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ORIGINAL ARTICLE

Zakharov-Kuznetsov-Burgers equation in superthermal electron-positron-ion plasma

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Abstract Properties of three-dimensional ion-acoustic solitary and shock waves accompining electron-positron-ion magnetoplasma with high-energy (superthermal) electrons and positrons are investigated. For this purpose, a Zakharov-Kuznetsov-Burgers (ZKB) equation is derived from the ion continuity equation, ion momentum equation with kinematic viscosity among ions fluid, electrons, and positrons having kappa distribution together with the Poisson equation. The dependence of the solitary and shock excitations characteristics on the parameter measuring the superthermality κ, the ion gyrofrequency Ω, the unperturbed positrons-to-ions density ratio ν, the viscosity parameter η, the direction cosine ℓ, the ion-to-electron temperature ratio σ_i, and the electron-to-positron temperature ratio σ_p have been investigated. Moreover, it is found that the parameters κ, Ω, ν, η, and ℓ lead to accelerate the particles, whereas the parameters σ_i and σ_p would lead to decelerate them. Numerical calculations reveal that the nonlinear pulses polarity are always positive. This study could be useful to understand the nonlinear electrostatic excitations in interstellar medium.

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Keywords Ion-acoustic waves · Superthermal electrons and positrons · Zakharov-Kuznetsov-Burgers equation · Solitary and shock waves

1 Introduction

In the last decades, numerous observations have provided consistent data and confirmed that there are deviations from Maxwellian equilibrium are expected to produce in space plasmas, which are sufficiently dilute and low collisional (Lazar et al. 2010a, 2010b). For example, observations and in-situ measurements have confirmed the wide-spread existence of superthermal populations at different altitudes in the solar wind plasma (Feldman et al. 1975; Pilipp et al. 1987; Fisk and Gloeckler 2006) and probably in the solar corona (Scudder 1992a, 1992b; Pierrard et al. 1999), Earth's magnetospheric plasma sheet (Christon et al. 1988), Jupiter (Leubner and Geophys 1982), Saturn (Armstrong et al. 1983), etc. One of these non-Maxwellian distributions is the superthermal (kappa) distribution. The latter assumes that the superthermal particles may arise due to the effect of external forces acting on the natural space environment plasmas or due to wave particle interaction. Plasma with an excess of superthermal particles are generally characterized by long tail in the high energy region. It has been noticed that a plasma in the presence of superthermal particles suffers velocity-space diffusion (Hasegawa et al. 1985). Importantly, the kappa (κ) distribution obeys an inverse power law at high velocities. For all velocities, in the limit when the spectral index κ becomes very large ($\kappa \to \infty$), the distribution function approaches the Maxwellian one. The κ distribution was first suggested by Vasyliunas (1968) to model space plasmas. The advantage of employing κ distribution lies in the fact that the Maxwellian distribution is a special case of the κ function in the limit of $\kappa \to \infty$. Based on superthermal assumption, many authors have been studied the effect of superthermal particles (electrons, positrons, and ions) in different types of plasma environments (e.g., Hellberg and Mace 2002; Leubner and Schupfer 2002; Leubner 2004; Abbasi and Pajouh 2007; Saini and Kourakis 2008; Chuang and Hau 2009; Tribeche and Boubakour 2009; El-Awady et al. 2010; Sultana et al. 2010).

An electron-positron-ion (e-p-i) plasma is a fully ionized gas consisting of electrons and positrons having equal masses and charges with opposite polarity. The e-p-i plasmas are found not only in early universe (Rees et al. 1983) but also in astrophysical environments such as in magnetosphere of pulsars (Michel 1982), active galactic nuclei (Miller and Witta 1987), interstellar medium (Moskalenko and Strong 1998; Adriani et al. 2009), etc. Also, e-p-i plasma can be artificially created in laboratories (Surko et al. 1986; Surko and Murphy 1990; Tinkle et al. 1994; Greaves and Surko 1995; Gahn et al. 2000). Indeed, the presence of ions leads to existence of several low frequency waves which otherwise do not propagate in electron-positron plasmas. One of these low frequency waves is the nonlinear ion-acoustic wave. The nonlinearity could produce various structures (Hirota 2004; Wazwaz 2009), such as solitons, shocks, peakons, cuspons, etc. However, we will pay attention to investigate both the solitons and shocks. Briefly, the solitary waves appear because of the balance between the dispersion (caused by charge separation) and the nonlinearity (because of convection of mobile particles). However, the shocks which are sometimes called kinks or double layers are monotonically change in the physical parameter from one value at one extreme to another at the other end, hence "kinks". This is associated with adjacent positive and negative charge regions, which give rise to the name "double layers". Such double layers are more difficult to generate and require a fine tuning of the plasma parameters, hence a more complicated plasma composition with enough leeway to obey the necessary constraints (Verheest et al. 2006). During the last decades, there has been increasing interest in interpreting low-frequency nonlinear ion-acoustic structures (solitons and shocks) with non Maxwellian particles both in electron-ion plasma and in e-p-i plasmas (e.g. Vasyliunas 1968; Hasegawa et al. 1985; Hellberg et al. 2000, 2005; Aoutou et al. 2008; Baluku and Hellberg 2008; El-Awady et al. 2010; Lazar et al. 2010a, 2010b; Sultana et al. 2010). To the best of our knowledge, most of these studies did not consider the propagation of ion-acoustic structures (solitons and shocks) in three component e-p-i magnetoplasma and taking into account the effects of superthermal particles (both electrons and positrons), as well as ion kinematic viscosity. Therefore, the goal of the present work is to tackle extension of the nonlinear ion-acoustic structures (solitons and shocks) in magnetoplasma composed of three distinct particle populations, namely inertial ions, as well as electrons and positrons obeying a kappa distribution taking into account the kinematic viscosity of the ion fluid.

The organization of the paper is as follows: The basic equations for the nonlinear electrostatic excitations in an e-p-i magnetoplasma are presented in Sect. 2. In Sect. 3 the reductive perturbation method is used to reduce the basic equations to Zakharov-Kuznetsov-Burgers (ZKB) equation. In Sect. 4, the ZKB equation is solved analytically to obtain both solitary and shock solutions. The latter are used to study the behavior of these nonlinear structures. The results are summarized in Sect. 5.

2 Model equations

Let us consider a three-dimensional, collisionless, magnetized e-p-i plasma consisting of inertialess superthermal electrons and positrons, as well as inertial single-charge adiabatic positive ions. The nonlinear dynamics of the ion-acoustic waves in such e-p-i plasma is described by the following normalized equations

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i \cdot \nabla)\mathbf{u}_i = -\nabla\varphi + \Omega(\mathbf{u}_i \times \hat{x}\,) - \frac{5}{3}\sigma_i n_i^{-1/3}\nabla n_i + \eta_i \nabla^2 \mathbf{u}_i, \tag{2}$$

$$\nabla^2\varphi = n_e - n_i - n_p, \tag{3}$$

where n_j is the density of jth species (where $j = e$, p, and i stands for electrons, positrons, and ions, respectively), $\mathbf{u}_i$ is the ion fluid velocity, φ is an electrostatic potential, and η_i is the normalized kinematic viscosity. We shall adopt kappa-distribution for electrons and positrons, by relying on a similar notations in Chuang and Hau (2009), wherein the fundamental algebra is expressed in detail. Therefore, the normalized number densities of electrons and positrons are given by

$$n_e = \mu\left(1 - \frac{\varphi}{\kappa}\right)^{-\kappa+\frac{1}{2}}, \tag{4}$$

$$n_p = \nu\left(1 + \sigma_p\frac{\varphi}{\kappa}\right)^{-\kappa+\frac{1}{2}}, \tag{5}$$

where κ is a real parameter measuring deviation from Maxwellian equilibrium. In the limit $\kappa \to \infty$, superthermal distribution reduces to Maxwell-Boltzmann distribution. At equilibrium, we have

$$\mu - \nu = 1, \tag{6}$$

where $\mu = n_e^{(0)}/n_i^{(0)}$ and $\nu = n_p^{(0)}/n_i^{(0)}$ denote the unperturbed density ratios of electrons and positrons-to-ions, respectively. Furthermore, we introduce the following notations:

$$\Omega = \frac{\omega_{ci}}{\sqrt{4\pi n_i^{(0)} e^2/m_i}}, \quad \omega_{ci} = eB_0/m_i c,$$

$$\sigma_i = T_i\,/T_e, \qquad \sigma_p = T_e\,/T_p, \quad \text{and} \quad \eta_i = \frac{\eta_0}{\omega_{pi}\,\lambda_{Di}^2},$$

where ω_{ci} is the ion gyrofrequency, σ_i is the ratio of ion-to-electron temperature, and σ_p is the ratio of electron-to-positron temperature. Here, e is the magnitude of the electron charge, B_0 is an external static magnetic field, m_i is the ion mass, c is the velocity of the light in vacuum, and η_0 is the unnormalized kinematic viscosity.

The variables appearing in (1)–(5) have been appropriately normalized. Thus $n_{i,e,p}$ is normalized by the unperturbed ion density $n_i^{(0)}$, $\mathbf{u}_i$ by the ion-acoustic speed $C_{si} = (k_B T_e/m_i)^{1/2}$, φ by the electrostatic potential $k_B T_e/e$, the space and time variables are in units of the ion Debye radius $\lambda_{Di} = (k_B T_e/4\pi n_i^{(0)} e^2)^{1/2}$ and the ion plasma period $\omega_{pi}^{-1} = (4\pi n_i^{(0)} e^2/m_i)^{-1/2}$, respectively. k_B is the Boltzmann constant.

3 Reduction to Zakharov-Kuznetsov-Burgers equation

To study the nonlinear ion-acoustic waves of small, but finite, amplitude, we use reductive perturbation method (Washimi and Taniuti 1966), which leads to a scaling of the independent variables through the stretched coordinates

$$X = \varepsilon^{1/2}(x - \lambda t), \qquad Y = \varepsilon^{1/2} y, \qquad Z = \varepsilon^{1/2} z, \quad \text{and} \quad T = \varepsilon^{3/2} t, \tag{7}$$

where λ is a wave propagation speed to be determined later and ε is a small parameter measuring the weakness of the dispersion and nonlinearity. Furthermore, the variables n_i, $\mathbf{u}_i$, and φ are expanded as

$$n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \cdots, \tag{8}$$

$$u_{ix} = \varepsilon u_{ix}^{(1)} + \varepsilon^2 u_{ix}^{(2)} + \cdots, \tag{9}$$

$$u_{iy} = \varepsilon^{3/2} u_{iy}^{(1)} + \varepsilon^2 u_{iy}^{(2)} + \cdots, \tag{10}$$

$$u_{iz} = \varepsilon^{3/2} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + \cdots, \tag{11}$$

$$\varphi = \varepsilon \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \cdots. \tag{12}$$

The value of η_i is assumed to be small, so that we may set $\eta_i = \varepsilon^{1/2}\eta$ where η is finite quantity of the order of unity.

Applying Binomial series of (4) and (5), and substituting the expansion (8)–(12), as well as the stretching (7) into (1)–(5), then collecting the terms in different powers of ε. The lowest-order in ε gives the following relations

$$n_i^{(1)} = \frac{1}{\Delta}\varphi^{(1)}, \tag{13}$$

$$u_{ix}^{(1)} = \frac{\lambda}{\Delta}\varphi^{(1)}, \tag{14}$$

$$u_{iy}^{(1)} = -\frac{\lambda^2}{\Omega\Delta}\frac{\partial \varphi^{(1)}}{\partial Z}, \tag{15}$$

$$u_{iz}^{(1)} = \frac{\lambda^2}{\Omega\Delta}\frac{\partial \varphi^{(1)}}{\partial Y}, \tag{16}$$

where the Poisson equation gives the compatibility condition

$$\frac{1}{\Delta} - \left(1 - \frac{1}{2\kappa}\right)(\mu + \nu\sigma_p) = 0, \tag{17}$$

with

$$\Delta = \lambda^2 - \frac{5}{3}\,\sigma_i. \tag{18}$$

The next-order in ε yields a system of equations in the second-order perturbed quantities. Solving this system with the aid of (13)–(18), we finally obtain the Zakharov-Kuznetsov-Burgers (ZKB) equation

$$\frac{\partial \varphi^{(1)}}{\partial T} + A\varphi^{(1)}\frac{\partial \varphi^{(1)}}{\partial X} + B\frac{\partial^3 \varphi^{(1)}}{\partial X^3} + C\frac{\partial}{\partial X}\left(\frac{\partial^2 \varphi^{(1)}}{\partial Y^2} + \frac{\partial^2 \varphi^{(1)}}{\partial Z^2}\right) - D\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}\right)\varphi^{(1)} = 0, \tag{19}$$

where

$$A = B\left\{3\frac{(\lambda^2 - \frac{5}{27}\sigma_i)}{\Delta^3} - \left(1 - \frac{1}{4\kappa^2}\right)(\mu - \nu\sigma_p^2)\right\}, \tag{20}$$

$$B = \tfrac{1}{2}\left(\frac{\lambda}{\Delta^2}\right)^{-1}, \qquad C = B\left(1 + \frac{\lambda^4}{\Omega^2\Delta^2}\right), \quad \text{and} \quad D = \frac{\eta}{2}. \tag{21}$$

It is worth mentioning here that the viscosity parameter η is contained only in the last term of (19). It is noticed that for $\eta \to 0$, the dissipative term (i.e. last term in (19)) disappears, yielding the formation of solitary pulses only, (due to the balance between the nonlinearity and dispersion). However, in our model the dissipative term is presented, which leads to lose the system energy, and thus leads to the formation of a shock. Thus, we pointout that the dissipative coefficient plays an important role to change the profile of the nonlinear structure, i.e. soliton to shock and vis versa, so the soliton existence is expected to be strongly affected by the viscosity term.

4 Solutions of ZKB equation and parametric study

To obtain the solution of (19), we introduce the transformation

$$\chi = \ell X + mY + nZ - UT, \tag{22}$$

where ℓ, m, and n are the direction cosines between the wave propagation vector k with the X, Y, and Z axes, respectively. U is the velocity of the moving frame normalized by C_{si}. Considering $\Phi(\chi) = \varphi^{(1)}(X, Y, Z, T)$, (19) takes the form of an ordinary differential equation as follows:

$$-U\frac{d\Phi}{d\chi} + A\ell\Phi\frac{d\Phi}{d\chi} + \ell F\frac{d^3\Phi}{d\chi^3} - D\frac{d^2\Phi}{d\chi^2} = 0, \tag{23}$$

where

$$F = B\ell^2 + C(m^2 + n^2).$$

If we emphasize on a limiting case when the dispersion is dominant cases (i.e., $D \to 0$), then (23) reduces to the well-known ZK equation. Integrating the latter and using the boundary conditions $\Phi \to 0$, $d\Phi/d\chi, d^2\Phi/d\chi^2 \to 0$ at $\chi \to \pm\infty$, we obtain the solitary wave solution

$$\Phi = \Phi_0 \operatorname{sech}^2(\chi/\delta), \tag{24}$$

where the maximum amplitude Φ_0 and the width δ of the solitary waves are given, respectively, by

$$\Phi_0 = 3U/A\ell, \tag{25}$$

and

$$\delta = 2(\ell F/U)^{1/2}. \tag{26}$$

Firstly, we will investigate the nature of the solitary structure (represented by (24)). We have numerically analyzed the potential amplitude Φ_0 and width δ and investigate how the ratio of ion-to-electron temperature σ_i, the parameter measuring deviation from Maxwellian equilibrium κ, the unperturbed density ratio of positrons-to-ions ν, the direction cosine ℓ, and the ion gyrofrequency Ω change the profile of the pulse structure.

From Figs. 1 and 2, it is seen that the increase of the parameter measuring the deviation from Maxwellian equilibrium κ, the ratio of ion-to-electron temperature σ_i, the direction cosine ℓ, the unperturbed density ratio of positrons-to-ions ν, leads to make the amplitude shorter. However, for $\ell < 0.4$ the decrease in the amplitude is much smaller than

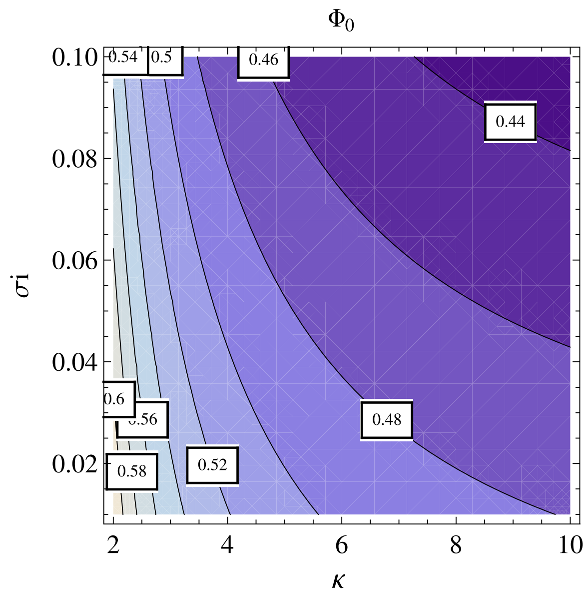

Fig. 1 The contour plot of the amplitude Φ_0 (given by (25)) with parameter measuring deviation from Maxwellian equilibrium κ and the ion-to-electron temperature ratio σ. The *numbers* on the contour curves indicate the values of the corresponding amplitude, where $\nu = 0.2$, $\ell = 0.9$, $U = 0.2$, and $\sigma_p = 0.9$

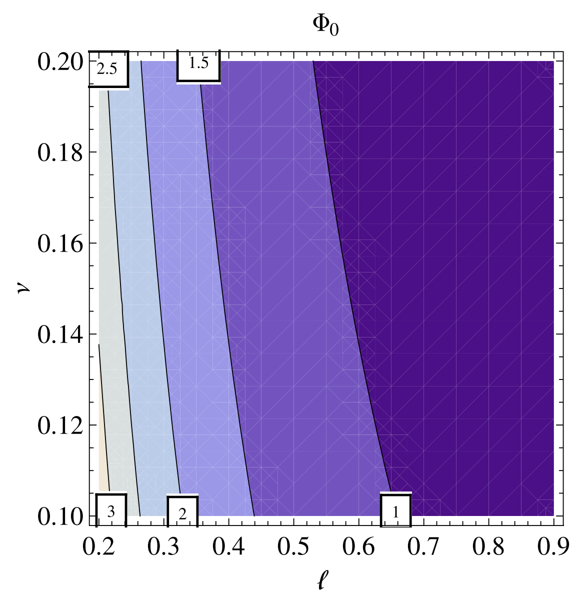

Fig. 2 The contour plot of the amplitude Φ_0 (given by (25)) with direction cosine ℓ and unperturbed density ratio of positrons-to-ions ν. The *numbers* on the contour curves indicate the values of the corresponding amplitude, where $\kappa = 2$, $\sigma_i = 0.05$, $U = 0.2$, and $\sigma_p = 0.9$

for $\ell > 0.4$. That is indicate that for large direction cosine ℓ, the pulse amplitude does not suffer significant change. The behavior of electron-to-positron temperature ratio σ_p is the same as the behavior of ion-to-electron temperature ratio σ_i, therefore we do not include it in the figures. The dependence of the spatial extension (width) δ on the parameter measur-

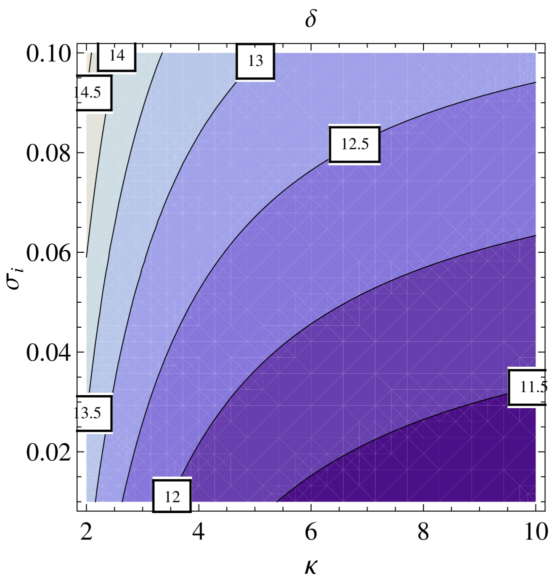

Fig. 3 The contour plot of the width δ (given by (26)) with parameter measuring deviation from Maxwellian equilibrium κ and the ion-to-electron temperature ratio σ_i. The *numbers* on the contour curves indicate the values of the corresponding width, where $\nu = 0.2$, $\ell = 0.9$, $\Omega = 0.1$, $U = 0.2$, and $\sigma_p = 0.9$

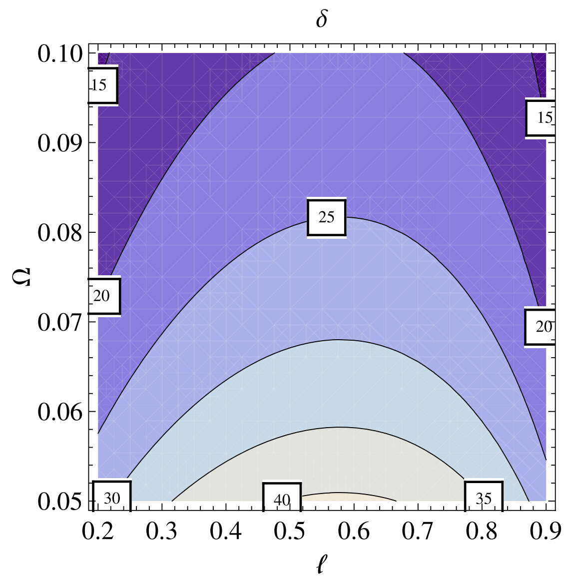

Fig. 5 The contour plot of the width δ (given by (26)) with direction cosine ℓ and ion gyrofrequency Ω. The *numbers* on the contour curves indicate the values of the corresponding width, where $\nu = 0.2$, $\kappa = 2$, $\sigma_i = 0.05$, $U = 0.2$, and $\sigma_p = 0.9$

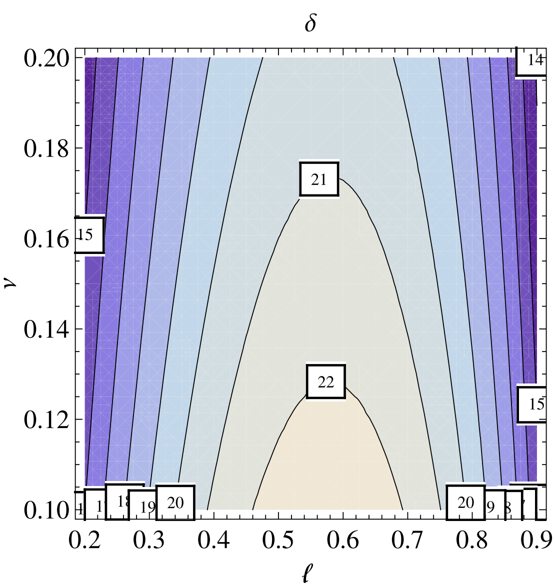

Fig. 4 The contour plot of the width δ (given by (26)) with direction cosine ℓ and unperturbed density ratio of positrons-to-ions ν. The *numbers* on the contour curves indicate the values of the corresponding width, where $\kappa = 2$, $\sigma_i = 0.05$, $\Omega = 0.1$, $U = 0.2$, and $\sigma_p = 0.9$

ing deviation from Maxwellian equilibrium κ, the ratio of ion-to-electron temperature σ_i, the direction cosine ℓ, the unperturbed density ratio of positrons-to-ions ν, and the ion gyrofrequency Ω are displayed in Figs. 3–5. In Fig. 3 it is noticed that the width decreases with increasing κ but it increases with the increase of σ_i. However, for small κ (i.e. $\kappa = 2$–6) the width changes with σ_i faster than for large κ (i.e. $\kappa > 6$). That is indicate that the index κ plays an important role in the pulse profile. For certain value of ℓ the width changes its profile/behavior; see Figs. 4 and 5. On the other hand, the pulse width becomes wider for $\ell < 0.58$ whereas it becomes narrower for $\ell > 0.58$. Furthermore, the increase of the unperturbed density ratio of positrons-to-ions ν, and the ion gyrofrequency Ω would lead to shrinks the solitary pulse width.

Now, we will obtain the general/exact solution of (23) involving both the dissipative and dispersion terms. We will employ the hyperbolic tangent (tanh) method (Malfliet 1992; Malfliet and Hereman 1996), which is a tool for finding the travelling wave solutions of peculiar type nonlinear evolution equations. Introducing new independent variable $W = \tanh(\rho\chi)$ to (23), we get

$$\begin{aligned} &-U\rho(1-W^2)\frac{d\Phi}{dW} + A\ell\rho(1-W^2)\Phi\frac{d\Phi}{dW} \\ &\quad + F\ell\rho^3(1-W^2) \\ &\quad \times \frac{d}{dW}\left\{(1-W^2)\frac{d}{dW}\left[(1-W^2)\frac{d}{dW}\right]\right\}\Phi \\ &\quad - D\rho^2(1-W^2)\frac{d}{dW}\left[(1-W^2)\frac{d}{dW}\right]\Phi = 0. \end{aligned} \tag{27}$$

Assume the solution is as a series in the form

$$\Phi(W) = a_0 + a_1 W + a_2 W^2. \tag{28}$$

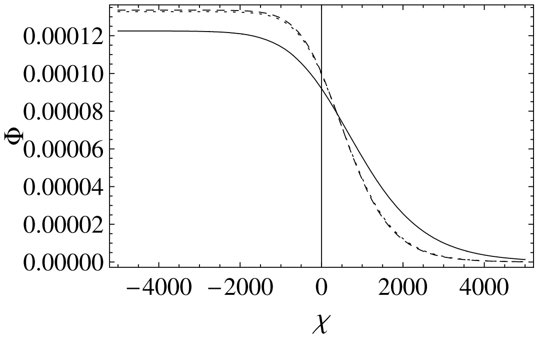

Fig. 6 The shock profiles Φ (given by (30)) against χ for different values of κ where $\kappa = 2$ (*solid line*), $\kappa = 20$ (*dotted line*), $\kappa = 50$ (*dashed line*), $\nu = 0.2$, $\ell = 0.9$, $\Omega = 0.1$, $\sigma_i = 0.05$, $\sigma_p = 0.9$, and $\eta = 0.1$

Using (28) into (27), we obtain a system of algebraic equations. Solving this system, we can finally obtain

$$a_0 = \frac{9}{25}\frac{D^2}{FA\ell^2}, \qquad a_1 = \mp\frac{6}{25}\frac{D^2}{FA\ell^2},$$
$$a_2 = -\frac{3}{25}\frac{D^2}{FA\ell^2}, \qquad \rho = \pm\frac{D}{10F\ell}, \qquad U = \frac{6D^2}{25F\ell}. \tag{29}$$

Therefore, (28) can be rewritten as

$$\Phi(\chi) = \frac{3}{25}\frac{D^2}{FA\ell^2}\left[2 - 2\tanh\left(\frac{D}{10F\ell}\chi\right) + \operatorname{sech}^2\left(\frac{D}{10F\ell}\chi\right)\right]. \tag{30}$$

Notice that the exact solution of the ZKB equation (23) contains the contribution from both dispersion and dissipative effects which influence the eventual shape of the wave potential. It may be appropriate to point out that the above solution has been obtained in the region of parameter values where the nonlinearity, dispersion, and dissipative coefficients in the ZKB equation (23) bear either positive or negative values. One sees in the solution (30) that the amplitude coefficient (i.e., where $\chi \to 0$) has ever positive value, independent on the intrinsic plasma parameters values. Therefore, the shock pulse polarity is always positive in the present system. Also, we have examined if $A \to 0$ for possible plasma parameters values. It is found that A cannot be either vanish or has negative sign. Therefore, the solution obtained above cover the entire range of plasma parameters.

The profile of the double layers (shock waves) is depicted against various plasma parameters, namely the parameter measuring deviation from Maxwellian equilibrium κ, the ion gyrofrequency Ω, the unperturbed density ratio of positrons-to-ions ν, the viscosity parameter η, the direction cosine ℓ, the ratio of ion-to-electron temperature σ_i, and the ratio of electron-to-positron temperature σ_p. It is obvious from Figs. 6, 7, 8, 9, 10 that the shock amplitude (width) increases (decreases) with the increase of κ, Ω, ν, η, and

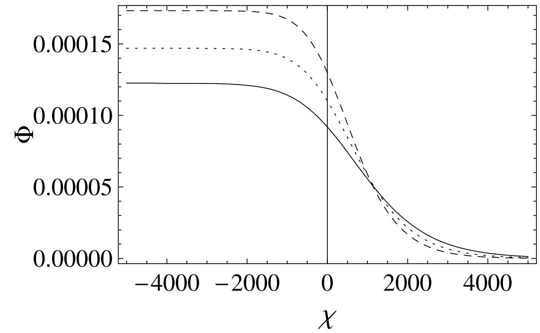

Fig. 7 The shock profiles Φ (given by (30)) against χ for different values of Ω where $\Omega = 0.1$ (*solid line*), $\Omega = 0.11$ (*dotted line*), $\Omega = 0.12$ (*dashed line*), $\kappa = 2$, $\nu = 0.2$, $\ell = 0.9$, $\sigma_i = 0.05$, $\sigma_p = 0.9$, and $\eta = 0.1$

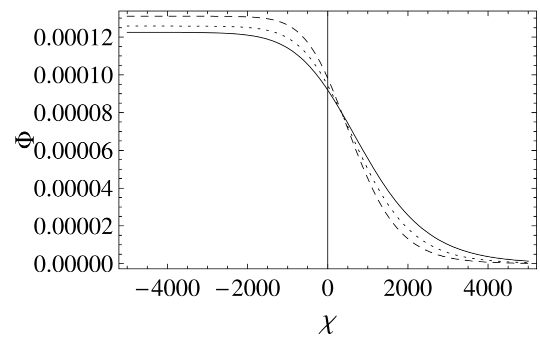

Fig. 8 The shock profiles Φ (given by (30)) against χ for different values of ν where $\nu = 0.2$ (*solid line*), $\nu = 0.3$ (*dotted line*), $\nu = 0.4$ (*dashed line*), $\kappa = 2$, $\ell = 0.9$, $\Omega = 0.1$, $\sigma_i = 0.05$, $\sigma_p = 0.9$, and $\eta = 0.1$

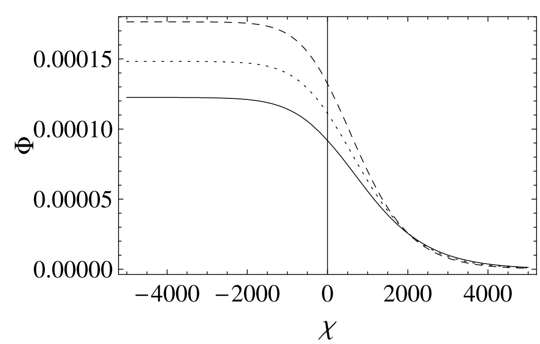

Fig. 9 The shock profiles Φ (given by (30)) against χ for different values of η where $\eta = 0.1$ (*solid line*), $\eta = 0.11$ (*dotted line*), $\eta = 0.12$ (*dashed line*), $\kappa = 2$, $\nu = 0.2$, $\ell = 0.9$, $\Omega = 0.1$, $\sigma_i = 0.05$, and $\sigma_p = 0.9$

ℓ. Physically, increasing the double layers amplitude means that the potential drop across the double layer enhances, and then more particles will be accelerated. Therefore, the physical parameters κ, Ω, ν, η, and ℓ lead to much potential drop and accelerate electrons and positrons/ions in opposite directions. The effects of σ_i and σ_p on the double

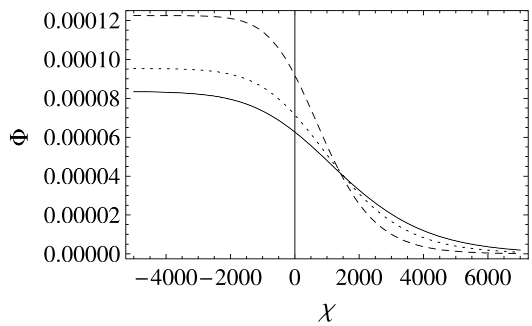

Fig. 10 The shock profiles Φ (given by (30)) against χ for different values of ℓ where $\ell = 0.8$ (*solid line*), $\ell = 0.85$ (*dotted line*), $\ell = 0.9$ (*dashed line*), $\kappa = 2$, $\nu = 0.2$, $\Omega = 0.1$, $\sigma_i = 0.05$, $\sigma_p = 0.9$, and $\eta = 0.1$

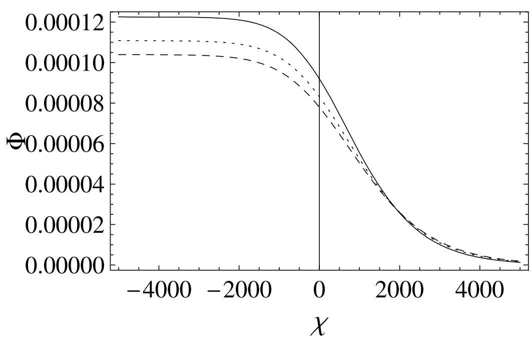

Fig. 11 The shock profiles Φ (given by (30)) against χ for different values of σ_i where $\sigma_i = 0.05$ (*solid line*), $\sigma_i = 0.08$ (*dotted line*), $\sigma_i = 0.1$ (*dashed line*), $\kappa = 2$, $\nu = 0.2$, $\ell = 0.9$, $\Omega = 0.1$, $\sigma_p = 0.9$, and $\eta = 0.1$

layer amplitude and width are displayed in Figs. 11 and 12. It is seen that the wave amplitude (width) shrinks (enhances) with the increase of σ_i, whereas the wave amplitude and width shrinks with the increase of σ_p. However, increasing ν (σ_p) leads to smaller (larger) and narrower (wider) excitations. Importantly, the parameters σ_i and σ_p would lead to decelerate the electrons and positrons/ions. Finally, in the limit $\lambda \to \sqrt{5\sigma_i/3}$ (fixed temperature ratio), no double layer excitations are expected to occur.

5 Summary

To summarize, we have investigated the propagation of three-dimensional ion-acoustic solitary and shock waves in e-p-i magnetoplasma with superthermal electrons and positrons (represented by kappa distribution). The dissipation in the system is introduced by taking into account the kinematic viscosity among the ions fluid. The evolution of the system is investigated by deriving ZKB equation. The analytical solutions of the latter are obtained, which indicate that either solitary or shock pulses can exist depending on the dissipation in the system. The dependence of the solitary

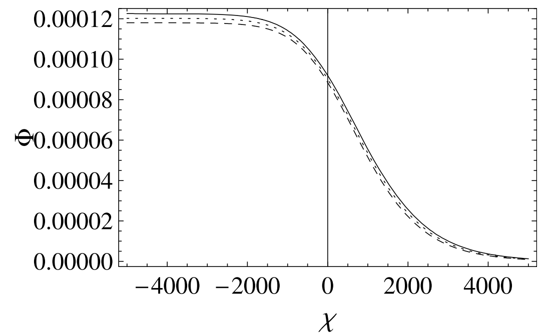

Fig. 12 The shock profiles Φ (given by (30)) against χ for different values of σ_p where $\sigma_p = 0.9$ (*solid line*), $\sigma_p = 1.05$ (*dotted line*), $\sigma_p = 1.2$ (*dashed line*), $\kappa = 2$, $\nu = 0.2$, $\ell = 0.9$, $\Omega = 0.1$, $\sigma_i = 0.05$, and $\eta = 0.1$

and shock excitations characteristics on the parameter measuring deviation from Maxwellian equilibrium κ, the ion gyrofrequency Ω, the unperturbed density ratio of positrons-to-ions ν, the viscosity parameter η, the direction cosine ℓ, the ratio of ion-to-electron temperature σ_i, and the ratio of electron-to-positron temperature σ_p have been investigated. Furthermore, the parameters κ, Ω, ν, η, and ℓ lead to accelerate the particles, whereas the parameters σ_i and σ_p would lead to decelerate them. Also, the numerical calculations reveal that the nonlinear pulses polarity are always positive.

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