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ORIGINAL ARTICLE

Closed form Vaidya-Tikekar type charged fluid spheres with pressure

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Abstract Recently, Bijalwan (Astrophys. Space Sci. doi:10.1007/s10509-011-0691-0, 2011) discussed all important solutions of charged fluid spheres with pressure and Gupta et al. (Astrophys. Space Sci. doi:10.1007/s10509-010-0561-1, 2010) found first closed form solutions of charged Vaidya-Tikekar (V-T) type super-dense star. We extend here the approach evolved by Bijalwan (Astrophys. Space Sci. doi:10.1007/s10509-011-0691-0, 2011) to find all possible closed form solutions of V-T type super-dense stars. The existing solutions of Vaidya-Tikekar type charged fluid spheres considering particular form of electric field intensity are being used to model massive stars. Infact at present maximum masses of the star models are found to be $8.223931 M_{\Theta}$ and $8.460857 M_{\Theta}$ subject to ultra-relativistic and non-relativistic conditions respectively. But these stars with such are large masses are not well behaved due to decreasing velocity of sound in the interior of star. We present new results concerning the existence of static, electrically charged perfect fluid spheres that have a regular interior. It is observed that electric intensity used in this article can be used to model superdense stars with ultrahigh surface density of the order 2×10^{14} gm/cm^3 which may have maximum mass $7.26368240 M_{\Theta}$ for ultra-relativistic condition and velocity of sound found to be decreasing towards pressure free interface. We solve the Einstein-Maxwell equations considering a general barotropic equation of state with pressure. For brevity we don't present a detailed analysis of the derived solutions in this paper.

Keywords Charge fluids · Reissner-Nordstrom · General relativity · Exact solution

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1 Introduction

It is well known that static, spherically symmetric, uncharged perfect fluids cannot be held in equilibrium below a certain radius without developing singularities inside. The possibility of holding a non-singular object in stable equilibrium but compact enough to be close (in fact arbitrarily close) to a black-hole state, is of great interest not only in order to judge the state of matter in this condition, that is being about to turn into a black-hole, but also to yield a classic model of charged massive particles which might have astrophysical and cosmological implications. Although one can reach this goal with non-perfect fluids, a perfect fluid solution of the type mentioned was recently found (Gupta and Kumar 2005a, 2005b) but with the presence of an electric charge. Electric charges inhibit the growth of space-time curvature and therefore they are an efficient means of avoiding singularities inside matter and enhance stability.

The charged analogues of Vaidya-Tikekar types of fluid spheres have been considered by many workers (Patel et al. 1997; Tikekar and Singh 1998; Patel and Pandya 1986a, 1986b). The solutions were then utilized to depict the charged superdense star models with ultra high surface density (Vaidya and Tikekar 1982). The Vaidya-Tikekar fluid spheres are described by space-time with the hypersurfaces $t = const.$ as 3-spheroids. The Vaidya-Tikekar type charged fluid spheres have been derived considering the electric field intensity that has positive gradient. In the past (Sharma et al.

2001; Gupta and Kumar 2005a, 2005b) have presented all the charged analogues of Vaidya-Tikekar type fluid spheres with said electric field intensity. Gupta et al. (2010) found some new closed form solutions of V-T type stars. The positive gradient consideration is too restrictive on the accumulation of charge. Taking notice of this fact in the present article the authors have successfully chosen a more general electric intensity and have obtained charged Vaidya-Tikekar fluid spheres joining smoothly with the Reissner-Nordstrom Metric at the pressure free boundary and utilized them to represent superdense star with surface density of the order 2×10^{14} gm/cm^3. The latter are analyzed numerically subject to the energy conditions throughout the star. Consequently, the maximum mass of the star models was found to be $7.26368240 M_\Theta$.

2 Field equations

Let us take the following spherically symmetric metric to describe the space-time of a charged fluid sphere

$$ds^2 = -e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\upsilon} dt^2. \tag{1a}$$

Let us take the static spherically symmetric space-time with $t =$ constant hypersurfaces as spheroids as

$$ds^2 = -\frac{K(1+Cr^2)}{(K+Cr^2)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + y^2(r) dt^2, \tag{1b}$$

where C and $K < 1$ are constant parameters.

If (1) describes charged fluid distribution then the space-time (1) has to satisfy the Einstein-Maxwell equations

$$R^i_j - \frac{1}{2} R\delta^i_j = -\kappa\left[(c^2\rho + p)v^i v_j - p\delta^i_j + \frac{1}{4\pi}\left(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F_{mn}F^{mn}\right)\right], \tag{2}$$

where $\kappa = \frac{8\pi G}{c^4}$, ρ, p and v^i denote matter density, fluid pressure and the unit time-like flow vector of the fluid respectively and F_{ik} being the skew symmetric electromagnetic field tensor satisfying the Maxwell equations

$$F_{ik;j} + F_{kj;i} + F_{ji;k} = 0, \tag{3}$$

$$\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi\sqrt{-g}\, j^i, \tag{4}$$

where $j^i = \sigma v^i$ represents the four-current vector of charged fluid while the charged density is denoted by σ.

The field equations (2) with respect to (1a) reduce to (Dionysiou 1982)

$$\kappa T^1_1 = -\frac{\upsilon'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = -\kappa p + \frac{q^2}{r^4}, \tag{5a}$$

$$\kappa T^2_2 = \kappa T^3_3 = -\left[\frac{\upsilon''}{2} - \frac{\lambda'\upsilon'}{4} + \frac{\upsilon'^2}{4} + \frac{\upsilon' - \lambda'}{2r}\right]e^{-\lambda} = -\kappa p - \frac{q^2}{r^4}, \tag{6a}$$

$$\kappa T^4_4 = \frac{\lambda'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \kappa c^2\rho + \frac{q^2}{r^4}. \tag{7a}$$

The field equations (5a), (6a) and (7a) with respect to (1b) reduce to

$$\frac{(K+Cr^2)}{K(1+Cr^2)}\left[\frac{-2y'}{ry} + \frac{C(K-1)}{K+Cr^2}\right] = -\kappa p + \frac{q^2}{r^4}, \tag{5b}$$

$$-\left[\frac{y''}{y} + \frac{y'}{ry} - \frac{C(K-1)ry'}{(K+Cr^2)(1+Cr^2)y} - \frac{C(K-1)}{(K+Cr^2)(1+Cr^2)}\right]\frac{(K+Cr^2)}{K(1+Cr^2)} = -\kappa p - \frac{q^2}{r^4}, \tag{6b}$$

$$\frac{C(K-1)(3+Cr^2)}{K(1+Cr^2)^2} = \kappa c^2\rho + \frac{q^2}{r^4}, \tag{7b}$$

where

$$q(r) = 4\pi\int_0^r \sigma r^2 e^{\lambda/2} dr = r^2\sqrt{-F_{14}F^{14}} = r^2 F^{41} e^{(\lambda+\upsilon)/2} \tag{8}$$

represents the total charge contained with in the sphere of radius 'r'. Equation (4) reduces to

$$\frac{\partial}{\partial r}(e^{(\lambda+\upsilon)/2} r^2 F^{41}) = -4\pi e^{(\lambda+\upsilon)/2} r^2 j^4. \tag{9}$$

Beyond the pressure free interface '$r = a$' the charged fluid sphere is expected to join with the Reissner-Nordstrom metric:

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)dt^2, \tag{10}$$

where M is the gravitational mass of the distribution such that

$$M = \mu(a) + \varepsilon(a)$$

while

$$\mu(a) = \frac{\kappa}{2}\int_0^a \rho r^2 dr, \quad \varepsilon(a) = \frac{\kappa}{2}\int_0^a r\sigma q e^{\lambda/2} dr, \quad e = q(a), \tag{11}$$

$\varepsilon(a)$ is the mass equivalence of the electromagnetic energy of distribution while $\mu(a)$ is the mass and 'e' is the total charge inside the sphere (Florides 1983)

Let us consider the barotropic equation of state $\kappa c^2\rho = g(p)$. Further, assume that the metric components (e^λ and e^υ) and the electric intensity are arbitrary functions of pressure $p(\omega)$ such that ω is some function of r i.e.

$$e^{-\lambda} = s(p(\omega)), \quad e^\upsilon = h(p(\omega)), \quad \kappa c^2\rho = g(p(\omega)). \tag{5c}$$

On subtracting (5a) from (7a) gives

$$\left(\frac{\upsilon' + \lambda'}{r}\right)e^{-\lambda} = \kappa(c^2\rho + p). \tag{5d}$$

Substituting (5c) in (5d) leads to

$$\frac{(\overline{\upsilon} + \overline{\lambda})}{(c^2\rho + p)} e^{-\lambda}\frac{dp}{dr} = r, \tag{5e}$$

where overhead dash denotes derivative w.r.t. p or ω. Equation (5e) yields

$$p = p(\omega), \quad \omega = c_1 + c_2 r^2, \tag{5f}$$

where c_1 and $c_2(\neq 0)$ are arbitrary constants.

$$e^\lambda = s(p(\omega)) = \frac{K\omega}{(K^2 - K + \omega)}, \tag{5g}$$

where $c_1 = K$ and $c_2 = KC$.

Matter density and velocity of sound can be expressed using (5c) in (5e) and (5a) as

$$c^2\rho = c_2\left(\frac{\overline{h}}{h}s - \overline{s}\right) - p, \tag{5h}$$

$$\sqrt{\frac{dp}{c^2 d\rho}} = \frac{1}{\sqrt{c_2((\overline{\frac{\overline{h}}{h}s}) - \overline{\overline{s}}) - 1}}. \tag{5i}$$

The velocity of sound less than unity in the interior of star using (5i) translates to

$$2 < c_2\left(\overline{\left(\frac{\overline{h}}{h}s\right)} - \overline{\overline{s}}\right). \tag{5j}$$

Further, in order to have physically viable solutions, from (5h) $c^2\rho + p (= c_2(\frac{\overline{h}}{h}s - \overline{s}))$ should be monotonically decreasing i.e.

$$2c_2^2\left(\overline{\left(\frac{\overline{h}}{h}s\right)} - \overline{\overline{s}}\right)r < 0. \tag{5k}$$

From (5j) and (5k) can only be satisfied if and only if $((\overline{\frac{\overline{h}}{h}s}) - \overline{\overline{s}}) < 0$ and $c_2 < 0$.

We consider $K < 0$ and $C > 0$ Gupta and Kumar (2005a, 2005b). Now, for the given expression of 'q', the expressions for the pressure and energy density can be had from ((5a)–(5k)) and (7a, 7b) subject to the consistency of (6), which requires elimination of 'p'.

$$\frac{(K + Cr^2)}{K(1 + Cr^2)}\left[\frac{y''}{y} + \frac{y'}{ry} - \frac{C(K-1)r(Cr - \frac{y'}{y})}{(K + Cr^2)(1 + Cr^2)}\right] = \frac{2q^2}{r^4}. \tag{12}$$

If we let

$$X = \sqrt{\frac{K}{K-1}}\sqrt{\frac{K^2 - K + \omega}{K^2}} = \sqrt{\frac{K}{K-1}}\sqrt{1 + \frac{Cr^2}{K}}, \tag{13}$$

then (12) transforms to a simple form

$$(1 - X^2)\frac{d^2y}{dX^2} + X\frac{dy}{dX} + \alpha y = 0 \tag{14}$$

where $\alpha = 1 - K + 2Kq^2\frac{(Cr^2+1)^2}{C^2r^6}$.

In order to solve the equation in closed form, let us take

$$y = (1 - X^2)^{1/4}\upsilon \tag{15}$$

and (14) reduces to

$$\upsilon'' + \left[\frac{\alpha}{1 - X^2} - \frac{2 + 3X^2}{4(1 - X^2)^2}\right]\upsilon = 0. \tag{16}$$

Equation (16) can easily be integrated if, we let

$$\frac{\alpha}{1 - X^2} - \frac{2 + 3X^2}{4(1 - X^2)^2} = A_1,$$

where A_1 can take positive, negative or zero constant value.

The electric intensity can be explicitly determined as

$$\frac{q^2}{r^4}=\frac{C^2r^2}{2(Cr^2+1)^2}\left[\frac{A_1(1+Cr^2)}{K(1-K)}+\frac{5(1-K)}{4K(1+Cr^2)}+1-\frac{7}{4K}\right]. \tag{17}$$

The expressions for density and pressure are expressed as

$$\kappa c^2\rho=\frac{C(K-1)}{K}\frac{(3+Cr^2)}{(1+Cr^2)^2}-\frac{C^2r^2}{2(1+Cr^2)^2}\times\left[\frac{A_1(1+Cr^2)}{K(1-K)}+\frac{5(1-K)}{4K(1+Cr^2)}-\frac{7}{4K}+1\right], \tag{18}$$

$$\kappa p=\frac{2y'}{ry}\frac{(K+Cr^2)}{K(1+Cr^2)}-\frac{C(K-1)}{K(1+Cr^2)}+\frac{C^2r^2}{2(1+Cr^2)^2}\times\left[\frac{A_1}{K}\frac{(1+Cr^2)}{(1-K)}+\frac{5(1-K)}{4K(1+Cr^2)}-\frac{7}{4K}+1\right]. \tag{19}$$

And the expressions for the density gradient and pressure gradient can be written as:

$$\kappa c^2\frac{d\rho}{dr}=\frac{C^2r}{(1+Cr^2)^4}\left[\frac{1}{2}\left(\frac{21}{K}-\frac{39}{2}\right)-\frac{A_1}{K(1-K)}+\left\{\frac{29}{2}\left(\frac{1}{K}-1\right)-\frac{2A_1}{K(1-K)}\right\}Cr^2+\left\{\frac{1}{4K}-1-\frac{A_1}{K(1-K)}\right\}C^2r^4\right], \tag{20}$$

$$\kappa\frac{dp}{dr}=\mathrm{Pr}-\frac{1}{2}e^{\lambda}(c^2\rho+p)\left[\frac{M(r)}{r^2}-\frac{q^2}{r^3}+\frac{\kappa pr}{2}\right], \tag{21}$$

where

$$\mathrm{Pr}=\frac{C^2r}{(1+Cr^2)^2}\left[\frac{A_1}{K(1-K)}(3+2Cr^2)+\frac{15}{4}\frac{(1-K)}{K(1+Cr^2)^2}+\left(1-\frac{7}{4K}\right)\frac{(3+Cr^2)}{(1+Cr^2)^2}\right]$$

$$M(r)=\frac{r}{2}\left[\frac{(K-1)Cr}{K(1+Cr^2)}+\frac{C^2r^4}{2(1+Cr^2)^2}\times\left\{\frac{A_1(1+Cr^2)}{K(1-K)}+\frac{5(1-K)}{4K(1+Cr^2)}+1-\frac{7}{4K}\right\}\right]$$

such that $e^{-\lambda}=1-\frac{2M(r)}{r}+\frac{q^2}{r^2}$.

3 The speed of sound

The sphere being charged, it is not possible on the present naive phenomenological level to say what the speed of propagation of a sound wave of arbitrary frequency will be. However, if the frequency is sufficiently large the adiabatic speed of propagation by the fluid is presumably by

$$v^2=\frac{dp}{d\rho}=\frac{c^2\left[C^2\left\{\frac{A_1(3+2Cr^2)(1+Cr^2)}{K(1-K)}+\frac{15}{4}\frac{(1-K)}{K(1+Cr^2)}+(1-\frac{7}{4K})(3+Cr^2)\right\}-e^{\lambda}\kappa(c^2\rho+p)\left\{\frac{M(r)}{r^3}-\frac{q^2}{r^4}+\frac{\kappa p}{2}\right\}(1+Cr^2)^3\right](1+Cr^2)}{C^2\left[\left\{\frac{-39}{4}+\frac{29}{2K}-\frac{A_1}{K(1-K)}\right\}+\left\{\frac{-2A_1}{K(1-K)}-\frac{29}{2}+\frac{29}{2K}\right\}Cr^2+\left\{\frac{-A_1}{K(1-K)}+\frac{1}{4K}-1\right\}C^2r^4\right]} \tag{22}$$

Also if the ratio of the surface density ρ_a to central density ρ_0 is $L=\frac{\rho_a}{\rho_o}$.

From (18)

$$\kappa c^2\rho_a=\frac{C(K-1)}{K}\frac{(3+Ca^2)}{(1+Ca^2)^2}-\frac{C^2a^2}{2(1+Ca^2)^2}\times\left[\frac{A_1(1+Ca^2)}{K(1-K)}+\frac{5(1-K)}{4K(1+Ca^2)}-\frac{7}{4K}+1\right], \tag{23}$$

$$\kappa c^2\rho_0=\frac{3C(K-1)}{K} \tag{24}$$

and

$$L=\frac{\rho_a}{\rho_0}=\frac{(3+Ca^2)}{3(1+Ca^2)^2}-\frac{KCa^2}{6(K-1)(1+Ca^2)^2}\times\left[\frac{A_1(1+Ca^2)}{K(1-K)}+\frac{5(1-K)}{4K(1+Ca^2)}-\frac{7}{4K}+1\right]. \tag{25}$$

On simplification of (25), we get

$$\left[\frac{-A_1}{2K(1-K)}-3\frac{(K-1)}{K}L\right](1+Ca^2)^3+\left[\frac{A_1}{2K(1-K)}+\frac{1}{2}-\frac{1}{8K}\right](1+Ca^2)^2$$

$$+\left[\frac{25}{8}-\frac{7}{2K}\right](1+Ca^2)+\frac{5(1-K)}{8K}=0, \tag{26}$$

where surface density ρ_a is taken out to be 2×10^{14} gm cm^{-3} and $C=\frac{8\pi GK\rho_0}{3c^2(K-1)}=\frac{1.24479\times10^{-13}}{(K-1)L}$.

One can easily get value of $1+Ca^2$ on solving the cubic equation (26) for positive, negative and zero value of A_1. For $A_1\geq 0$, it follows from Riccati rule that (26) has one negative and two positive roots for all values of K and L. Also one root lies between 0 and 1. So, we are left with only one root for $1+Ca^2>1$. However for $A_1<0$, nothing can be said analytically but numerically calculated roots have same behaviour as in the case of $A_1\geq 0$.

Further, the expression for the pressure can be derived as follows:

Case 1. $A_1=-\beta^2$

$$y=A(1-X^2)^{1/4}(e^{\beta X}+Be^{-\beta X}),$$

$$\begin{aligned}\kappa p=C\Bigg[&\sqrt{\frac{K}{K-1}}\sqrt{1+\frac{Cr^2}{K}}\frac{2\beta(e^{\beta X}-Be^{-\beta X})}{K(1+Cr^2)(e^{\beta X}+Be^{-\beta X})}\\&+\frac{1}{(1+Cr^2)^2}+\frac{(1/2+1/(8K))Cr^2}{(1+Cr^2)^2}\\&-\frac{(K-1)}{K(1+Cr^2)}-\frac{\beta^2Cr^2}{2K(1-K)(1+Cr^2)}\\&+\frac{5(1-K)Cr^2}{8K(1+Cr^2)^3}\Bigg].\end{aligned} \tag{27}$$

Case 2. $A_1=0$

$$y=A(1-X^2)^{1/4}(X+B),$$

$$\begin{aligned}\kappa p=C\Bigg[&\sqrt{\frac{K}{K-1}}\sqrt{1+\frac{Cr^2}{K}}\frac{2}{K(1+Cr^2)(X+B)}\\&+\frac{1}{(1+Cr^2)^2}+\frac{(1/2+1/(8K))Cr^2}{(1+Cr^2)^2}\\&-\frac{(K-1)}{K(1+Cr^2)}+\frac{5(1-K)Cr^2}{8K(1+Cr^2)^3}\Bigg].\end{aligned} \tag{28}$$

Case 3. $A_1=\beta^2$

$$y=A(1-X^2)^{1/4}(\cos(\beta X)+B\sin(\beta X)),$$

$$\begin{aligned}\kappa p=C\Bigg[&\frac{-2\beta(\sin(\beta X)-B\cos(\beta X))}{K(1+Cr^2)(\cos(\beta X)+B\sin(\beta X))}\sqrt{\frac{K}{K-1}}\\&\times\sqrt{1+\frac{Cr^2}{K}}+\frac{1}{(1+Cr^2)^2}+\frac{(1/2+1/(8K))Cr^2}{(1+Cr^2)^2}\\&-\frac{(K-1)}{K(1+Cr^2)}+\frac{\beta^2Cr^2}{2K(1-K)(1+Cr^2)}\\&+\frac{5(1-K)Cr^2}{8K(1+Cr^2)^3}\Bigg].\end{aligned} \tag{29}$$

4 Physical analysis of the solutions

The physical validity of the charged fluid sphere (CFS) depends upon the following conditions (called reality conditions or energy conditions) inside and on the sphere '$r=a$' such that

(i) $\rho>0, 0\leq r\leq a$.
(ii) $p>0, r<a$.
(iii) $p=0, r=a$.
(iv) $dp/dr<0, d\rho/dr<0, 0<r<a$.
(v) $c^2\rho\geq p$ non-relativistic (NR) or $c^2\rho\geq 3p$ ultra-relativistic (UR).
(vi) The velocity of sound $(dp/d\rho)^{1/2}$ should be less than that of light throughout the CFS ($0\leq r\leq a$).

Beside the above the CFS is expected to join smoothly with the Nordstrom metric, which requires the continuity of e^λ, e^ν and q across the pressure free interface '$r=a$'.

$$\frac{(K+Ca^2)}{K(1+Ca^2)}=1-\frac{2m(a)}{a}+\frac{e^2}{a^2}, \tag{30}$$

$$y^2=\left(1-\frac{2m(a)}{a}+\frac{e^2}{a^2}\right), \tag{31}$$

$$q(a)=e, \tag{32}$$

$$p_{(r=a)}=0. \tag{33}$$

The conditions (30) and (32) are automatically satisfied to the preposition (11) however (31) and (33) can provide the unique values of arbitrary constants A and Bin each of the cases 1 to 3.

From the numerical analysis of $M(r)$ and C, it is observed that M decreases with increase in L and increases with increase in β^2. However the physical constraints in terms of the reality conditions have severely restricted the magnitude of mass. An attempt has been made to get maximum mass (effectively M/M_Θ) in each case. M and M_Θ denote mass of the model and solar mass respectively. Various physical quantities such as ρ, p, dp/dr, $d\rho/dr$, $c^2\rho-p$ (NR), $c^2\rho-3p$ (UR), $(dp/d\rho)^{1/2}$ (velocity of sound), j^4 (the only surviving component of the 4-current vector) and M/M_Θ have been calculated numerically for $0\leq x\leq 1$,

Table 1 The star of maximum mass with its radius corresponding to various values of the guiding parameters K, B and L

For $0 \le p \le c^2 p\&dp/dp \le c^2$ (NR)					For $0 \le 3p \le cp\&dp/dp \le c$ (UR)			
K	L	β	Radius (Km)	Max. M/M_Θ	L	β	Radius (Km)	Max. M/M_Θ
Case 1								
−6	0.29000	2.68000	16.83018	4.08517340	0.29000	2.68000	16.83018	4.0851734
−5.5	0.23000	2.40000	17.01503	4.87694340	0.23000	2.40000	17.01503	4.87694340
−5	0.06010	2.09000	14.72360	7.40523480	0.07180	2.09000	15.21157	7.26368240
−3	0.06900	2.18600	12.79387	7.40523480	0.06900	2.19700	12.53931	7.26368240
−2.5	0.10000	1.97420	13.72565	7.40523480	0.10000	1.99990	13.43589	7.26368240
−2.4	0.09000	2.12798	12.83409	7.40523480	0.09000	2.12798	12.58877	7.26368240
−2	0.11000	2.11000	12.81786	7.40523480	0.11000	2.11000	12.57285	7.26368240
−1.5	0.16500	1.95570	13.63324	7.40523480	0.16500	1.95570	13.37264	7.26368240
−1	0.24015	2.00300	13.77914	7.40523480	0.24015	2.00300	13.51575	7.26368240
−.8	0.29800	1.99000	14.31522	7.40523480	0.29800	1.99000	14.04159	7.26368240
−.791	**0.29900**	**2.00400**	**14.25290**	**7.40523480**	**0.29900**	**2.00400**	**13.98045**	**7.26368240**
−.700	0.34000	1.96000	14.89129	7.40523480	0.34000	1.96000	14.60664	7.26368240
−.400	0.50000	2.05900	16.53079	7.40523480	0.50000	2.05900	16.21480	7.26368240
Case 2								
−1.99	**0.290**	–	**19.34143**	**5.9391024**	**0.349**	–	**18.99689**	**5.0592442**
−1.5	0.350	–	19.92668	5.9019845	0.410	–	19.39786	4.9629789
−1	0.490	–	20.13283	5.0724358	0.510	–	19.83958	4.7402463
−.8	0.540	–	20.49044	5.0841766	0.560	–	20.13679	4.7221282
−.400	0.760	–	19.17011	3.4523376	0.760	–	19.17011	3.4523376
Case 3								
−1.99	**0.258**	**0.001**	**19.45322**	**6.4587585**	**0.3480**	**0.001**	**19.00406**	**5.0733663**
−1.5	0.3500	0.001	19.92668	5.9019844	0.4050	0.001	19.44768	5.0373489
−1	0.4700	0.001	20.40950	5.4170270	0.5010	0.001	19.97359	4.8882233
−.8	0.5400	0.310	20.54452	5.0884818	0.5570	0.001	20.19098	4.7755715
−.400	0.7481	0.090	19.60503	3.7251923	0.7481	0.090	19.60503	3.7251923

$M_\Theta = 1.475$ Km, $G = 6.673 \times 10^8$ cm^3/gm sec^2, $c = 2.997 \times 10^{10}$ cm/sec

$(x = r/a)$ and the data so obtained is analysed subject to the reality conditions. The data for the various cases reveals the following informations:

Case 1.

It is observed that the maximum mass first increases and then decreases with the increase of β for $0 < L < mL$. However it decreases with the increase of β when $mL < L < 1$, where $.4 < mL < .53$. mL increases with decrease in K. In order to keep j^4 physically valid the K and β form a relation $\beta^2 > (2+K)(K-1)/4$. It is easily seen that for large negative K, β has to be taken large but reality conditions severely restricts the Maximum mass. Hence, Maximum mass decreases appreciably for $K \ll 0$. Pressure and density become very large negative values of K. The maximum mass $7.26368240 M_\Theta$ is possible for (UR) $K = -0.7910$, $L = 0.299$, $\beta = 2.004$, $a = 14.25290$ Km. When maximum mass is greater than $7.26368240 M_\Theta$ then velocity of sound decreases towards pressure free interface.

Case 2.

The j^4 remains physically valid only for $K > -2$. This case provides maximum mass e.g. $5.9391024 M_\Theta$ (NR) for $K = -1.99$, $L = 0.290$, $a = 19.34143$ Km and $5.0592442 M_\Theta$ (UR) for $K = -1.99$, $L = 0.349$, $a = 18.99689$ Km. Although the maximum mass increases with decrease in K.

Case 3.

Mass decreases with increase in charge for $L < 0.5$ while increases with increase in charge for $L > 0.5$. In order to keep j^4 physically valid the K and β form a relation $\beta^2 < (2+K)(1-K)/4$. This implies that physical solutions are valid only for $K > -2$. This case provides maximum mass e.g. $6.4587585 M_\Theta$ (NR) for $K = -1.99$, $L = 0.258$,

$\beta = .001$, $a = 19.45322$ Km. and $5.0733663 M_{\Theta}$ (UR) for $K = -1.99$, $L = 0.3480$, $\beta = .001$, $a = 19.00406$ Km. As in previous case the maximum mass increases with decrease in K.

5 Conclusion

The authors have tried to develop some models of Vadiya-Tikekar type charged fluid spheres considering more general electric field intensity. The three solutions have been analysed numerically after joining them smoothly with the Reissner-Nordstrom Metric at the pressure free boundary. And hence discussed the limitations on the guiding parameters, K, β and L subject to the prescribed energy conditions. The sample values of mass M/M_{Θ}, and radii are depicted on Table 1 under the heading ultra-relativistic and non-relativistic' conditions. One of the graphs shows variation of M/M_{Θ} for different values of K. Also the graphs for pressure, density, j^4 and velocity of sound are being traced for each case. Overall maximum mass for the charged fluid spheres of maximum mass M found to be $7.26368240 M_{\Theta}$ for strong energy condition. The corresponding radius is given as 14.25290 Km. When maximum mass is greater than $7.26368240 M_{\Theta}$ then velocity of sound decreases towards pressure free interface.

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