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ORIGINAL ARTICLE

Extremization of mass of charged superdense star models describe by Durgapal type space-time metric

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Abstract In the present article, we have obtained a class of charged superdense star models, starting with a static spherically symmetric metric in curvature coordinates by considering Durgapal (J. Phys. A 15:2637, 1982) type metric i.e. $g_{44} = B(1 + Cr^2)^n$, where n being any positive integer. It is observed that the maximum mass of the charged fluid models is monotonically increasing with the increasing values of $n \leq 4$. For $n \geq 4$, the maximum mass of the charged fluid models is throughout monotonically decreasing and over all maximum mass is attained at $n = 4$. The present metric tends to another metric which describes the charged analogue of Kuchowicz neutral solution as $n \to \infty$. Consequently the lower limit of maximum mass of the charged fluid models could be determined and found to be 5.1165 solar mass with corresponding radius 18.0743 Km. While the upper limit of maximum mass of the model of this category is already known to be 5.7001 solar mass with corresponding radius 17.1003 Km for $n = 4$. The solutions so obtained are well behaved.

Keywords Canonical coordinates · Charged fluids · Superdense star · General relativity

1 Introduction

Relevance of the study of charged fluid distributions is based upon the facts:

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(1) Charged fluids are more capable of averting the gravitational collapse in comparison to their neutral counterpart.
(2) A non well behaved fluid model may be made well behaved after the due charging.
(3) A charged fluid which reduces to a flat space after the removable of charge may be useful to construct an electronic model.
(4) Charged dust models are also of great hope as regard the structure of electron is concerned.

In the present article we have obtain charged perfect fluid models describe by Durgapal (1982) type metric i.e. $g_{44} = B(1 + Cr^2)^n$ for every integral value of n (neutral analogue of such solutions are already published by Maurya and Gupta 2011a, 2011b). The charged fluid models for some integral value of n have already derived by Pant et al. (2010, 2011a) for $n = 3$, Pant (2011) for $n = 4$, Gupta and Maurya (2011) for $n = 5$, Maurya and Gupta (2011a, 2011b) for $n = 6$, and Pant et al. (2011b) for $n = 1, 2$ and 7.

While obtaining the charged model it is observed that the maximum mass of the models keeps on increasing with increasing values of $n \leq 4$. On the other hand for $n \geq 4$, the maximum mass of the models keeps on decreasing throughout. Therefore the maximum mass is achieved at $n = 4$. It is worth pointing out here that the solutions so obtained start merging with that of charged Kuchowicz (1967) solution for higher values of n say $n \geq 3470$. The fact is supported by the behaviour of n and C which is of the type $n.C =$ constant and consequently the present metric tends to the charged Kuchowicz's metric as $n \to \infty$ and hence the lower limit of the maximum mass of the model could be determined which is 5.1165 solar mass. In the process of investigations of present models, we come across various types of stars such as white dwarfs (Mass $<$ 1.44 solar mass), Quark star (2 solar mass–3 solar mass), Neutron star (1.35 solar mass–2.1 solar mass).

2 Field equations

Let us consider a spherically symmetric metric curvature coordinates as

$$ds^2 = -e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu} dt^2 \tag{1}$$

where the functions $\lambda(r)$ and $\nu(r)$ satisfy the Einstein-Maxwell equations

$$\begin{aligned} -\kappa T_j^i &= R_j^i - \frac{1}{2} R \delta_j^i \\ &= -\kappa \left[(c^2\rho + p) v^i v_j - p\delta_j^i \right. \\ &\quad \left. + \frac{1}{4\pi}\left(-F^{im} F_{jm} + \frac{1}{4}\delta_j^i F_{mn} F^{mn} \right) \right] \end{aligned} \tag{2}$$

with $\kappa = \frac{8\pi G}{c^4}$ while ρ, p, v^i, F_{ij} denote energy density, fluid pressure, flow vector and skew-symmetric electromagnetic field tensor respectively. The resulting field equations are (Dionysiou 1982)

$$\frac{\lambda'}{r} e^{-\lambda} + \frac{(1 - e^{-\lambda})}{r^2} = \kappa c^2 \rho + \frac{q^2}{r^4} \equiv \kappa T_4^4, \tag{3}$$

$$\frac{\nu'}{r} e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa p - \frac{q^2}{r^4} = \kappa T_1^1, \tag{4}$$

$$\begin{aligned} &\left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right] e^{-\lambda} \\ &\quad = \kappa p + \frac{q^2}{r^4} = \kappa T_2^2 = \kappa T_3^3. \end{aligned} \tag{5}$$

Equations (4) and (5) give

$$e^{-\lambda}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'}{2r} - \frac{1}{r^2} \right] - e^{-\lambda}\lambda'\left[\frac{\nu'}{4} + \frac{1}{2r} \right] = \frac{2q^2}{r^4}. \tag{6}$$

Let us now assume that the value of ν is given by a general expression

$$e^{\nu} = B(1 + Cr^2)^n, \quad C > 0,\ B > 0 \tag{7}$$

where B and C are constant.

Substitution of (7) in to (3)–(6) leads to

$$\frac{2nY}{(1+x)} - \frac{(1-Y)}{x} + \frac{Cq^2}{x^2} = \frac{\kappa p}{C}, \tag{8a}$$

$$\frac{(1-Y)}{x} - 2\frac{dY}{dx} - \frac{Cq^2}{x^2} = \frac{\kappa c^2 \rho}{C} \tag{8b}$$

and

$$\frac{dY}{dx} + P(x) Y = f(x) \tag{9}$$

where

$$x = Cr^2, \qquad e^{-\lambda} = Y,$$

$$P(x) = \frac{-[1 + 2x + (1 - 2n - n^2)x^2]}{x(1+x)[1 + (n+1)x]},$$

$$f(x) = \frac{[(2q^2 C/x) - 1](1+x)}{x[1 + (n+1)x]}.$$

The solution of (9) is given by

$$Y = FA \dashv FI \tag{10}$$

where

$$F = -\int P(x)dx = \frac{x}{(1+x)^{n-2}[1 + (n+1)x]^{2/(n+1)}},$$

$$I = \int \frac{[(2q^2 C/x) - 1](1+x)^{n-1} dx}{x^2 [1 + (n+1)x]^{(n-1)/(n+1)}}$$

and A is arbitrary constant of integration.

3 New class of solutions

In order to solve the integral I for obtaining the new class of solution, we consider the electric intensity E of the form

$$\frac{E^2}{C} = \frac{Cq^2}{x^2} = \frac{n^2 K}{2} x[1 + (n+1)x]^{(n-1)/(n+1)} \tag{11}$$

where $K \geq 0$ is constant. The electric field intensity given by (11) is physically palatable since E^2 remains regular and positive throughout the sphere. In addition, the electric field given by (11) vanishes at the centre of the star.

With reference of (11), (9) yields following solution

$$\begin{aligned} Y = \Bigg[&\frac{n.Kx(1+x)^2}{[1 + (n+1)x]^{2/(n+1)}} - \frac{B_1.x}{2(1+x)^{n-2}} \\ &+ \frac{[1 + (n+1)x]}{(1+x)^{n-2}} - f(x).g(x) \\ &+ \frac{Ax}{(1+x)^{n-2}.[1 + (n+1)x]^{2/(n+1)}} \Bigg] \end{aligned} \tag{12}$$

where

$$B_1 = \frac{(n-1).(n-2)}{2} - \frac{a_{n-4}}{(n+1)} + 2n + 2,$$

$$f(x) = \frac{A_{n-4}}{(n+3)} + \sum_{i=0}^{n-5} \frac{A_i.x^{n-4-i}}{n^2 - (i+2).n - (i+1)},$$

$$A_{i+1} = a_{i+1} - \frac{(n-4-i).A_i}{n^2 - (i+2).n - (i+1)},$$

$$a_{i+1} = \frac{(n-1)!}{(i+1)!(n-i-2)!} - \frac{a_i}{(n+1)},$$

$$A_0 = a_0 = 1, \qquad g(x) = \frac{x[1+(n+1)x]}{(n+1)(1+x)^{n-1}},$$

$$A_{n-j} = a_{n-j} = 0 \quad \text{for all } n < j.$$

Using (12), into (8a) and (8b), we get the following expressions for pressure and energy density

$$\frac{\kappa p}{C} = \left[\begin{array}{l} \frac{[1+(2n+1)x]}{(1+x)}.\left[\frac{A}{(1+x)^{n-2}.[1+(n+1)x]^{2/(n+1)}} \right. \\ \left. - \frac{[1+(n+1).x].f(x)}{(n+1).(1+x)^{n-2}} - \frac{B_1}{2.(1+x)^{n-2}}\right] \\ + \frac{N_7(x)}{(1+x)^{n-1}} + \frac{n.K.[2+(5n+4).x+(n^2+5n+2).x^2]}{2.N_3(x)} \end{array}\right], \tag{13}$$

$$\frac{\kappa c^2 \rho}{C} = \left[\begin{array}{l} \frac{F_1(x)}{(1+x)^{n-1}} + \frac{B_1..N_1(x)}{2.(1+x)^{n-1}} - \frac{A..N_2(x)}{(1+x)^{n-1}.N_3(x)} \\ + \frac{f(x)..N_4(x)}{(n+1).(1+x)^{n-1}} + 2f_2(x) - \frac{n.K.N_6(x)}{2.N_3(x)} \end{array}\right] \tag{14}$$

where

$$F_1(x) = [f_1(x) - (n+1)].(1+x) - 2[3+(3+2.n-n^2).x],$$

$$N_1(x) = [3 + x + 2.(3-n).x],$$

$$N_2(x) = [3 + (6+n).x + (3+5.n-2.n^2).x^2],$$

$$N_4(x) = [3 + (3n+12).x + (9+7n-2n^2).x^2],$$

$$N_6 = 6 + (7n+18).x + (2n^2+22n+18).x^2 + (n^3+2n^2+15n+6).x^3,$$

$$N_3(x) = [1+(n+1).x]^{(n+3)/(n+1)},$$

$$N_7(x) = [(3n+2) - f_3(x) + (2n^2+3n+1).x],$$

$$f_1(x) = \sum_{i=1}^{n-2} \frac{(n-2)!.x^{i-1}}{i!.(n-2-i)!}, \qquad f_3(x) = \sum_{i=1}^{n-1} \frac{(n-1)!x^{i-1}}{i!.(n-1-i)!},$$

$$f_2(x) = \frac{[1+(n+1).x]}{(n+1).(1+x)^{n-2}}.\sum_{i=0}^{n-5} \frac{(n-4-i).A_i.x^{n-4-i}}{n^2-(i+2).n-(i+1)}.$$

Consequently the expressions for pressure and density gradients read as

$$\frac{\kappa}{C}\frac{dp}{dx} = \left[A.\frac{dp_1}{dx} - \frac{dp_2}{dx} - \frac{dp_3}{dx} + \frac{dp_4}{dx} + \frac{n^2.K}{2}.\frac{dp_5}{dx}\right] \tag{15}$$

where

$$\frac{dp_1}{dx} = \frac{n[1+(2-n)x+(1+n-2n^2).x^2]}{(1+x)^n.[1+(n+1)x]^{(n+3)/(n+1)}},$$

$$\frac{dp_2}{dx} = \frac{f(x).N_9(x) + (1+x).[1+(2n+1)x].[1+(n+1).x].f_4(x)}{(n+1).(1+x)^n},$$

$$\frac{dp_3}{dx} = \frac{B_1}{2}.\frac{[(n+2)+(2+3n-2n^2).x]}{(1+x)^n},$$

$$\frac{dp_4}{dx} = \frac{(n-1).f_3(x) - (1+x).f_6(x) + (3+4n-n^2) + (2+5n+n^2-2n^3).x}{(1+x)^n},$$

$$\frac{dp_5}{dx} = \frac{[5+(7n+9).x+(2n^2+10n+4)x^2]}{[1+(n+1)x]^{(n+3)/(n+1)}},$$

$$\frac{\kappa c^2}{C}.\frac{d\rho}{dx} = \left[\frac{d\rho_1}{dx} + \frac{d\rho_2}{dx} - A.\frac{d\rho_3}{dx} + \frac{d\rho_4}{dx} + 2.\frac{d\rho_5}{dx} - \frac{n^2.K}{2}.\frac{d\rho_6}{dx}\right] \tag{16}$$

where

$$\frac{d\rho_1}{dx} = \frac{(1+x)^2 g_1(x) + (2-n).(1+x).[f_1(x)-(n+1)] - 2.(3+2n-n^2).[1+(2-n).x] + 6.(n-1)}{(1+x)^n},$$

$$\frac{d\rho_2}{dx} = \frac{B_1}{2}.\frac{[(2n^2-11n+14).x + (10-5n)]}{(1+x)^n},$$

$$\frac{d\rho_3}{dx} = \frac{n.(1-8n)x + (9n-10n^2-n^3).x^2 - 5n + (2n^4-7n^3+2n^2+3n).x^3}{(1+x)^n.[1+(n+1).x]^{(2n+4)/(n+1)}},$$

$$\frac{d\rho_4}{dx} = \frac{(1+x).[f_4(x).N_4(x) + f(x).N_8(x)] - (n-1).f(x).N_4(x)}{(n+1).(1+x)^n},$$

$$\frac{d\rho_5}{dx}=\frac{[(n-4)+(n^2-2n-5).x-(n+1).x^2]}{(n+1).(1+x)^{n-1}}.f_4(x)-\frac{[1+(n+1)x]}{(n+1).(1+x)^{n-2}}f_5(x),$$

$$\frac{d\rho_6}{dx}=\frac{[1+(4n+30)x+(5n^2+26n+41).x^2+(2n^3+4n^2+30n+12)x^3]}{[1+(n+1)x]^{(2n+4)/(n+1)}},$$

with

$$f_4(x)=\sum_{i=0}^{n-5}\frac{(n-4-i).A_i.x^{n-5-i}}{n^2-(i+2).n-(i+1)},$$

$$f_5(x)=\sum_{i=1}^{n-5}\frac{i.(n-4-i).A_i.x^{n-5-i}}{n^2-(i+2).n-(i+1)},$$

$$g_1(x)=\sum_{i=1}^{n-3}\frac{(n-2)!.i.x^{i-1}}{(i+1)!(n-i-3)!},$$

$$f_6(x)=\sum_{i=2}^{n-1}(i-1)\frac{(n-1)!x^{i-2}}{i!.(n-1-i)!},$$

$$N_8(x)=(3n+12)+2(9+7n-2n^2)x,$$

$$N_9(x)=(2n+3)+(6+10n+n^2)x+(3+8n+3n^2-2n^3)x^2.$$

4 Conditions for regular and well behaved model

For well behaved nature of the solution implies in curvature coordinates, the following conditions should be satisfied (augmentation of Delgaty and Lake 1998 and Pant et al. 2010, 2011a conditions).

(i) The solution should be free from physical and geometric singularities and non zero positive values of e^λ and e^ν i.e. $(e^\lambda)_{r=0}=1$ and $e^\nu>0$.

(ii) Pressure p should be zero at boundary $r=a$.

(iii) $c^2\rho\ge p>0$ or $c^2\rho\ge 3p>0$, $0\le r\le a$, where former inequality denotes weak energy condition (WEC), while the later inequality implies strong energy condition (SEC).

(iv) $(dp/dr)_{r=0}=0$ and $(d^2p/dr^2)_{r=0}<0$ so that pressure gradient dp/dr is negative for $0<r\le a$.

(v) $(d\rho/dr)_{r=0}=0$ and $(d^2\rho/dr^2)_{r=0}<0$ so that density gradient $d\rho/dr$ is negative for $0<r\le a$.

The condition (iii) and (iv) imply that pressure and density should be maximum at the centre and monotonically decreasing towards the surface.

(vi) The casualty condition $(dp/c^2d\rho)^{1/2}<1$ i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e. $\frac{d}{dr}(\frac{dp}{d\rho})<0$ or $(\frac{d^2p}{d\rho^2})>0$ for $0\le r\le a$ i.e. the velocity of sound is increasing with the increase of density.

(vii) The ratio of pressure to the density $(p/c^2\rho)$ should be monotonically decreasing with the increase of r i.e. $\frac{d}{dr}(\frac{p}{c^2\rho})_{r=0}=0$ and $\frac{d^2}{dr^2}(\frac{p}{c^2\rho})_{r=0}<0$ and $\frac{d}{dr}(\frac{p}{c^2\rho})$ is negative valued function for $r>0$.

(viii) The central red shift Z_0 and surface red shift Z_a should be positive and finite i.e. $Z_0=[(e^{-\nu/2}-1)_{r=0}]>0$ and $Z_a=[e^{\lambda(a)/2}-1]>0$ and both should be bounded.

(ix) Electric intensity E, such that $E(0)=0$ is taken to be monotonically increasing i.e. $(dE/dr)>0$ for $0<r<a$.

5 Properties of new class of solution

$$\left[\frac{\kappa p}{C}\right]_{r=0}=-\frac{A_{n-4}}{(n+1).(n+3)}-\frac{a_{n-3}}{2}+(n+2)+A+n.K, \tag{17a}$$

$$\left[\frac{\kappa c^2\rho}{C}\right]_{r=0}=\frac{3.A_{n-4}}{(n+1).(n+3)}+\frac{3.a_{n-3}}{2}+(3n-6)-3A-3n.K, \tag{17b}$$

$$\left[\frac{\kappa}{C}\frac{dp}{dr}\right]_{r=0}=2Cr\left[\frac{\kappa}{C}\frac{dp}{dx}\right]_{x=0}=0, \tag{18a}$$

$$\left[\frac{\kappa c^2}{C}\frac{d\rho}{dr}\right]_{r=0}=2Cr\left[\frac{\kappa}{C}\frac{d\rho}{dx}\right]_{x=0}=0, \tag{18b}$$

$$\left[\frac{\kappa}{C}\frac{d^2p}{dr^2}\right]_{r=0}=2C\left[\frac{n}{2}.\frac{a_{n-4}}{(n+4)}+\frac{n}{2}.\frac{A_{n-5}}{(n+1).(n+2).(n+3)}-\frac{n^3+5n^2-6n}{4}+A.n+\frac{5n^2.K}{2}\right], \tag{19a}$$

$$\left[\frac{\kappa c^2}{C}\frac{d^2\rho}{dr^2}\right]_{r=0} = 2C\left[\frac{5n}{2}.\left\{-\frac{n^2+3n-2}{2}+\frac{a_{n-4}}{(n+3)} + \frac{A_{n-5}}{(n+1).(n+2).(n+3)}\right\} + 5n.A + \frac{n^2K}{2}\right]. \tag{19b}$$

For $p_{r=0}$ and $\rho_{r=0}$ must be positive, $\frac{p_{r=0}}{c^2\rho_{r=0}} \leq 1$ and $(d^2p/dr^2)_{r=0} < 0$, $(d^2\rho/dr^2)_{r=0} < 0$. Consequently we have

$$\frac{A_{n-4}}{(n+1).(n+4)} + \frac{a_{n-3}}{2} - (n+2) - nK < A < -\frac{a_{n-4}}{2.(n+4)} - \frac{A_{n-5}}{2.(n^3+6n^2+11n+6)} + \frac{n^2+5n-6}{4} - \frac{5.nK}{2}. \tag{20}$$

Hence velocity of sound at the centre given by

$$\left[\frac{dp}{c^2d\rho}\right]_{r=0} = \left[\frac{\frac{n}{2}\cdot\frac{a_{n-4}}{(n+4)} + \frac{n}{2}\cdot\frac{A_{n-5}}{(n+1).(n+2).(n+3)} - \frac{n^3+5n^2-6n}{4} + A.n + \frac{5n^2.K}{2}}{\frac{5n}{2}.\{-\frac{n^2+3n-2}{2} + \frac{a_{n-4}}{(n+3)} + \frac{A_{n-5}}{(n+1).(n+2).(n+3)}\} + 5n.A + \frac{n^2K}{2}}\right] \tag{21}$$

which should be

$$\left[\frac{dp}{c^2d\rho}\right]_{r=0} < 1,$$

for all values of $K \geq 0$ and A.

Using (13) and (14)

$$\left[\frac{p}{c^2\rho}\right] = \frac{I(x)}{J(x)} \tag{21a}$$

where

$$I(x) = \left[\begin{array}{l} \frac{[1+(2n+1)x]}{(1+x)}.\left[\frac{A}{(1+x)^{n-2}.[1+(n+1)x]^{2/(n+1)}}\right. \\ \quad \left. - \frac{[1+(n+1).x].f(x)}{(n+1).(1+x)^{n-2}} - \frac{B_1}{2.(1+x)^{n-2}}\right] \\ \quad + \frac{N_7(x)}{(1+x)^{n-1}} + \frac{n.K.[2+(5n+4).x+(n^2+5n+2).x^2]}{2N_3(x)} \end{array}\right],$$

$$J(x) = \left[\begin{array}{l} \frac{F_1(x)}{(1+x)^{n-1}} + \frac{B_1..N_1(x)}{2.(1+x)^{n-1}} - \frac{A..N_2(x)}{(1+x)^{n-1}.N_3(x)} \\ \quad + \frac{f(x)..N_4(x)}{(n+1).(1+x)^{n-1}} + 2f_2(x) - \frac{n.K.N_6(x)}{2N_3(x)} \end{array}\right].$$

Differentiating (21a) w.r.t. x

$$\frac{d}{dx}\left[\frac{p}{c^2\rho}\right] = \frac{J(x)\frac{dI(x)}{dx} - I(x)\frac{dJ(x)}{dx}}{\{J(x)\}^2}, \tag{21b}$$

$$\left[\frac{d^2}{dr^2}\left(\frac{p}{c^2\rho}\right)\right]_{r=0} = 2C.\frac{\alpha.\beta - \gamma..\delta}{\alpha^2} \tag{21c}$$

where

$$\alpha = \frac{3.A_{n-4}}{(n+1).(n+3)} + \frac{3.a_{n-3}}{2} + (3n-6) - 3A - 3n.K,$$

$$\beta = \left[\frac{n}{2}.\frac{a_{n-4}}{(n+4)} + \frac{n}{2}.\frac{A_{n-5}}{(n+1).(n+2).(n+3)} - \frac{n^3+5n^2-6n}{4} + A.n + \frac{5n^2.K}{2}\right],$$

$$\gamma = -\frac{A_{n-4}}{(n+1).(n+3)} - \frac{a_{n-3}}{2} + (n+2) + A + n.K,$$

$$\delta = \left[\frac{5n}{2}.\left\{-\frac{n^2+3n-2}{2} + \frac{a_{n-4}}{(n+3)} + \frac{A_{n-5}}{(n+1).(n+2).(n+3)}\right\} + 5n.A + \frac{n^2.K}{2}\right].$$

The expression of right hand side of (21c) is negative for all values of $K \geq 0$ and A. Then pressure-density ratio $\frac{p}{c^2\rho}$ is maximum at the centre.

The expression for gravitational red-shift Z is given by

$$Z = \frac{(1+x)^{-n/2}}{\sqrt{B}} - 1. \tag{22}$$

The central value of gravitational red-shift to be non zero positive finite, we have

$$1 > B > 0.$$

Differentiating (22) w.r.t. x, we get,

$$\left[\frac{d^2Z}{dr^2}\right]_{r=0} = -\frac{nC}{\sqrt{B}} < 0. \tag{23}$$

The expression of right hand side of (23) is negative, and then the gravitational red shift is the maximum at centre and monotonically decreasing towards the surface.

Differentiating (11) w.r.t. x, we get,

$$\frac{d}{dx}\left(\frac{E^2}{C}\right) = \frac{n^2.K}{2}\left[\frac{(1+2n.x)}{[1+(n+1)x]^{2/(n+1)}}\right],$$

$$\frac{d}{dr}\left(\frac{E^2}{C}\right) = Cr\frac{n^2.K}{2}\left[\frac{(1+2n.x)}{[1+(n+1)x]^{2/(n+1)}}\right] = +ve \quad \text{for } 0<r<a.$$

Thus the electric intensity is zero at the centre and monotonically increasing towards the pressure free interface for all values of $K>0$.

6 Boundary conditions

Besides the above, the charged fluid spheres is expected to join smoothly with the Reissner-Nordstrom metric at the pressure free boundary $r=a$

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{e^2}{r^2}\right)^{-1}dr^2 - r^2(d\theta^2+\sin^2\theta d\phi^2) + \left(1-\frac{2M}{r}+\frac{e^2}{r^2}\right)dt^2 \tag{24}$$

which requires the continuity of e^λ, e^ν and q across the boundary

$$e^{-\lambda(a)} = 1-\frac{2M}{a}+\frac{e^2}{a^2}, \tag{25}$$

$$e^{\nu(a)} = y^2_{(r=a)} = 1-\frac{2M}{a}+\frac{e^2}{a^2}, \tag{26}$$

$$q(a)=e, \tag{27}$$

$$p_{(r=a)}=0. \tag{28}$$

The condition (28) can be utilized to compute the values of arbitrary constants A as follows: On setting $x_{r=a}=X=Ca^2$ (a being the radius of the charged sphere). Pressure at $p_{(r=a)}=0$ gives

$$A = \left[\begin{array}{l} \frac{N_3(X).f(X)}{(n+1)} + [1+(n+1)X]^{2/(n+1)} \\ \quad\times\left[\frac{B_1.}{2} + \frac{.[f_3(X)-(3n+2)-(2n^2+3n+1).X]}{[1+(2n+1)X]}\right] \\ \quad - \frac{n.K.(1+X)^{n-1}.[2+(5n+4)X+(n^2+5n+2)X^2]}{2.[1+(2n+1)X]} \end{array}\right] \tag{29}$$

where

$$N_3(X) = [1+(n+1).X]^{(n+3)/(n+1)},$$

$$f(X) = \frac{A_{n-4}}{(n+3)} + \sum_{i=0}^{n-5}\frac{A_i.X^{n-4-i}}{n^2-(i+2).n-(i+1)},$$

$$f_3(X) = \sum_{i=1}^{n-1}\frac{(n-1)!X^{i-1}}{i!.(n-1-i)!}.$$

The expression for mass can be written as

$$m(a) = \frac{a}{2}\left[\frac{2nX}{[1+(2n+1)X]} + n^2.K.\frac{X^2.[1+(n+1)X]^{2n/(n+1)}}{[1+(2n+1)X]}\right] \tag{30}$$

such that $e^{-\lambda(a)} = 1-\frac{2M}{a}+\frac{e^2}{a^2}$, where $M=m(a)$ and $y^2_{(r=a)} = 1-\frac{2M}{a}+\frac{e^2}{a^2}$ gives

$$B = \left[\frac{G(X)+X.H(X)}{(1+X)^{2n-2}.[1+(2n+1)X]} + \frac{n.K}{2}.\frac{X(1+X).\{2(1+X).[1+(2n+1)X]-E(X)\}}{(1+X)^n.[1+(n+1)X]^{2/(n+1)}[1+(2n+1)X]}\right] \tag{31}$$

with

$$E(X) = [2+(5n+4)X+(n^2+5n+2)X^2],$$

$$G(X) = 1+(3n+2).X+(2n^2+3n+1)X^2,$$

$$H(X) = [f_3(X)-(3n+2)-(2n^2+3n+1).X].$$

Also, if the surface density ρ_a is prescribed as $2\times 10^{14}\ \mathrm{g\,cm^{-3}}$ (super dense star case) then value of constant C can be calculated for a given X $(=Ca^2)$, using the following expression

$$\kappa c^2\rho_a = C\left[\begin{array}{l}\frac{F_1(X)}{(1+X)^{n-1}} + \frac{B_1.N_1(X)}{2.(1+X)^{n-1}} - \frac{A..N_2(X)}{(1+X)^{n-1}.N_3(X)} \\ \quad + \frac{f(X)..N_4(X)}{(n+1).(1+X)^{n-1}} + 2f_2(X) - \frac{n.K.N_6(X)}{2.N_3(X)}\end{array}\right]$$

with

$$F_1(X) = [f_1(X)-(n+1)].(1+X) - 2[3+(3+2.n-n^2).X],$$

$$N_1(X) = [3+X+2.(3-n).X],$$

$$N_2(X) = [3+(6+n).X+(3+5.n-2.n^2).X^2],$$

$$N_4(X) = [3+(3n+12).X+(9+7n-2n^2).X^2],$$

$$N_6 = 6+(7n+18).X+(2n^2+22n+18).X^2 + (n^3+2n^2+15n+6).X^3,$$

$$N_3(x) = [1+(n+1).X]^{(n+3)/(n+1)},$$

$$f_2(X) = \frac{[1+(n+1).X]}{(n+1).(1+X)^{n-2}}.\sum_{i=0}^{n-5}\frac{(n-4-i).A_i.X^{n-4-i}}{n^2-(i+2).n-(i+1)},$$

$$f(X) = \frac{A_{n-4}}{(n+3)} + \sum_{i=0}^{n-5}\frac{A_i.X^{n-4-i}}{n^2-(i+2).n-(i+1)}.$$

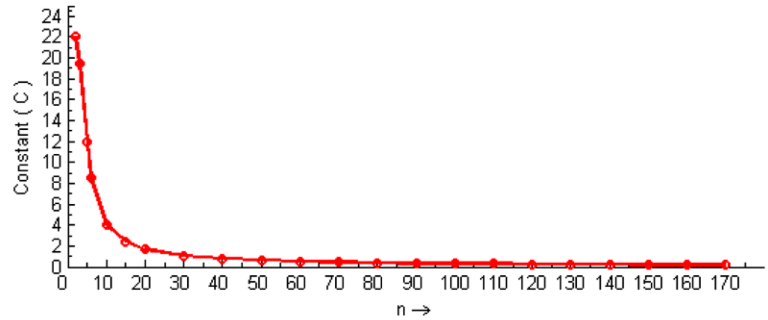

Fig. 1 Behaviour of constant (C) versus n

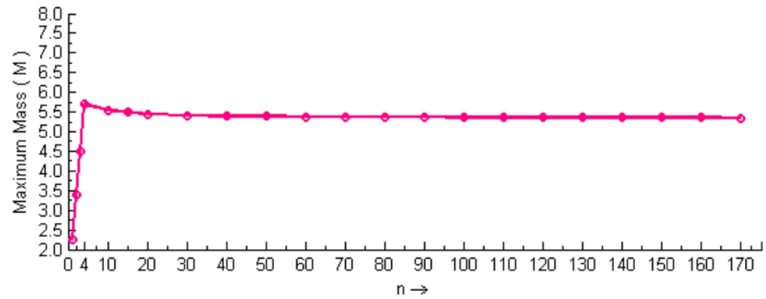

Fig. 2 Behaviour of mass (M) versus n

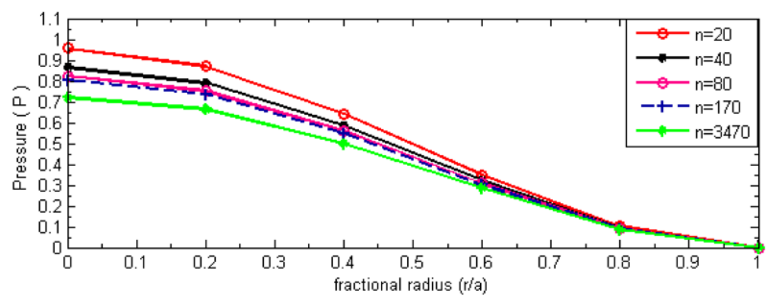

Fig. 3 Behaviour of pressure (P) versus radius

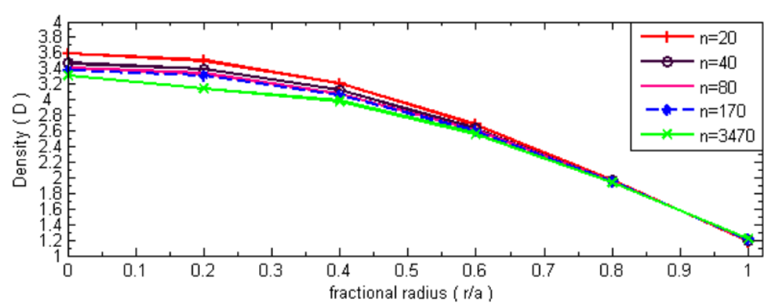

Fig. 4 Behaviour of density (D) versus radius

7 Conclusions

In the present article, we have obtained the well behaved charged solutions for the perfect fluid balls described by a static spherically symmetric metric in canonical form with the metric potential $g_{44} = B(1 + Cr^2)^n$ (which is essentially due to Durgapal 1982) for every positive integral value of n. However the detailed analysis for the well behaved charged models is done for $n \leq 3470$. For $n \geq 1$, the family of superdence star models satisfy the energy conditions $c^2\rho \geq p > 0$, $dp/dr < 0$, $d\rho/dr < 0$, adiabatic index $\gamma = ((p + c^2\rho)/p)(dp/(c^2 d\rho)) > 1$, and the causality condition $dp/c^2 d\rho < 1$. Also velocity of sound, the ratio $p/c^2\rho$ and red shift are monotonically decreasing towards the pressure free interface for $n \geq 1$. The Maximum mass and corresponding radius have obtained for each model correspond-

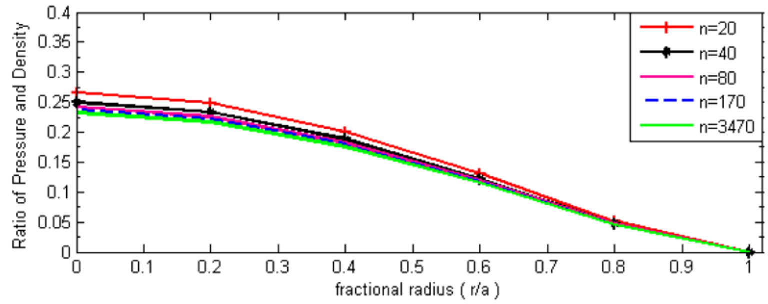

Fig. 5 Behaviour of ratio of pressure and density versus radius

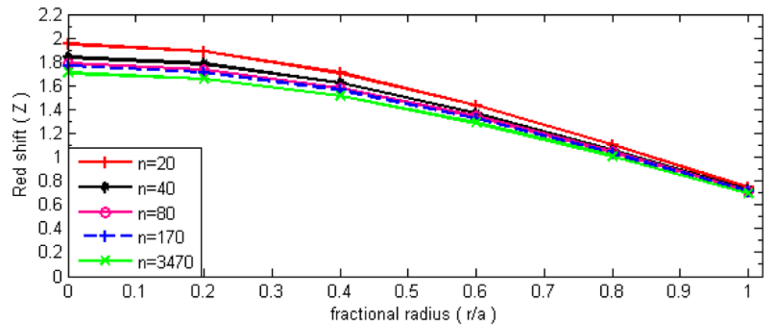

Fig. 6 Behaviour of red shift (Z) versus radius

Table 1 Maximum mass (M/M_Θ) and radius (a) for different n, K and Ca^2 for WEC and SEC

n	K	$0 \le p \le c^2\rho$ (WEC)			$0 \le 3p \le c^2\rho$ (SEC)		
		Ca^2	Radius (Km)	Max M/M_Θ	Ca^2	Radius (Km)	Max M/M_Θ
1	0.379000	0.4650	14.5108	2.2349	0.4650	14.5108	2.2349
4	0.0405625	0.5283	17.1003	5.7001	–	–	–
10	0.118000	0.1241	17.6071	5.5467	0.1241	17.6071	5.5467
20	0.1510000	0.05388	17.8115	5.4495	0.05388	17.8115	5.4495
40	0.1682500	0.02528	17.9025	5.3994	0.02528	17.9025	5.3994
100	0.1790000	0.009746	17.9533	5.3675	0.009746	17.9533	5.3675
170	0.18194117	0.005677	17.9671	5.3507	0.005677	17.9671	5.3507
3470	0.18790000	0.00027	18.0324	5.1985	0.0002700	18.0324	5.1985

ing to each integral value of n, which is considered in the text. Over all maximum mass for the whole family turns out to be 5.7001 M_Θ corresponding to the radius 17.1003 Km with red shift at centre $Z_0 = 3.9326$ and red shift at surface $Z_a = 1.1118$ for $n = 4$. Further the values of the parameter C for each integral values of n are studied. Consequently we come across an interesting result which leads to the relation $nC = \text{constant} = k$ (say) or a rectangular hyperbola on the $n-C$ plane as given by Fig. 1. The above discussion helps us to write $g_{44} = B(1 + Cr^2)^n$ as $g_{44} = B(1 + \frac{k}{n}r^2)^n$ which tends to $g_{44} = Be^{kr^2}$ as $n \to \infty$. The metric potential $g_{44} = Be^{kr^2}$ same as metric potential of Kuchowicz solution which suggests that the family of the solution tends that of charged Kuchowicz solution as $n \to \infty$ and there-

Table 2 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 1$, $K = 0.379$, $Ca^2 = 0.4650$)

$n = 1$, $K = 0.379$, $Ca^2 = 0.4650$, Radius (a) = 14.5108 Km, $M = 2.2349\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.1559	2.3224	2.1630	0.0000	0.3688	0.0671	0.6936
0.2	0.1386	2.1952	2.2754	0.0235	0.3676	0.0632	0.6780
0.4	0.0961	1.8755	2.6599	0.1880	0.3601	0.0513	0.6339
0.6	0.0484	1.4851	3.5447	0.6345	0.3346	0.0326	0.5674
0.8	0.0132	1.1118	6.1288	1.5039	0.2678	0.0118	0.4867
1.0	0.0000	0.7858	∞	2.9373	0.0487	0.0000	0.3992

Table 3 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 4$, $K = 0.0405625$, $Ca^2 = 0.5283$)

$n = 4$, $K = 0.04056$, $Ca^2 = 0.5283$, Radius (a) = 17.1003 Km, $M = 5.7001\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	2.5243	5.1062	3.0129	0.0000	0.9984	0.4944	3.9326
0.2	2.2291	4.8042	3.0222	0.0421	0.9787	0.4640	3.7305
0.4	1.5146	4.0136	3.0896	0.3552	0.9201	0.3774	3.1936
0.6	0.7424	2.9965	3.3313	1.2827	0.8133	0.2478	2.4821
0.8	0.2013	1.9867	4.2538	3.2574	0.6256	0.1013	1.7548
1.0	0.0000	1.0913	∞	6.7884	0.1947	0.0000	1.1118

Table 4 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 10$, $K = 0.1180$, $Ca^2 = 0.1241$)

$n = 10$, $K = 0.1180$, $Ca^2 = 0.1241$, Radius (a) = 17.6071 Km, $M = 5.5467\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	1.1862	3.8875	4.2711	0.0000	0.9993	0.3051	2.2147
0.2	1.0746	3.7696	4.0495	0.0434	0.9478	0.2851	2.1361
0.4	0.7806	3.3965	3.7305	0.3683	0.8349	0.2298	1.9137
0.6	0.4143	2.7723	3.7616	1.3501	0.6993	0.1494	1.5836
0.8	0.1158	1.9841	4.8649	3.5133	0.5179	0.0583	1.1937
1.0	0.0000	1.1570	∞	7.5479	0.0164	0.0000	0.7911

Table 5 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 20$, $K = 0.151000$, $Ca^2 = 0.05388$)

$n = 20$, $K = 0.1510$, $Ca^2 = 0.05388$, Radius (a) = 17.8115 Km, $M = 5.4495\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.9565	3.5960	4.7433	0.0000	0.9983	0.2660	1.9492
0.2	0.8712	3.5051	4.4493	0.0430	0.9411	0.2486	1.8864
0.4	0.6426	3.2064	4.0082	0.3639	0.8180	0.2004	1.7066
0.6	0.3490	2.6772	3.9673	1.3296	0.6764	0.1304	1.4337
0.8	0.1002	1.9673	5.1005	3.4548	0.4972	0.0509	1.1012
1.0	0.0000	1.1840	∞	7.4271	0.0093	0.0000	0.7450

Table 6 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 40$, $K = 0.1682500$, $Ca^2 = 0.02528$)

$n = 40$, $K = 0.16825$, $Ca^2 = 0.02528$, Radius (a) = 17.9025 Km, $M = 5.3994\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.8659	3.4694	5.0002	0.0000	0.9994	0.2496	1.8406
0.2	0.7905	3.3889	4.6666	0.0428	0.9395	0.2332	1.7838
0.4	0.5868	3.1201	4.1598	0.3615	0.8115	0.1881	1.6203
0.6	0.3220	2.6313	4.0801	1.3188	0.6670	0.1224	1.3699
0.8	0.0936	1.9574	5.2264	3.4244	0.4883	0.0478	1.0607
1.0	0.0000	1.1961	∞	7.3641	0.0127	0.0000	0.7241

Table 7 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 80$, $K = 0.1773$, $Ca^2 = 0.01225$)

$n = 80$, $K = 0.17725$, $Ca^2 = 0.01225$, Radius (a) = 17.9468 Km, $M = 5.3714\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.8240	3.4081	5.1291	0.0000	0.9993	0.2418	1.7888
0.2	0.7529	3.3323	4.7765	0.0427	0.9383	0.2260	1.7346
0.4	0.5607	3.0770	4.2378	0.3601	0.8082	0.1822	1.5787
0.6	0.3092	2.6075	4.1392	1.3126	0.6624	0.1186	1.3387
0.8	0.0905	1.9516	5.2916	3.4071	0.4842	0.0464	1.0405
1.0	0.0000	1.2020	∞	7.3277	0.0200	0.0000	0.7136

Table 8 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 170$, $K = 0.18194117$, $Ca^2 = 0.005677$)

$n = 170$, $K = 0.18194117$, $Ca^2 = 0.005677$, Radius (a) = 17.9671 Km, $M = 5.3588\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.8039	3.3788	5.2017	0.0000	0.9999	0.2379	1.7645
0.2	0.7350	3.3052	4.8377	0.0426	0.9381	0.2224	1.7116
0.4	0.5481	3.0566	4.2804	0.3595	0.8068	0.1793	1.5592
0.6	0.3029	2.5963	4.1712	1.3100	0.6601	0.1167	1.3240
0.8	0.0888	1.9490	5.3283	3.3999	0.4820	0.0456	1.0311
1.0	0.0000	1.2048	∞	7.3129	0.0150	0.0000	0.7086

Table 9 Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($n = 3470$, $K = 0.187900$, $Ca^2 = 0.00027$)

$n = 3470$, $K = 0.1879$, $Ca^2 = 0.00027$, Radius (a) = 18.0324 Km, $M = 5.1985\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.7198	3.3119	5.2156	0.0000	0.9918	0.2324	1.7077
0.2	0.6647	3.1414	4.8608	0.0422	0.9317	0.2174	1.6575
0.4	0.4998	2.9834	4.3114	0.3554	0.8028	0.1758	1.5122
0.6	0.2846	2.5619	4.1985	1.2934	0.6581	0.1150	1.2876
0.8	0.0868	1.9373	5.3224	3.3530	0.4829	0.0458	1.0064
1.0	0.0000	1.2135	∞	7.2060	0.0807	0.0000	0.6951

Table 10 Table for charged Kuchowicz's solution. Behaviour of pressure (P), density (D), adiabatic index (γ), red shift (Z), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.035$, $ka^2 = 1.74456$)

$K = 0.035$, $ka^2 = 1.74456$, Radius (a) = 18.0743 Km, Maximum mass $M = 5.1165\ M_\Theta$

r/a	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.7081	3.1093	5.3564	0.0000	0.9968	0.2277	0.9805
0.2	0.6492	3.0542	4.9908	0.0345	0.9354	0.2126	0.9462
0.4	0.4883	2.8600	4.4238	0.3019	0.8032	0.1707	0.8470
0.6	0.2736	2.4793	4.3107	1.1496	0.6545	0.1104	0.6927
0.8	0.0818	1.9116	5.5082	3.1071	0.4754	0.0428	0.4981
1.0	0.0000	1.2192	∞	6.9104	0.0123	0.0000	0.2804

fore the maximum mass of models for any n can not be less than the maximum mass of the charged Kuchowicz' model. The maximum mass of charged Kuchowicz' superdense star model turns out to be 5.1165 M_Θ with corresponding radius 18.0743 Km, also the central red shift $Z_0 = 0.9805$ and the surface red shift $Z_a = 0.2804$ for $ka^2 = 1.74456$.

Detailed physically behaviour of the models for various integral values n are displayed by means of graphs and tables.

8 Tables for numerical values of physical quantities

In Tables 1–10: Z_0 = red shift at the centre, Z_a = red shift at the surface, Solar mass $M_\Theta = 1.475$ km, $G = 6.673 \times 10^{-8}$ cm^3/gs^2, $c = 2.997 \times 10^{10}$ cm/s, $D = (8\pi G/c^2)\rho a^2$, $Q = q$, $P = (8\pi G/c^4)pa^2$, $\gamma = \frac{p+c^2\rho}{p}\frac{dp}{c^2d\rho}$, $R = \frac{p}{c^2\rho}$, $V = (dp/c^2d\rho)^{1/2}$.

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