are constants at $t = 0$. The directional scale factors and volume of the universe increase exponentially with the cosmic time whereas the mean Hubble parameter and expansion scalar are constants throughout the evolution. Therefore, uniform exponential expansion takes place. Further, we see that the DE term appears in exponential form in the scale factors and thus effects their evolution significantly. Thus, the spatial geometry of the universe is affected by the anisotropic DE. The difference between the directional EoS parameters of the DE and hence the pressures of the DE, along $x$-axis and $y$-axis (or $z$-axis) is $3\alpha D$, which is constant throughout the evolution of the universe. Therefore, the anisotropy of the DE does not vanish during the evolution of universe.

As $t \to \infty$, the scale factors and volume of the universe become infinitely large whereas the skewness and directional EoS parameters, directional Hubble parameters become constants. The pressure and energy density of the DE drops to zero in quintessence model (i.e. $\omega > -1$). Figure 6 shows the plot of anisotropy parameter ($\bar{A}$) versus time, in both quintessence model and phantom model. We see that in quintessence model, the anisotropy parameter decreases as time increases and finally drops to zero at late time. But in phantom model the anisotropy parameter does not vanish during the evolution of Universe. The left hand side of energy conditions have graphed in Figs. 7 and 8 for quintessence model and phantom model respectively. It is observed that the quintessence model violates strong energy condition whereas phantom model violate weak energy condition as well as strong energy condition, as expected.

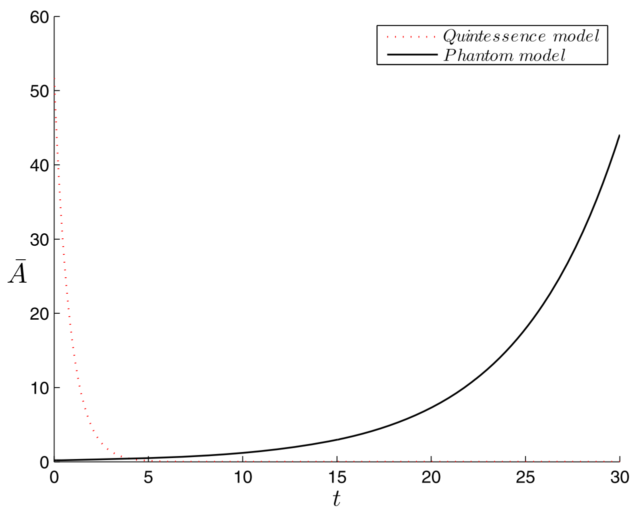



**Fig. 6** Plot of Anisotropic parameter ($\bar{A}$) versus time ($t$) for $n = 0$

For $n = 0$, we get $q = -1$; incidentally this value of deceleration parameter leads to $\frac{dH}{dt} = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the universe. Therefore, the derived model can be utilized to describe the dynamics of the late time evolution of the actual Universe. So, in what follows, we emphasize upon the late time behaviour of the derived model. Figure 5 depicts the variation of directional EoS parameter versus time for $n = 0$. We observe that $\omega_x, \omega_y$ or $\omega_z$ are evolving with negative sign, as expected.

From (81), it is observed that for sufficiently large time, the overall density parameter ($\Omega$) approaches to 1 in quintessence model. Thus the derived model predicts a flat Universe at late time.

## 4 Concluding remarks

In this paper, some spatially homogeneous and anisotropic DE models in Bianchi type V space-time have been studied. The main features of the work are as follows

**Fig. 7** Single plot of energy conditions in quintessence model for $n = 0$

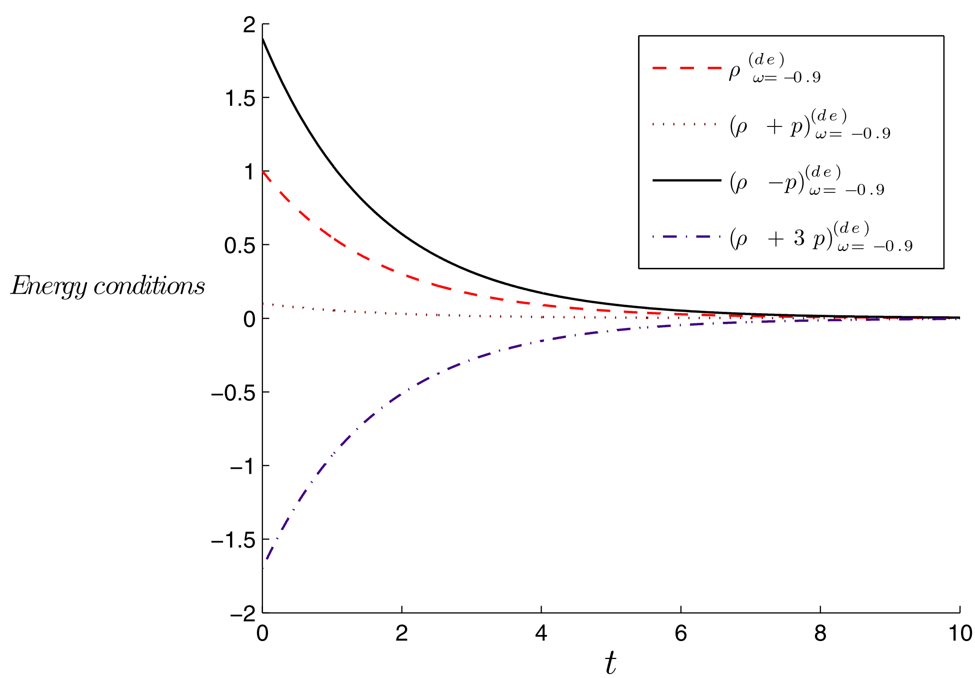
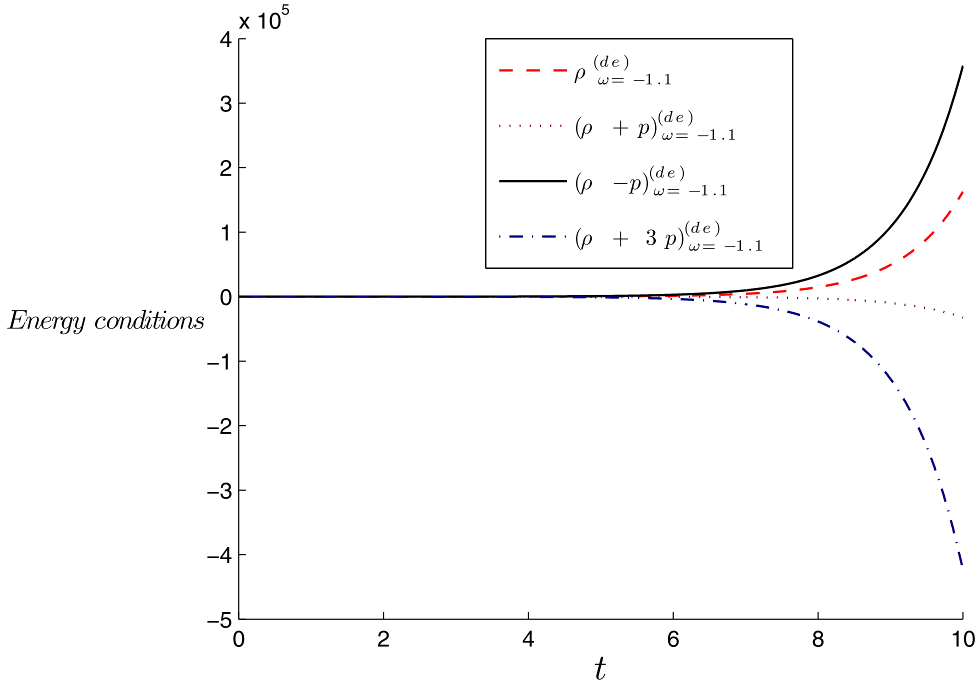


**Fig. 8** Single plot of energy conditions in phantom model for $n = 0$




- The models are based on the exact solution of Einstein's field equations for anisotropic Bianchi type V space-time filled with DE.
- The singular model ($n \neq 0$) seems to describe the dynamics of the Universe from big bang to the present epoch while non singular model ($n = 0$) seems reasonable to project dynamics of the future Universe.
- The directional EoS parameters (i.e. $\omega_x$, $\omega_y$ or $\omega_z$) evolve with in the range predicted by observations.
- In the present models, we do not rule out the anisotropic nature of DE. The anisotropic DE contributes to the expansion of one (or two) of the scale factors while it opposes the expansion of the other two (or one) scale factors leading the geometry of Universe. The anisotropy of DE vanishes at late time for singular model (see Fig. 1) whereas for non singular model, anisotropy occurs at early stage (i.e. quintessence model) or at later time of Universe (i.e. phantom model) (see Fig. 6).
- The derived quintessence models violate the strong energy condition whereas the phantom models violate the weak energy condition as well as strong energy condition (see Figs. 3, 7, 8).
- The flatness of Universe can be achieved in quintessence model (i.e. $\omega > -1$) for sufficiently large time because the overall density parameter ($\Omega$) approaches to 1. Thus in our analysis, the quintessence model is turning out as a suitable model for describing the late time evolution of Universe.

**Acknowledgements** Author would like to thanks the anonymous learned referee for his/her valuable comments which improved the paper in this form. Also Author thanks to S. Kumar for helpful discussions.

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