Astrophys Space Sci (2011) 334:225–230
DOI 10.1007/s10509-011-0725-7

ORIGINAL ARTICLE

Some enigmatic aspects of the early universe

C. Sivaram · Kenath Arun

Received: 29 March 2011 / Accepted: 27 April 2011 / Published online: 28 May 2011

Abstract Matter collapsing to a singularity in a gravitational field is still an intriguing question. Similar situation arises when discussing the very early universe or a universe recollapsing to a singularity. It was suggested that inclusion of mutual gravitational interactions among the collapsing particles can avert a singularity and give finite value for various physical quantities. We also discussed how inclusion of large dark energy term compensates for the net gravity. The discussion is taken further by including the effects of charge, magnetic fields and rotation. The role of large extra dimensions under the extreme initial conditions is discussed and possible connection with the cyclic brane theory is explored. We constrain various cosmic quantities like the net charge, number density of magnetic monopoles, primordial magnetic fields, size of the extra dimensions, etc. We are also able to arrive at the parameters governing the observed universe.

Keywords Singularity · Mutual gravitational interactions · Dark energy · Magnetic monopoles · Primordial magnetic fields

It is now generally accepted that the universe went through a very hot dense phase and many of the predictions of this standard evolutionary model, including the anisotropies (Mather 2007; Smoot 2007) and the light element abundances (Steigman 2007) are now well established (Peacock 1991). However there are still some enigmatic (Sivaram 1988) aspects of the earliest phase of the universe, which are only partially explained by models such as those involving inflation (Guth 1981).

In a recent paper (Sivaram and Arun 2011) it was also suggested that inclusion of mutual gravitational interactions among the particles in the early dense universe can lead to a 'pre-big bang' scenario, with particle masses greater than the Planck mass implying an accelerating phase of the universe, which then goes into the radiation phase when the masses fall below the Planck mass.

The existence of towers of states of such massive particles (i.e. multiples of Planck mass) as implied in various unified theories, provides rapid acceleration in the early universe, similar to the usual inflation scenario, but here the expansion rate goes over 'smoothly' to the radiation dominated universe when temperature becomes lower than the Planck temperature.

In order to include the possible effects of dark energy on charge, rotation and magnetic fields of individual objects we consider a few exact solutions of Einstein's equation. Now the exact solution for the metric of a particle of mass m in general relativity is given by the Schwarzschild solution as:

$$dS^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{dr^2}{(1 - \frac{2GM}{rc^2})} - r^2 d\Omega^2 \qquad (1)$$

The solution for charged massive particle is given by the Reissner-Nordstrom solution as (Misner et al. 1973):

$$dS^2 = \left(1 - \frac{2GM}{rc^2} + \frac{Ge^2}{c^4 r^2}\right) c^2 dt^2 - \frac{dr^2}{(1 - \frac{2GM}{rc^2} + \frac{Ge^2}{c^4 r^2})} - r^2 d\Omega^2 \qquad (2)$$

C. Sivaram
Indian Institute of Astrophysics, Bangalore, 560 034, India
e-mail: sivaram@iiap.res.in

K. Arun (✉)
Christ Junior College, Bangalore, 560 029, India
e-mail: kenath.arun@cjc.christcollege.edu

From the above solution, for a massless charged particle, $g_{00} = 1 + \frac{Ge^2}{c^4 r^2} = 0$ would imply that $r = \sqrt{-G}e/c^2$, that is a naked singularity will arise. Due to this it is argued that a massless particle cannot have charge (Xiang and Shen 2005).

But these zero rest mass particles will have a mass due to their electric charge (Landau and Lifshitz 1963). The electric energy for a charge e of size r is given by: $E = \frac{e}{r^2}$.

And the corresponding energy density is given by: $\frac{E^2}{8\pi} = \frac{e^2}{8\pi r^4}$.

The total energy density associated with this charge is: $\varepsilon = \int_r^\infty \frac{e^2}{8\pi r^4} 4\pi r^2 dr = \frac{e^2}{2r}$.

Therefore the mass due to the electric charge is given by: $M = \frac{e^2}{2rc^2}$.

From this we have: (cf. (2))

$$g_{00} = 1 - \frac{2G}{rc^2}\left(\frac{e^2}{2rc^2}\right) + \frac{Ge^2}{c^4 r^2} \tag{3}$$

The last two terms cancels and hence we have in this case: $g_{00} = 1$, that is it gives a flat space-time. Thus if a particle's mass is only due to its electric charge (like perhaps for the classical electron), then it will not bend the space-time around it!

Including the dark energy term, assumed to be just the cosmological constant, Λ, (Sivaram 1979) the solution is given by the Reissner-Nordstrom de-sitter solution (also called the Kottler metric) as:

$$dS^2 = \left(1 - \frac{2GM}{rc^2} + \frac{Ge^2}{c^4 r^2} - \frac{\Lambda r^2}{3}\right) c^2 dt^2 - \frac{dr^2}{(1 - \frac{2GM}{rc^2} + \frac{Ge^2}{c^4 r^2} - \frac{\Lambda r^2}{3})} - r^2 d\Omega^2 \tag{4}$$

where,

$$g_{00} = 1 - \frac{2GM}{rc^2} + \frac{Ge^2}{c^4 r^2} - \frac{\Lambda r^2}{3} \tag{5}$$

If the cosmological constant term compensates for the charge in (5), then as for the electron $\frac{2GM}{rc^2} \ll 1$, we have, $r = (\frac{3Ge^2}{c^4 \Lambda})^{1/4}$.

That is, the last two terms in (5) become equal at this value of r (where $\Lambda = 10^{-56}$ cm^{-2}). For an electron this works out to be of the order of 10^{-3} cm.

This suggests that the electrostatic energy density around a single electron becomes comparable to the cosmological constant energy density at sub-millimetre distances. This could be testable in an experiment involving single electrons in devices like an ion trap.

If we consider the effects of quantum vacuum fluctuation term, where energy density is given by $\frac{\beta \hbar c}{r^4}$, where $\beta \sim 1$ (like in the case of Casimir force), the metric then becomes

$$g_{00} = 1 - \frac{2GM}{rc^2} + \frac{\beta G \hbar c}{c^4 r^2} - \frac{\Lambda r^2}{3}$$

As in the earlier case, the last two terms become comparable for a radial distance given by:

$$r = \left(\frac{3G\hbar}{c^3 \Lambda}\right)^{1/4} \tag{6}$$

Which again works out to be $\sim 3 \times 10^{-3}$ cm.

For a spinning body we get a similar expression as (6), that is,

$$r = \left(\frac{3GJ}{c^3 \Lambda}\right)^{1/4} \tag{7}$$

where J is the angular momentum.

From (7) we see that:

$$\frac{J}{r^4} = \frac{c^3}{3G}\Lambda \tag{8}$$

which is a constant. We have already shown (for a very large range of structures in the universe) that the following relation holds. That is (Sivaram 1992, 2008):

$$\frac{M}{r^2} = \frac{c^2}{G}\sqrt{\Lambda} \tag{9}$$

which is again a constant. Therefore (8) and (9) together imply that $J \propto M^2$. This is consistent with what is actually observed for a wide range of astronomical structures (Sivaram 1984, 1987a).

For the cosmological case (where the total action is $\sim J = 10^{120}\hbar$), r then corresponds to the size of the universe, that is:

$$r = \left(\frac{3GJ}{c^3 \Lambda}\right)^{1/4} = (3 \times 10^{-3}\ \text{cm})(10^{120})^{1/4} \sim 10^{28}\ \text{cm} \tag{10}$$

In the case of a typical galaxy, $J = 10^{100}\hbar$, this implies:

$$r = \left(\frac{3GJ}{c^3 \Lambda}\right)^{1/4} = (3 \times 10^{-3}\ \text{cm})(10^{100})^{1/4} \sim 3 \times 10^{22}\ \text{cm} \tag{11}$$

In the case of a star, $J = 10^{76}\hbar \Rightarrow r \sim 10^{16}$ cm, which is the expected size of the nebula (interstellar cloud) from which the star formed (Sivaram 1984, 1994a, 1994b; Taylor 1994).

We can see that the dark energy term goes as $1/r^4$ in the metric, so for the recollapsed universe as a whole at $r \sim$

10^{-3} cm, the cosmological constant term is of the order of $\Lambda = \Lambda_{Pl} \sim 10^{66}$ cm^{-2}, (that is in the final stage of collapse) and this is consistent with earlier results (Sivaram 1986; Sivaram et al. 2008a).

At the present epoch, according to the above relation:

$$\Lambda = \Lambda_{Pl}\left(\frac{3\times 10^{-3}}{10^{28}}\right)^4 \sim 10^{-56}\ \mathrm{cm}^{-2} \tag{12}$$

which corresponds to the observed value (Krause 2004)!

But r scales with time as: $r \propto t^{1/2}$ in the radiation era, this implies that $\Lambda \propto \frac{1}{t^2}$.

At the present epoch, that is 10^{18} s (Hubble time), the dark energy term is given by:

$$\Lambda = \Lambda_{Pl}\left(\frac{10^{-43}}{10^{18}}\right)^2 \sim 10^{-56}\ \mathrm{cm}^{-2} \tag{13}$$

This is again in agreement with what is observed. We can think of what the implications for a closed universe, recollapsing to a finite size of $r \approx 10^{-3}$ cm are, as implied by the above equations and also elaborated in earlier parts (Sivaram 1986; Sivaram et al. 2008b).

We can try and interpret in what follows. We note that in many current models of unification of interactions, there are large extra dimensions of sub-millimetre ($r \approx 10^{-3}$ cm) size (Arkani-Hamed et al. 1998).

The zero point energy (ZPE) fluctuations at the boundary between our 3-space and the extra dimension can give rise to a vacuum energy density in our universe.

L^{-1} gives a wave number (k) cut-off so that vacuum energy density propagating into our universe gives rise to an effective cosmological constant Λ obtained as (Sivaram 1999):

$$\Lambda \approx \frac{L_{Pl}^2}{L_{EW}^4 (L_{EW}/L_{Pl})^{8/n}} \tag{14}$$

where n is the number of extra dimensions and L_{Pl} is the familiar Planck length. For $n = 2$, we have (Sivaram 1992, 1999):

$$\Lambda \approx \frac{L_{Pl}^6}{L_{EW}^8} = \frac{\hbar^7 G^3}{G_F^4 c^5} \sim 10^{-56}\ \mathrm{cm}^{-2} \tag{15}$$

where, $L_{Pl} = (\hbar G/c^3)^{1/2}$, $L_{EW} = (G_F/\hbar c)^{1/2} = 7\times 10^{-17}$ cm is the beta decay length and $G_F = 1.5\times 10^{-49}$ erg cm^3 is the Fermi constant.

This precisely gives the size of the extra dimensions as $L \approx 0.01$ cm. This value of L can also be understood as the Casimir energy between two branes (separated by L) balancing the repulsive cosmic density (Sivaram 1992; Weinberg 1996):

$$L \approx \left(\frac{8\pi G\hbar c}{240\Lambda c^4}\right)^{1/4} \sim 0.01\ \mathrm{cm} \tag{16}$$

Size of the extra dimension L, and their number n, are independent parameters but in order to achieve equality of strength of gravity and electroweak force at $L_{EW} \sim 10^{-19}$ m, they become constrained by $L = 10^{(32/n)-19}$. This is in the spirit of models with large extra dimensions.

For large n, strength of gravity grows very rapidly at microscopic length and becomes comparable to the electroweak force at $\sim 10^{-19}$ m (Arkani-Hamed et al. 1998; Randall and Sundrum 1999).

The size of the extra dimensions is given by:

$$R_C = \frac{\hbar c}{M_W c^2}\left(\frac{M_{Pl}}{M_{EW}}\right)^{2/n} \sim L_{EW}^n \times 10^{32} \tag{17}$$

For $n = 1$, the size of the extra dimension is given by, $R_C \sim 10^{-17}\times 10^{32} = 10^{15}$ cm, which is about the size of the solar system and there are no observed deviations from the usual 4-dimensional space in this scale, hence $n = 1$ is not a possible number for a large extra dimension.

For $n = 2$, the size of the extra dimension is given by, $R_C \sim 10^{-3}$ cm, which is testable and is indeed being explored in numerous experiments (Hoyle et al. 2001).

So if the universe recollapses to $L \sim 10^{-3}$ cm, it can tunnel into this extra dimension given by (17) to another spacetime and reexpand.

The force per unit area or the energy density between two branes given by (Randall and Sundrum 1999; Manzoni 2001):

$$\frac{F}{A} = \varepsilon = \frac{\hbar c}{240\pi d^4} \tag{18}$$

In terms of the cosmological constant it is given by: $\varepsilon = \frac{\Lambda c^4}{8\pi G}$, where we have effectively:

$$\Lambda = \frac{\hbar^7 G^3}{G_F^4 c^5} \tag{19}$$

Therefore the energy density is given by:

$$\varepsilon = \frac{\hbar^7 G^2}{8\pi G_F^4 c} \tag{20}$$

This has the value of $\sim 10^{-8}$ erg s/cm^3, precisely what is observed from the dark energy.

According to general relativity, the maximum force is $F_{\max} = \frac{GM^2}{R_{\min}^2}$, where, $R_{\min} \approx \frac{GM}{c^2}$. This implies that $F_{\max} = \frac{c^4}{G}$.

Therefore the maximum area to which it can expand, under the pressure (energy density) given by (20) is given by maximum force/energy density. That is given by

$$A_{\max} = \frac{8\pi G_F^4 c\cdot \frac{c^4}{G}}{\hbar^7 G^2} = \frac{8\pi G_F^4 c^5}{\hbar^7 G^3} \approx 10^{56}\ \mathrm{cm}^2 \tag{21}$$

And the corresponding size is given by $R = 10^{28}$ cm which is the Hubble radius.

So we have a possible scenario to explain the present scale of the expanding universe.

From (20), the energy density is given by: $\varepsilon = \frac{\hbar^7 G^2}{8\pi G_F^4 c} \sim 10^{-8}$ erg s/cm^3.

The energy associated with the total volume is therefore given by:

$$E_\Lambda = \frac{\hbar^7 G^2}{8\pi G_F^4 c} \times 2\pi^2 \Lambda^{-3/2} = \frac{\pi G_F^2 c^{13/2}}{4\hbar^{7/2} G^{5/2}} \sim 2 \times 10^{77}\ \text{erg s} \tag{22}$$

where $2\pi^2\Lambda^{-3/2}$ is the volume. This energy quantifies the total dark energy in the universe. The corresponding mass is given by:

$$M_\Lambda = \frac{E_\Lambda}{c^2} = \frac{\pi G_F^2 c^{9/2}}{4\hbar^{7/2} G^{5/2}} \sim 2 \times 10^{56}\ \text{g} \tag{23}$$

The maximum power associated with the baryonic matter is given by general relativity as c^5/G. This can be seen as follows.

The total energy released is $\sim \frac{GM^2}{R}$, and the total power is given by:

$$P \approx \frac{GM^2}{Rt} \tag{24}$$

In general relativity, the smallest time scale (or smallest length scale) associated with a given mass is $t_{\min} \approx \frac{GM}{c^3}$ (for R corresponding to the gravitational radius of $R \approx \frac{GM}{c^2}$).

This gives the maximal power as:

$$P \approx \frac{GM^2}{Rt} \approx \frac{c^5}{G} \tag{25}$$

(Substituting the minimal values for R and t as given above.)

So if we have a given set of objects of total baryonic mass (ΣM), generating radiation energy we can write for the combined maximal Eddington luminosity as (Sivaram 1982; Rees 1978):

$$\frac{4\pi G(\Sigma M) c m_P}{\sigma_T} = \frac{c^5}{G} \tag{26}$$

where, m_P is the proton mass and σ_T is the Thomson cross section and $\Sigma M = M_b$ gives the total baryonic mass, which generates the radiation luminosity.

This gives (Sivaram 1982):

$$M_b = \frac{\sigma_T c^4}{4\pi G^2 m_P} \sim 9 \times 10^{54}\ \text{g} \tag{27}$$

From (23) and (27) we can see that:

$$\frac{M_\Lambda}{M_b} \sim 25 \tag{28}$$

This is consistent with observations (Sivaram 2006).

The maximum area to which this mass can expand is given by (21) as $A_{\max} = \frac{8\pi G_F^4 c^5}{\hbar^7 G^3}$. And the mass by area is given by:

$$\frac{M_\Lambda}{A} = \frac{\pi G_F^2 c^{9/2}}{4\hbar^{7/2} G^{5/2}} \times \frac{\hbar^7 G^3}{8\pi G_F^4 c^5} = \frac{\hbar^{7/2} G^{1/2}}{32 G_F^2 c^{1/2}} \sim 1\ \text{g/cm}^2 \tag{29}$$

This is consistent with the scaling relations for the mass as obtained for a wide range of self-similar structures in the universe, right from the scale of the electron to that of the entire universe (Sivaram 1992, 2006, 2008).

We can also arrive at the maximum magnetic field possible at the present epoch based on the results obtained above. The maximum energy density due to the magnetic field at the Planck epoch is given by:

$$\frac{B^2}{8\pi} = \rho_{Pl} = \frac{(\frac{\hbar c^5}{G})^{1/2}}{\frac{4}{3}\pi(\frac{\hbar G}{c^3})^{3/2}} \approx 10^{114}\ \text{erg s/cm}^3 \tag{30}$$

The corresponding maximum field is then given by (Sivaram 2000; Zel'dovich et al. 1966):

$$B_{\max} = (8\pi \times 10^{114})^{1/2} \approx 10^{57}\ \text{G} \tag{31}$$

Since the flux (BR^2) is conserved we can calculate the magnetic field for the present epoch. That is:

$$B_0 R_0^2 = B_{\max} R_{\min}^2 \tag{32}$$

where $R_{\min} = 10^{-3}$ cm is the size corresponding at the Planck epoch.

The magnetic field at the present epoch is then given by:

$$B_0 = 10^{57}\left(\frac{R_{\min}}{R_0}\right)^2 \approx 10^{-6}\ \text{G} \tag{33}$$

This is the maximum possible magnetic field at the present epoch. But the microwave background sets constraints on the energy density due to this magnetic field. At BBN era corresponding to time of 1 second, the size of the universe and the temperature are of the order of 10^{18} cm and 10^{10} K.

The corresponding energy density due to the radiation is given by:

$$\rho_{rad} = aT^4 \approx 10^{26}\ \text{erg s/cm}^3 \tag{34}$$

The microwave background sets a constraint that the energy density due to the magnetic field be less than 5% of that given by (34) (Sivaram 2006). That is

$$\rho_{mag} = \frac{B^2}{8\pi} \approx 10^{24}\ \mathrm{erg\,s/cm^3} \tag{35}$$

The maximum magnetic field of 10^{11} G corresponds to temperature of 10^{10} K and size of 10^{18} cm. Since the flux is conserved, the field at the present epoch is given by:

$$B_0 = 10^{11}\left(\frac{10^{18}}{10^{28}}\right)^2 \approx 10^{-9}\ \mathrm{G} \tag{36}$$

This is consistent with the observed Faraday rotation, in extragalactic structures (Zel'dovich et al. 1966).

We have seen from (31) that the maximum magnetic field at the Planck epoch is of the order of $\sim 10^{57}$ G and the corresponding flux is given by:

$$BR^2_{\min} \approx 10^{52}\ \mathrm{G\,cm^2} \tag{37}$$

where $R_{\min} = 10^{-3}$ cm is the size corresponding at the Planck epoch.

The unit quantum of flux is given by (for example see Feynman Lectures Vol. III, 1965):

$$\frac{\hbar c}{2e} \sim 10^{-8} \quad \text{(in cgs units)} \tag{38}$$

From (37) and (38) we see that there are 10^{60} units of flux quanta. This can be interpreted as the maximum number of monopoles. This can be explained as follows. From (33) we see that the maximum magnetic field allowed at the present epoch is $\sim 10^{-6}$ G, but from the consideration of conservation of flux we arrived at the field at the present epoch as $\sim 10^{-9}$ G (36) which is consistent with observation. This could be because the remaining energy is trapped in these monopoles.

Therefore the 10^{60} units of flux quanta can be interpreted as the maximum number of monopoles. Since the flux (number of monopoles) is conserved, there are 10^{60} monopoles in the present volume of $2\pi^2\Lambda^{-3/2} \sim 2\times 10^{85}\ \mathrm{cm}^3$.

That is the monopole density at the present epoch is:

$$\sim \frac{10^{60}}{2\times 10^{85}} \sim 5\times 10^{-26}/\mathrm{cc} \tag{39}$$

Or the monopole flux is given by:

$$\sim 5\times 10^{-26} \times \frac{c}{4} \approx 10^{-16}\ \mathrm{cm^{-2}\,s^{-1}} \tag{40}$$

This matches with the bound on the monopole flux of $10^{-16}\ \mathrm{cm^{-2}\,s^{-1}}$ set by Parker from independent considerations (Sivaram 1987b).

We had for the single electron charge, the radius as:

$$r = \left(\frac{3Ge^2}{c^4\Lambda}\right)^{1/4} \approx 10^{-3}\ \mathrm{cm} \tag{41}$$

For electron charge e and $\Lambda \approx 10^{-56}\ \mathrm{cm}^{-2}$ (that is the present value).

In the initial stage, we had for the whole universe, $\Lambda = \Lambda_{Pl}$ and the corresponding size works out to be:

$$r = \left(\frac{3Gq^2}{c^4\Lambda_{Pl}}\right)^{1/4} \approx 10^{-3}\ \mathrm{cm} \tag{42}$$

Comparing (41) and (42), it follows that since r is of comparable magnitude, $q^2 \approx 10^{120}e^2$, giving:

$$q \approx 10^{60}e \tag{43}$$

This gives the maximal value of the total electric charge we can have. As the radius encompasses 10^{90} particles, this would imply that net charge (in electron unit) is 1 part in 10^{30} (Sivaram and Arun 2008; Altschul 2007).

So it appears interesting that the upper limit of the number of magnetic flux quanta ($\sim 10^{60}$) is also the same as the upper limit on the number of electric charge. This suggests a electric-magnetic charge duality in the spirit of Dirac (who explained quantification of electric charge on the basis of existence of magnetic monopole via the quantisation condition $eg \approx \hbar c$, where g is the magnetic charge) (Dirac 1931; Shellard 1994; Schwinger 1969).

References

Altschul, B.: Phys. Rev. Lett. **98**, 261801 (2007)

Arkani-Hamed, N., et al.: Phys. Lett. B **429**, 263 (1998)

Dirac, P.A.M.: Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. **133**, 60 (1931)

Guth, A.: Phys. Rev. D **23**, 347 (1981)

Hoyle, C.D., et al.: Phys. Rev. Lett. **86**, 418 (2001)

Krause, L.M.: Astrophys. J. **604**, 481 (2004)

Landau, L., Lifshitz, E.: Classical Theory of Fields. Pergamon, New York (1963)

Manzoni, L.: Phys. Lett. B **508**, 337 (2001)

Mather, J.C.: Rev. Mod. Phys. **79**, 1331 (2007)

Misner, C., Thorne, K., Wheeler, J.: Gravitation. Freeman, San Francisco (1973)

Peacock, J.: Cosmological Physics. Cambridge University Press, Cambridge (1991)

Randall, C., Sundrum, R.: Phys. Rev. Lett. **83**, 3370 (1999)

Rees, M.: Nature **275**, 35 (1978)

Schwinger, J.: Science **165**, 757 (1969)

Shellard, V.A.: Cosmic String and Other Topological Defects. Cambridge University Press, Cambridge (1994)

Sivaram, C.: Phys. Rep. **51**, 111 (1979)

Sivaram, C.: Astrophys. Space Sci. **86**, 501 (1982)

Sivaram, C.: In: de Sabbata, V., Karade, T.M. (eds.) Proc. of Sir Arthur Eddington Centenary Symposium, Rel. Astrophys. and Cosmology, pp. 228–244 (1984)

Sivaram, C.: Astrophys. Space Sci. **125**, 189 (1986)
Sivaram, C.: Nature **327**, 105 (1987a)
Sivaram, C.: Earth Moon Planets **37**, 155 (1987b)
Sivaram, C.: Astrophys. Space Sci. **140**, 403 (1988)
Sivaram, C.: Gravity Essay, Unpublished (1992)
Sivaram, C.: Astrophys. Space Sci. **215**, 186 (1994a)
Sivaram, C.: Astrophys. Space Sci. **219**, 135 (1994b)
Sivaram, C.: Mod. Phys. Lett. **14**, 2363 (1999)
Sivaram, C.: Curr. Sci. **79**, 413 (2000)
Sivaram, C.: In: Saha, S.K., Rastogi, V. (eds.) 21st Century Astrophysics (2006)
Sivaram, C.: preprint, arxiv:0801.1218 (astro-ph) (2008)
Sivaram, C., Arun, K.: preprint, arxiv:0802.1562 (astro-ph) (2008)
Sivaram, C., Arun, K.: Astrophys. Space Sci. (2011, in press). doi:10.1007/s10509-011-0670-5
Sivaram, C., Arun, K., Reddy, V.: preprint, arxiv:0804.2637 (astro-ph) (2008a)
Sivaram, C., Arun, K., Reddy, V.: preprint, arxiv:0806.4023 (astro-ph) (2008b)
Smoot, G.F.: Rev. Mod. Phys. **79**, 1349 (2007)
Steigman, G.: Primordial nucleosynthesis. In: Kayser et al. (eds.) Ann. Rev. Nucl. Part. Phys., p. 463 (2007)
Taylor, R.J.: The Stars: Their Structure and Evolution. Cambridge University Press, Cambridge (1994)
Weinberg, S.: Quantum Theory of Fields, vol. I. Cambridge University Press, Cambridge (1996) (see Casimir Effect)
Xiang, L., Shen, Y.: Mod. Phys. Lett. A **20**, 1823 (2005)
Zel'dovich, Ya.B., Ruzmaikin, A.A., Sokoloff, D.D.: Primordial Magnetic Field. Academic Press, New York (1966)