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ORIGINAL ARTICLE

Effect of q-nonextensive distribution of electrons on ion acoustic shock waves in dissipative plasma

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Abstract Ion acoustic shock waves (IASWs) are studied in a plasma consisting of nonextensive electrons and ions. The dissipation is taken into account the kinematic viscosity among the plasma constituents. The Korteweg-de Vries-Burgers (KdV-Burgers) equation is derived by reductive perturbation method. Shock waves are solutions of KdV-Burgers equation. It is shown that acceptable values of q-parameter (where q stands for the electron nonextensive parameter) are more than 3 in a weakly nonlinear analysis. We have found that the amplitude of shock waves decreases by an increasing q-parameter.

Keywords Shock waves · Ion acoustic · Nonextensive electrons

1 Introduction

Ion-acoustic shock waves and solitons in collisionless unmagnetized plasmas have been extensively studied in recent years. Nonlinear ion acoustic shock waves are amazing manifestation of nature, arising out of competition between properties like nonlinearity, dispersion and dissipation. The ion-acoustic solitons, which are formed due to balance between nonlinearity and dispersion, are described by the Korteweg–de Vries (KdV) equation, which contains the lowest order nonlinear and dispersive terms (Washimi and Taniuti 1966). The first experimental observation of ion-acoustic solitons was made by Ikezi et al. (1970) where the temperature ratio of electrons to ions (T_e/T_i) is high enough so that Landau damping could be safely neglected. However, in some situations it is difficult to neglect the energy dissipation mechanism caused either by collisionless Landau damping or by some collisional processes. Ion-acoustic shocklike waves were observed in the same double plasma device by applying a ramp voltage to the anode of the driver plasma (Taylor et al. 1970b). Observations made by the Viking spacecraft (Bostrom 1992) and Freja satellite (Dovner et al. 1994) have found electrostatic solitary structures in the magnetosphere with density depressions. Ion-acoustic shock waves were first observed in a novel plasma device called Double Plasma device (Taylor et al. 1970a). Asif Shah et al. (2009) have derived the Korteweg–de Vries–Burger (KdVB) equation ion acoustic shock waves in a weakly relativistic electron–positron–ion plasma. During the last two decades, Space plasma observations indicate clearly the presence of electron populations which are far away from their thermodynamic equilibrium. In a interesting and influential paper Gougam et al. (2011) showed that the presence of a nonextensive distribution of electrons changes the nature of ion acoustic solitary structures. Astrophysical electron-nuclear plasmas are properly described by nonextensive distributions of metastable states. Leubner (2002, 2004a, 2004b) shows that distributions very close to kappa-distributions are a consequence of the generalized entropy favored by nonextensive statistics, which provides the missing link for power-law models of non-thermal features from fundamental physics. Nonextensive statistics was successfully applied to a number of astrophysical and cosmological scenarios. Those include stellar polytropes (Plastino and Plastino 1993), the solar neutrino problem (Kaniadakis et al. 1996), peculiar velocity distribu-

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tions of galaxies (Lavagno et al. 1998) and generally systems with long range interactions and fractal like spacetimes. Cosmological implications were discussed (Torres et al. 1997) and recently an analysis of plasma oscillations in a collisionless thermal plasma was provided from q-statistics (Lima et al. 2000). On the other hand, kappa-distributions are highly favored in any kind of space plasma modeling (Mendis and Rosenberg 1994) where a reasonable physical background was not apparent. A comprehensive discussion of kappa distributions in view of experimentally favored non-thermal tail formations is provided by Leubner and Schupfer (2000) where also typical values of the index κ are quoted and referenced for different space plasma environments. In the present analysis the missing link to fundamental physics is provided within the framework of an entropy modification consistent with nonextensive statistics. The family of kappa distribution is obtained from the positive definite part $12 \le \kappa \le \infty$, corresponding to $-1 \le q \le 1$ of the general statistical formalism where in analogy the spectral index kappa is a measure of the degree of nonextensivity. Since the main theorems of the standard Maxwell-Boltzmann statistics admit profound generalizations within nonextensive statistics (Plastino et al. 1994; Rajagopal 1995, 1996; Chame and Mello 1997; Lenzi et al. 1998), a justification for the use of kappa-distributions in astrophysical plasma modeling is provided from fundamental physics. The aim of the present paper is study ion-acoustic shock waves in a plasma having nonextensive electrons and ions in the existence of the kinematic viscosity.

2 Derivation of KdV Burger equation and discussion

Consider a two-component plasma with cold ions and nonextensive electrons. We assume that the phase velocity of ion-acoustic wave is much smaller than the electron thermal velocities and larger than the ion thermal velocity, so we therefore ignore the electron inertia. The equilibrium charge neutrality condition is $n_{e0} = n_{i0}$ where n_{e0} and n_{i0} denotes unperturbed number densities of electrons and ions, respectively. The dynamics of ion acoustic waves in such plasmas are described by the following normalized equations

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0$$
$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} - \eta \frac{\partial^2 u_i}{\partial x^2} = 0 \tag{1}$$
$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i$$

where n_i and n_e are the ion and electron number densities, respectively. The densities of the plasma species are normalized by the unperturbed electron density n_{e0}. u and ϕ are the ion fluid velocity and the electrostatic potential, respectively. u is normalized by the ion acoustic speed $c_s = \sqrt{T_e/m_i}$ and ϕ is normalized by (T_e/e), where m_i is the ion mass, T_e is the electron temperature and e is the electron charge. The time t and the distance x are normalized by the ion plasma frequency $\omega_{pi}^{-1} = \sqrt{m_i/4\pi n_{i0} e^2}$ and by the electron Debye length $\lambda_D = \sqrt{T_e/4\pi n_{e0} e^2}$, respectively. The coefficient of kinematic viscosity η is incorporated in the parameter $\eta = \frac{\eta}{\lambda_D c_s}$.

To model the effects of nonextensive electrons, we have (Gougam and Tribeche 2011; Amour and Tribeche 2010)

$$n_e = [1 + (q-1)\phi]^{\frac{q+1}{2(q-1)}} \tag{2}$$

where, the parameter q stands for the strength of nonextensivity. In the extensive limiting case ($q = 1$), distribution (2) reduces to the well-known Maxwell-Boltzmann distribution.

In (1), the densities of the plasma species are normalized by their unperturbed densities, the ion velocity is normalized by the ion acoustic speed $c_i = \sqrt{T_e/m}$, space variables (x) are normalized by the electron Debye length $\lambda_D = \sqrt{T_e/4\pi n_\circ e^2}$, time variable is normalized by the electron plasma period $T = \sqrt{m_e/4\pi n_{e\circ} e^2}$ and electrostatic potential is normalized by the quantity (T_e/e). The coefficient of kinematic viscosity η is incorporated in the parameter, $\eta = \frac{\eta}{\lambda_{Dcs}}$. In order to investigate the propagation of ion acoustic shock waves and to derive the required *KdVB* equation in our e-p-i plasma, the independent variables are stretched as

$$\xi = \varepsilon^{1/2}(x - \lambda t), \qquad \tau = \varepsilon^{3/2} t, \qquad \eta = \varepsilon^{1/2}\eta_\circ \tag{3}$$

and the dependent variables are expanded as

$$n_i = 1 + \varepsilon n_1 + \varepsilon n_2$$
$$u_i = \varepsilon u_1 + \varepsilon u_2 \tag{4}$$
$$\phi = \varepsilon \phi_1 + \varepsilon \phi_2$$

where ε is a small parameter which characterizes the strength of the nonlinearity, and λ is the phase velocity of the wave. Now, substituting (3) and (4) into (1) and collecting the terms in the different powers of ε, we obtain the following equations at the lowest order of ε

$$n_1 = \frac{\phi_1}{\lambda^2}, \qquad u_1 = \frac{\phi_1}{\lambda}, \qquad \lambda = \sqrt{\frac{2}{q+1}} \tag{5}$$

and for the higher orders of ε, we have

$$-\lambda \frac{\partial n_2}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + \frac{\partial (n_1 u_1)}{\partial \xi} + \frac{\partial u_2}{\partial \xi} = 0$$

$$-\lambda\frac{\partial u_2}{\partial\xi}+\frac{\partial u_1}{\partial\tau}+\frac{\partial\phi_2}{\partial\xi}+u_1\frac{\partial u_1}{\partial\xi}-\eta_\circ\frac{\partial^2 u_1}{\partial\xi^2}=0 \tag{6}$$
$$\frac{\partial^2\phi_1}{\partial\xi^2}=\frac{(q+1)}{2}\phi_2+\frac{(3-q)}{8}\phi_1^2-n_2$$

Finally the KdV-Berger equation is derived from (5) and (6)

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}-C\frac{\partial^2\phi_1}{\partial\xi^2}=0 \tag{7}$$

where

$$A=\frac{3}{2\lambda}-\frac{(3-q)\lambda^3}{8},\qquad B=\frac{\lambda^3}{2},\qquad C=\frac{\eta_\circ}{2} \tag{8}$$

Equation (7) is the well known KdV Burger equation describing the nonlinear propagation of the ion acoustic shock waves in a plasma with q-nonextensive electrons. In this equation A and B are the nonlinear coefficient and dispersive term and the Burger term (C) arises due to the effect of ion kinematic viscosity. Note that in the absence of the viscosity term, (7) is reduced to a usual KdV equation for the propagation of ion acoustic solitary waves. The Burger term implies the possibility of the existence of a shock-like solution. If the dissipation term is negligible compared to the dispersion terms, then solitonic structure will be established by balancing the effects of dispersive and nonlinear terms. On the other hand, if the coupling becomes strong the shock waves will appear. The nature of these shock structures depends on the relative values between the dispersive and dissipative coefficients B and C, respectively. The KdV-Burger equation is widely used in plasma physics and theoretical physics.

The tangent hyperbolic (tanh) method is a powerful method for the computation of exact traveling wave solutions. This method analyzes treatment of these equations and analysis leads to find the traveling wave solutions of peculiar type nonlinear evolution equations.

More recently, Asif Shah et al. (2009) have calculated the monotonic shock waves solution theoretically by employing the tanh method (Wazwaz 2008). It is clear that we need to find traveling waves in the medium. The profile of a traveling wave does not change with a transformation like $\chi=\kappa(\xi-\upsilon\tau)$ in which "υ" is the wave velocity. Using this transformation in (7) changes the KdV-Burger equation to the following equation

$$-\upsilon\frac{\partial\phi_1}{\partial\chi}+A\phi_1\frac{\partial\phi_1}{\partial\chi}+B\frac{\partial^3\phi_1}{\partial\chi^3}-C\frac{\partial^2\phi_1}{\partial\chi^2}=0 \tag{9}$$

It is an ordinary differential equation. In the second step the transformation $Y=\tanh(\chi)$ is used and we have the following modified equation

$$\begin{aligned}&-\upsilon(1-Y^2)\frac{d\phi_1}{dY}+A\phi_1(1-Y^2)\frac{d\phi_1}{dY}\\&\quad+B(1-Y^2)\frac{d}{dY}\left[(1-Y^2)\frac{d}{dY}\left((1-Y^2)\frac{d}{dY}\right)\right]\phi_1\\&-C(1-Y^2)\frac{d}{dY}\left((1-Y^2)\frac{d}{dY}\right)\phi_1=0\end{aligned} \tag{10}$$

The above differential equation can be solved using regular series expansion as $\phi_1(Y)=\sum_{s=0}^{n=2}a_sY^s$. Now coefficients can be fixed with getting terms of left hand side terms equal to zero for different powers of "Y". Finally the following solution is derived

$$\phi_1=\frac{12B}{A}[1-\tanh^2\chi]-\frac{36C}{15A}\tanh\chi \tag{11}$$

Some numerical simulations have been performed for testing the evolution of (11) using (7). All simulations confirm validity of (11).

In order to study the transition from solitary to shock ion-acoustic waves, we can use (9) in our plasma model. Now we can investigate the properties of shock waves with numerical analysis of ϕ_1 using (9).

Because of using a weakly nonlinear analysis the potential (ϕ_1) has to ranging from -1 to $+1$. Therefore the shock amplitude does not exceed 1 and we have to choose the adequate parameters to maintain the weakly nonlinear nature of our analysis. The variation of the shock amplitude as a function of q can be studied by plotting the amplitude respect to q for the case of $\chi=0$ using (9). Figure 1 shows that the amplitude becomes more than 1 for $q<3$ as the shock wave structure will be failed to exist, thus the values $q<3$ are rule out from our model. Our results, plotted in Fig. 1, show that the admissible values of q are greater than 3 ($q>3$). It is also obvious that the amplitude decreases with an increasing value of q-parameter. This result consistent with the one made by Gougam et al. (2011) for ion acoustic solitary waves in e-i plasma with nonextensive electrons. It is shown that the amplitude of the solitary waves decreases in the electron-positron-ion plasmas (Shah and Saeed 2009). Therefore it can conclude the domain of allowable nonextensive q-parameters is expanded in the existence of positron. The effect of q-parameter on the behavior of shock profile has been shown in Fig. 2. It is clear the shock amplitude decreases when q increases. This figure also presents that the created peaks on the wave profile ($q=4$) disappear with greater values of q ($q=25$).

Figure 3 shows the behavior of shock profile with the fixed values of q-parameters and different values of $\eta_\circ$. This figure shows how soliton profile wave is reduced to shock structure. It is clear that the transformation from shock to

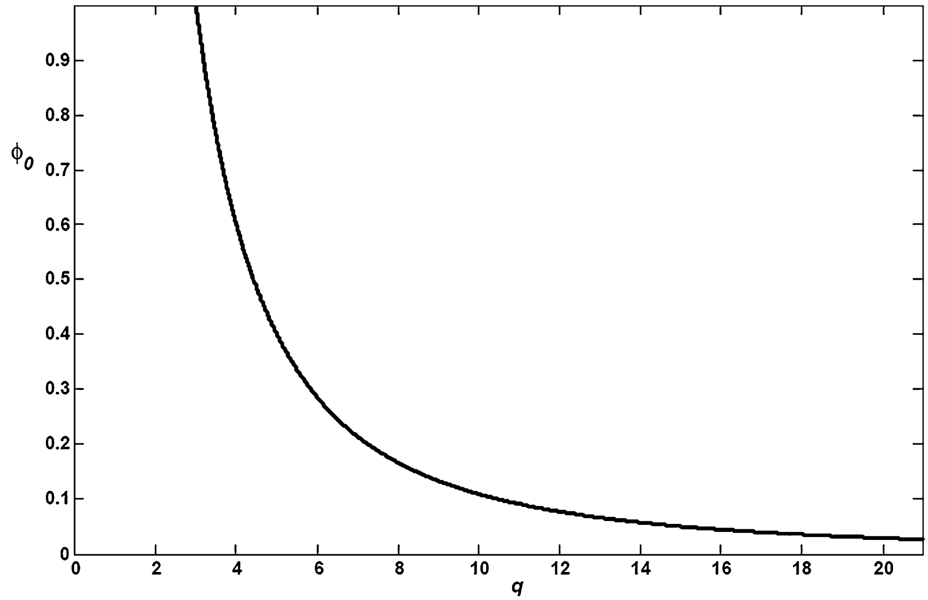


Fig. 1 The variation of the amplitude of shock wave with respect to q

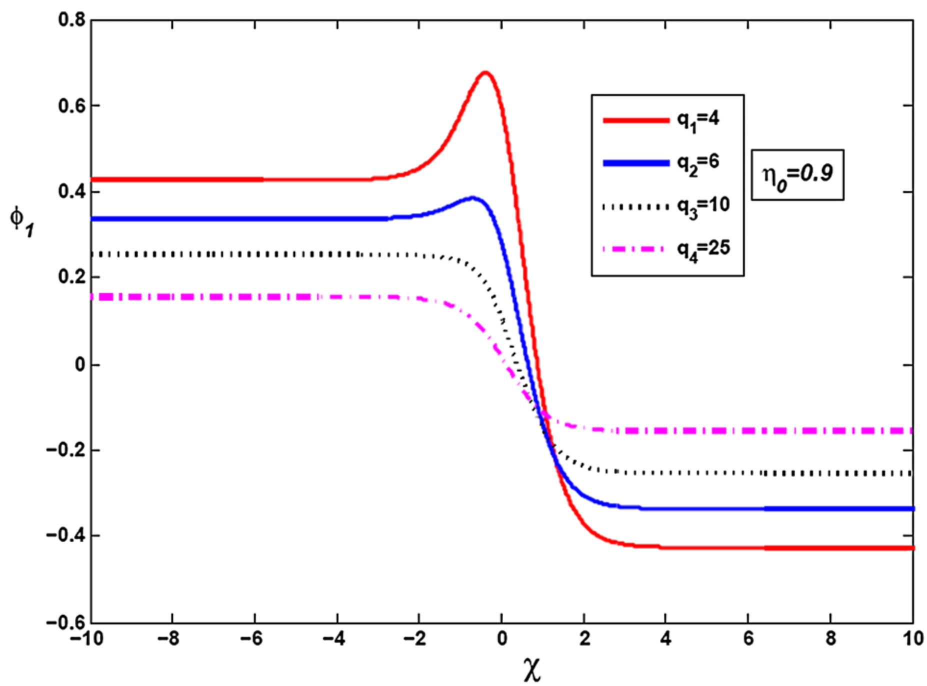


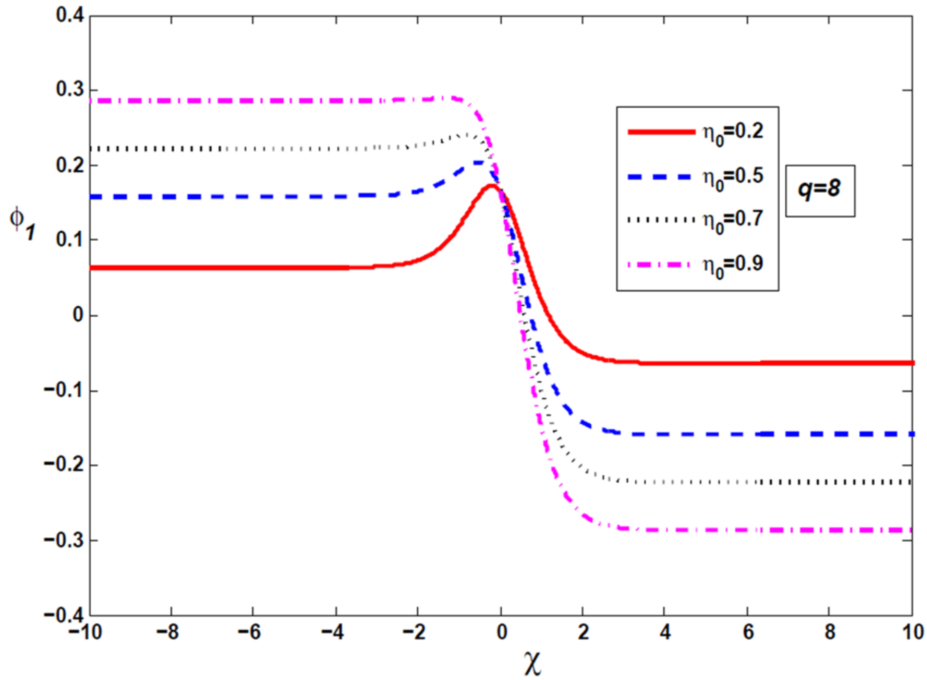
Fig. 2 Shock structure for the electrostatic potential ϕ_1 with respect to χ with $\eta_\circ = 0.9$ and different values of the $q = 4$, 6, 10 and 25

solitary waves is due to variation in $\eta_\circ$. In fact, when dissipation term is negligible compared to the nonlinearity and dispersion terms ($\eta_\circ \approx 0$), then solitonic structure will appear by balancing the effects of dispersive and nonlinear terms. On the other hand, if the kinematic viscosity becomes strong, the shock waves will appear by balancing the effects of dissipative and dispersive terms. So the presence of the Burgers term demonstrates any disturbance from developing into solitons and leads to the formation of a shock wave. It is found that an increasing in $\eta_\circ$ (meaning an increasing in dissipation) considerably enhances the amplitude as well as the steepness of the shock front. This result congruent with the one made by Masood et al. (2009) in e-p-i plasma consisting adiabatically hot positive ions and by Masood et al. (2008) in quantum e-p-i plasmas. This result also is in agreement with the result of Shah et al. (2009) for e-p-i plasma with the Boltzmann distribution of electron.

The existence of oscillatory shock waves also has been investigated using numerical simulation of (7). A solitary wave solution has been used as initial condition as follows

$$\phi_1 = \phi_\circ \sec h^2 \frac{(\xi - u\tau)}{w} \tag{12}$$

Fig. 3 Shock structure for the electrostatic potential ϕ_1 with respect to χ with $\eta_\circ = 0.9$ and different values of the $\eta_\circ = 0.2, 0.5, 0.7$ and 0.9



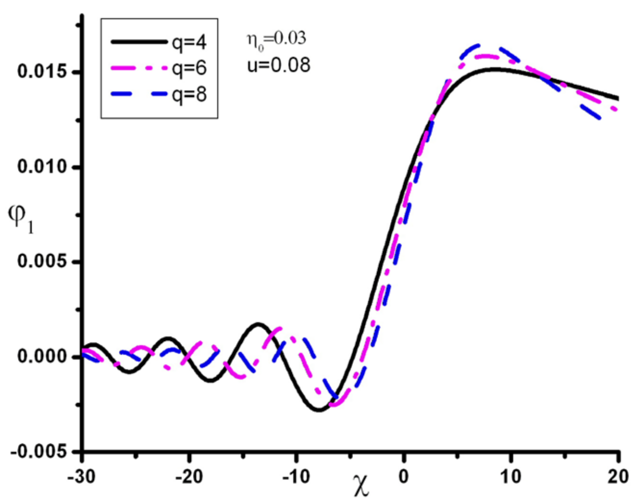


Fig. 4 Oscillatory shock wave profile propagated in the plasma with $u = 0.08$, $\eta_\circ = 0.03$ and different values of q-parameter

in which u is constant velocity of solitary wave. The amplitude $\phi_\circ$ and width w of the ion acoustic waves are given by

$$\phi_\circ = \frac{3u}{A}, \qquad w = 2\sqrt{\frac{B}{u}} \tag{13}$$

Equation (12) is not exact solution of (7) thus it crashes into shock waves. For a small $\eta_\circ$ oscillatory shock profiles have been appeared and greater values of $\eta_\circ$ provides monotonic shock waves.

Figure 4 presents oscillatory shock profiles as a function of χ for different values of "q".

Figure 4 shows that the amplitude of produced shock waves increases when "q" increases; but over shoot in the edge of the kink increases. Also one can find from Fig. 4 that the period of oscillation decreases with an increasing $\eta_\circ$.

3 Linear dispersion relation

In the view of standard normal-mode analysis, one can derive the dispersion relation of ion acoustic solitary waves. Dispersion relation has been obtained in different plasmas. We, in this section, study the dispersion properties of the linear waves. According to the standard normal-mode analysis, by linearization of dependent variables n_i, ϕ and u_i in terms of their equilibrium and perturbed parts (Annou 1998), we have

$$n_i = 1 + n_1, \qquad \phi = \phi_1, \qquad u_i = u_1 \tag{14}$$

Then, we may assume that all the perturbed quantities are proportional to $e^{i(kx-\omega t)}$ with k being the wave propagation constant in the direction of x-axis and so we have $\frac{\partial}{\partial t} = -i\omega$, $\frac{\partial}{\partial x} = ik$. Substituting (14) into (1) and using their linear terms one obtains linear dispersion relation as

$$\omega^2 = \frac{k^2}{k^2 + \frac{q+1}{2}} \tag{15}$$

For real values of ω, all perturbation variables oscillate harmonically and if any or all of the ω's have positive imaginary parts, then the system is unstable since those normal modes will grow in time (Samanta et al. 2007).

4 Summary

We have investigated the effect of nonextensive electrons on the nonlinear propagation of ion acoustic waves in electron-ion plasmas. The dissipative Burger term in the nonlinear KdVB equation arises by considering the kinematic viscosity among the plasma constituents. We have shown that how soliton profile is converted into shock structure when the viscosity force increases. Our results show that shock waves appear in such plasmas and the effect of the nonextensive electrons modifies the structure of the waves. We can draw the following conclusion from the investigation

(i) In a weakly nonlinear analysis of shock waves structure for electron-ion plasma with nonextensive electrons, the acceptable value of q-parameter is greater than 3.
(ii) The amplitude of shock wave decreases by increasing q-parameter.
(iii) The solitary wave profile is reduced to shock wave structure when the effective dissipation appears.
(iv) Oscillatory shock waves also can be propagated in described medium.
(v) Amplitude and the period of oscillatory shock waves decreases with an increasing "q" while overshoot in the edge of the kink in the shock profile increases.

Our investigation may be taken as a prerequisite for understanding the localized shock structures in e–i plasma with q-nonextensive electrons that has been observed in Saturn's magnetosphere (Schippers et al. 2008). In contrast to the usual two-component plasmas, it has been observed that the nonlinear waves in plasmas with an additional positron component behave differently. We can also consider ion acoustic shock waves in electron-positron-ion plasma with nonextensive positron. In fact, this kind of plasmas is frequently encountered in space and can be created in some situations, for example from the interaction of the solar wind (Gaelzer et al. 2008) with space plasma comprising electrons, positrons and ions in thermal equilibrium.

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