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ORIGINAL ARTICLE

Causal viscous universe coupled with zero-mass scalar field in higher derivative theory

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Abstract Cosmological solutions in the presence of an imperfect fluid and zero-mass scalar field are obtained in higher derivative theory. We investigate both power law and exponential expansion of the universe described by full causal theories proposed by Israel and Stewart. It is observed that energy density, co-efficient of bulk viscosity decrease with time in the presence of massless scalar field and temperature increase with expansion of universe.

Keywords Viscous fluid, zero-mass scalar field and higher derivative theory

1 Introduction

Studies of the dynamics of scalar field in connection with inflationary universe scenario have remained very attractive because it can solve some of the outstanding problems of standard big bang cosmology while at the same time the main underlying idea is quite simple. Recent observations from supernovae light curve data to Wilkinson Microwave Anisotropy Probe (WMAP) data (Spergel et al. 2003; Page 2003) indicate that the present universe is accelerating. To accommodate the present acceleration of the universe one needs either to modify Einstein's general theory of relativity (GR) or to include a time varying cosmological constant. One of the attempts to modify GR is based on adding a curvature squared term in the Einstein-Hilbert action which is known as the generalized theory of gravity. Starobinsky (1980) has shown that higher order theories of gravity admit inflation, which was proposed before the seminal work of Guth. However the efficacy of the theory became known only after the work of Guth (1981), where he used a temperature phase transition mechanism. It is interesting to note that higher order theories of gravity have a number of good features. It is known that with suitable counter terms, viz to $C^{\mu\nu\rho\delta}C_{\mu\nu\rho\delta}$, R^2, and the cosmological term (Λ), added to the Einstein action, one gets a perturbation theory which is well behaved, formally renormalizable, and asymptotically free (Stelle 1977). The standard approach to inflation starts from quantum field theory and finds the dynamics of the model by using the Einstein field equations. Although, it has some serious trouble of its own, there is a general belief that inflation, in some form or other, is indeed a part of cosmological evolution. There have been continuous effort to solve the various problems of inflation while keeping its important virtues alive by suggesting different modifications (Linde 1990; Kolb and Turner 1990). Perfect fluid cosmological models have been investigated in the context of higher derivative theory by Chimento and Jakubi (1996). Although the matter distribution in the universe is satisfactorily explained by a perfect fluid, a number of processes may have occurred in the early universe leading to viscosity. In the early universe, viscosity may be arise due to the decoupling of neutrinos during the radiation era, the decoupling of matter from radiation during the recombination era, creation of super strings during the quantum era, particle collisions involving gravitons, cosmological quantum particle creation processes, as well as during the formation of galaxies (Misner 1968; Barrow and Matzner 1977). Thus dissipation may play an important role in the evolution of the universe. We consider Eckart, truncated and full causal theories for describing the imperfect fluid to obtain cosmological solutions. Eckart theory by Eckart (1940) is used to describe viscous fluid in cosmology to model the universe. But they suffers

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from serious shortcomings viz., causality and stability as shown by Hiscock and Lindblom (1985). It is known that the above problems may be resolved by including higher order derivation terms in the transport equation (Muller 1967; Hiscock and Salmonson 1991). Israel (1976), Israel and Stewart (1979), and Pavon et al. (1982), Pavon (1991) developed a fully relativistic formulation of theory taking into account second order derivation terms in the theory which is termed extended irreversible thermodynamics (EIT). Using the transport equation obtained from EIT , cosmological solutions have been obtained in Einstein gravity by Ratra and Peebles (1988). A source of viscosity around the present time which gives rise to the observed acceleration could be the presence of dark matter by Pavon and Zimdahl (1993) or a cosmic antifriction force giving rise to an effective negative pressure as shown by Zindahl et al. (2001). Ram (1993) showed that one can have power law or exponential inflations consistent with an arbitrary potential driving inflation. Ellis and Madsen (1991)considered an FRW model with a minimally coupled scalar field along with a potential and a perfect fluid in the form of radiation. They solved for the scalar field and potential in terms of time t. Barrow and Saich (1993) assumed that the kinetic and potential terms for the scalar field are proportional to each other and obtained a solution for the scale factor in the case of minimally coupled scalar field in the case of non minimally coupled scalar fields, we refer to the investigations by Barrow and Mimoso (1994) and Mimiso and Wands (1995). Massless DKP field in a Lyra manifold is investigated by Casana et al. (2007).

In this paper, we investigate the evolution of zero-mass scalar field interacting with imperfect fluid in higher derivative theory. The objective of the work is to explore the effects of higher order terms in the evolution of the universe coupled with massless scalar field. The paper is presented as follows: we discussed model and field equations in Sect. 2 and solutions of field equations in Sect. 3. The conclusions are discussed in Sect. 4.

2 Model and field equations

We consider a gravitational action with higher order terms in the scalar curvature (R) which is given by

$$I = -\int \left[\frac{1}{2} f(R) + L_m\right] \sqrt{-g} d^4x \tag{1}$$

where $f(R)$ is a function of R and its higher powers, g is the determinant of the four dimensional metric. L_m is the matter Lagrangian, choosing $8\pi G = c = 1$. Several authors have studied classical solutions of this action without matter and have concluded that the big bang singularity may be avoided as shown by Kung (1996). Variation of action (1) with respect to $g_{\mu\nu}$ yields

$$f'(R)R_{\mu\nu} - \frac{1}{2} f(R) + f''(R)(\nabla_\mu \nabla_\nu R - \Box R g_{\mu\nu}) + f'''(R)(\nabla_\mu R \nabla_\nu R - \nabla^\sigma R \nabla_\sigma R g_{\mu\nu}) = -T_{\mu\nu} \tag{2}$$

where $\Box = g_{\mu\nu} \nabla^\mu \nabla^\nu$ and ∇_μ is the covariant differential operator, and prime represents the derivative with respect to R. The energy momentum tensor for the fluid coupled with massless scalar field is given as $T_{\mu\nu} = [T^F_{\mu\nu} + T^\phi_{\mu\nu}]$. The energy momentum tensor ($T^F_{\mu\nu}$) of a fluid with bulk viscosity is given by the equation

$$T^F_{\mu\nu} = (\rho + p + \pi) v_\mu v_\nu + (p + \pi) g_{\mu\nu} \tag{3}$$

where ρ and p are density and thermodynamic pressure of the fluid while π is the bulk viscous stress. In a co-moving system, the unit time vector v_μ is given by $v^\mu v_\mu = -1$ and $v^\mu = \delta^\mu_0$. The contribution to $T_{\mu\nu}$ from the minimally coupled zero-mass scalar field ϕ is

$$T^\phi_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{g_{\mu\nu}}{2} \phi_{,\alpha}\phi^{,\alpha} \tag{4}$$

ϕ being a zero-mass scalar field which satisfies the Klein-Gordon equation relation

$$g^{\mu\nu} \phi_{;\mu\nu} = \chi(r, t) \tag{5}$$

where $\chi(r, t)$ is the source density, the suffix comma and semicolon after a field variable represent ordinary and covariant differentiation w.r.t. 't' and '$g_{\mu\nu}$' respectively. We consider the flat Robertson-Walker spacetime given by the metric

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)] \tag{6}$$

where $a(t)$ is the scale factor of the universe. The scalar curvature of the universe is given as

$$R = -6[\dot{H} + 2H^2] \tag{7}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and an overdot represents a derivative with respect to time. The trace and $(0, 0)$ components of (2) are given by

$$f'(R)R_{00} + \frac{1}{2} f(R) - 3f''(R)\left(\ddot{R} + 3\frac{\dot{a}}{a}\dot{R}\right) + 3f'''(R)\dot{R} + T = 0, \tag{8}$$

$$Rf'(R) - 2f(R) + 3f''(R)\ddot{R} + 3\frac{\dot{a}}{a}\dot{R} + T_{00} = 0 \tag{9}$$

Now, we consider a higher derivative theory of gravity described by $f(R) = R + \alpha R^2$. Using (7) in (8) and (9), we get

$$3H^2 - 18\alpha[2H\ddot{H} - \dot{H}^2 + 6\dot{H}H^2] = \rho + \frac{\dot{\phi}^2}{2} \tag{10}$$

The conservation equation for the fluid is

$$\dot{\rho}+3H(\rho+p+\pi)=0 \tag{11}$$

From (5), we obtain the Klein-Gordon equation for the zero-mass scalar field as

$$\ddot{\phi}+3H\dot{\phi}=\chi \tag{12}$$

From the consideration of non equilibrium thermodynamics, the bulk viscous stress π given by the equation

$$\tau\dot{\pi}+\pi=-3\eta H-\frac{\epsilon}{2}\tau\pi\left(3H+\frac{\dot{\tau}}{\tau}-\frac{\dot{\eta}}{\eta}-\frac{\dot{T}}{T}\right) \tag{13}$$

where τ is the relaxation time,η is the coefficient of bulk viscosity and T is the temperature of the fluid. The parameter ϵ takes the value 0 or 1, the former represents the truncated Israel-Stewart causal theory and the latter represents the full causal theory. One recovers the noncausal Eckart theory for $\tau=0$. Equation (13) becomes the much simpler form "$\pi=-3\eta H$". The drawback of this simplified version is that this leads to infinite speed of propagation of signals violating the causality requirement and instability of the model against perturbations. Even in Truncated Israel-Stewart theory($\epsilon=0$), (13) becomes "$\pi+\tau\dot{\pi}=-3\eta H$", this break down of causality can be removed. We refer to the papers by Maartens (1995) and Lindblom (1996) for brief and comprehensive review. Zimdhal (1996) investigated how the truncated theory is indeed a good limit to the full causal theory in certain physical situations. An equation of state for the isotropic fluid is chosen in the form

$$p=(\gamma-1)\rho \tag{14}$$

where $\gamma(1\leq\gamma\leq 2)$ is a constant.

3 Solutions of the field equations

Only three among four equations i.e., (10)–(13) can be treated to be independent whereas there are seven variables, namely $a(t), \rho, p, \pi, \phi, \eta$ and T to be solved as functions of time. It may be observed here that the actual behavior of the various thermodynamic parameters in the theory may not be fully realized as the number of unknowns is more than the number of field equations. By choosing the "gamma law" equation of state and giving π as a function of ρ and a, the system of equations can be solved. In the following examples, this exact approach is resorted to. We assume a particular form of expansion of the universe, namely a power law expansion and exponential expansion, and try to solve for ϕ and thermodynamic temperature T.

3.1 A perfect fluid model revisited

For perfect fluid, one has $\pi=0$. Equation (11) leads to the continuity equation for a perfect fluid. For simplicity, we first consider power law inflation of the universe.

3.1.1 Power law cosmology

For power law expansion, we assume the scale factor in the form

$$a(t)=a_o t^n \tag{15}$$

where a_o and n are positive constants. The spatial volume is given by $V=a^3=a_o^3t^{3n}$ and scalar expansion factor $\theta=u^{\mu}_{;\mu}=3\frac{\dot{a}}{a}=\frac{3n}{t}$. We found the shear scalar(σ) as $\sigma^2=\frac{1}{12}[(\frac{g_{11;0}}{g_{11}}-\frac{g_{22;0}}{g_{22}})^2+(\frac{g_{22;0}}{g_{22}}-\frac{g_{33;0}}{g_{33}})^2+(\frac{g_{33;0}}{g_{33}}-\frac{g_{11;0}}{g_{11}})^2]=0$. It is observed that the spatial volume V is zero and expansion scalar is infinite at $t=0$ which shows that the universe starts evolving with zero volume at $t=0$ with an infinite rate of expansion. We also found that $\lim\frac{\sigma}{\theta}=0$ as $t\to\infty$. This relation confirms that the space time is isotropic. Equation (11) integrates to give

$$\rho=\rho_o t^{-3\gamma n} \tag{16}$$

where ρ_o is a constant of integration.

For $\chi=0$, from (12), one has

$$\phi-\phi_o=\frac{ct^{1-3n}}{1-3n} \tag{17}$$

where ϕ_o and c are constants of integration. Here the evolution of zero-mass scalar field ϕ is valid for $n\neq\frac{1}{3}$. ϕ is constant at $t=0$ and grows to infinity at $t\to\infty$. In this model, the deceleration parameter for an power law inflationary universe is negative i.e., $q=n>1$.

For $\chi\neq 0$, using (10) and (16), one has

$$\dot{\phi}^2=\frac{6n^2}{t^2}\left[1-\frac{18\alpha(1-2n)}{t^2}-\frac{\rho_o t^{2-3\gamma n}}{n^2}\right] \tag{18}$$

It is very difficult to integrate this equation, so we take $n=1/2$

$$\dot{\phi}^2=\frac{3}{2t^2}[1-4\rho_o t^{2-\frac{3\gamma}{2}}], \tag{19}$$

$$\phi-\phi_o=\frac{\delta}{\beta}\left[2\sqrt{1-bt^{\beta}}+\ln\left|\frac{\sqrt{1-bt^{\beta}}-1}{\sqrt{1-bt^{\beta}}+1}\right|\right] \tag{20}$$

where $\delta=\frac{\sqrt{6}}{2}$, $b=\frac{\rho_o}{4}$ and $\beta=2-\frac{3\gamma}{2}$. In this case, $\dot{\phi}^2$ is valid for $t\geq(4\rho_o)^{2/3\gamma}$. From (19), it is shown that this model is valid for a finite value of t. From (20), it is evident that once the solution is valid, it remains valid for subsequent times only if $\gamma>\frac{4}{3}$. The deceleration parameter is found as $q = 1$ which observes an expansion of universe. Thus it is evident that the presence of higher derivative term do not effect the behavior of the early universe in the presence of zero-mass scalar field. The cosmological evolution in this case is the same as that obtained in the Einstein gravity.

3.1.2 Exponential cosmology

For exponential inflation, one choose the scale factor of the universe in the form

$$a(t) = a_1 e^{\kappa t} \tag{21}$$

where κ and a_1 are constants. Here the deceleration parameter is $q = -\frac{a\ddot{a}}{\dot{a}^2} = -1$ which is consistent with the recent observations of supernovae Ia which require the present universe is accelerating. The volume V and scalar expansion θ are given by $A = a_1^3 e^{3\kappa t}$ and $\theta = 3\kappa$. At initial epoch, volume and scalar expansion are constant. Universe has Big-bang singularity with constant scalar expansion at $t \to \infty$. In this model also, $\sigma^2 = 0$ which observes that the universe is isotropic.

For $\chi = 0$, from (11) and (12), one has

$$\rho = \rho_1 e^{-3\kappa t}, \tag{22}$$

$$\phi - \phi_1 = \frac{c_o e^{-3\kappa t}}{-3\kappa} \tag{23}$$

where ϕ_1,ρ_1 and c_o are constants of integration. It is observed that ϕ is satisfied for all $\kappa \neq 0$. Here, the massless scalar field is constant at $t = 0$ and grows exponentially at $t \to \infty$.

For $\chi \neq 0$, using (10) and (22), one has

$$\phi - \phi_1 = \pm \int \left[2(\kappa - \rho_o e^{-3\gamma\kappa t})\right]^{1/2} dt \tag{24}$$

which means that there is no real solution. As we observed in Einstein gravity, this solution is discarded as unphysical.

3.2 A bulk viscous model

The bulk viscous stress for a causal viscous fluid is determined from (11). For the sake of simplicity, it is customary to assume the different thermodynamic quantities to be simple power functions of the fluid density ρ (Maartens 1995). Thus we assume the widely accepted *adhoc* relations:

$$\eta = \lambda \rho^s, \quad \tau = \lambda \rho^{s-1} \tag{25}$$

where $\eta \geq 0$, $\tau \geq 0$ and s, λ are all positive constants. Although, other behaviors are also conceivable, we restrict ourselves to this one.We discuss here the causal viscous cosmology i.e.full Israel-Stewart theory ($\epsilon = 1$) for $\chi = 0$. In the full causal theory (13) becomes

$$\tau\dot{\pi} + \pi = -3\eta H - \frac{1}{2}\tau\pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\eta}}{\eta} - \frac{\dot{T}}{T}\right) \tag{26}$$

3.2.1 Power law cosmology

In this case, we consider a power law expansion of the universe given by (15) with $n = \frac{1}{2}$. The energy density and bulk viscous stress are obtained (10) and (11) as

$$\rho = \frac{3}{4t^2} - \frac{c^2}{2t^3} \tag{27}$$

$$\pi = -\frac{(3\gamma - 4)}{4t^2} + \frac{(2-\gamma)c^2}{2t^3} \tag{28}$$

The scalar expansion, volume and shear scalar have the same signification as we observe in perfect fluid. In this case also, it is evident that the presence of higher derivative term do not effect the behavior of π and ρ in the presence of massless scalar field. The cosmological evolution in this case is same as that obtained in the Einstein gravity. Using relation (25), (26) can be written as

$$\frac{\dot{T}}{T} = 3H - \frac{\dot{\rho}}{\rho} + \frac{6H\rho}{\pi} + \rho^{1-s}\frac{2}{\lambda} + \frac{2\dot{\pi}}{\pi} \tag{29}$$

It is very difficult to integrate. For simplicity, we choose $s = \frac{1}{2}$. Using (15), (27) and (28) in (29), we obtain the thermodynamic temperature in the form

$$T = T_o a_o^3 \frac{[2(2-\gamma)c^2 - (3\gamma-4)t]^2}{4(3t - 2c^2)} \times \left[t - \frac{(2-\gamma)c^2}{3\gamma - 4}\right]^{\frac{12(5+3\gamma)}{(4-3\gamma)(2-\gamma)}} \times t^{3(n + \frac{\gamma-4}{2-\gamma})} e^{\frac{2(9t-2c^2)}{\lambda\sqrt{t(3t-2c^2)}}} \tag{30}$$

For $n = \frac{\gamma-4}{\gamma-2}$, T evolves as exponentially increasing function of time. It is observed that (i) for stiff fluid model ($\gamma = 2$), the result is discarded as unphysical, (ii) for radiation ($\gamma = \frac{4}{3}$), one has $n = 4$ yielding $a(t) \sim t^4$ which admits an accelerating universe, (iii) for matter dominated epoch ($\gamma = 1$) and in the presence of viscosity, one has $n = 3$ and the scale factor of the universe evolves as $a(t) \sim t^3$ which admits an accelerating universe. It is interesting as it accommodates the present accelerating phase of the universe. However, in the absence of massless scalar field ($c = 0$), the temperature is a power law function of time. So, the temperature increases with increase in time. But in the presence of scalar field, the temperature of the universe grows exponentially with increase in time.

3.2.2 Exponential cosmology

Using (21), one obtains the energy density and bulk viscous stress from (10) and (11) as

$$\rho = 3\kappa^2 - \frac{1}{2}c_o^2 e^{-6\kappa t} \tag{31}$$

$$\pi = -9\gamma\kappa^2 + \left(\frac{3\gamma}{2} - 1\right)c_o^2 e^{-6\kappa t} \tag{32}$$

In this case, the scalar expansion, volume and shear scalar have the same signification as we observe in perfect fluid. From (31) and (32), for simplicity $\gamma = 1$, we obtain a relation between π and ρ as

$$\frac{\rho}{\pi} = -\frac{1}{3} \tag{33}$$

It is very difficult to integrate (29). For simplicity, we choose $s = 1$. Using (31)–(33) in (29), we obtain the thermodynamic temperature in the form

$$T = T_o \frac{[-9\kappa^2 + \frac{1}{2}c_o^2 e^{-6\kappa t}]^2 e^{(\kappa+\frac{2}{\lambda})t}}{3\kappa^2 - \frac{1}{2}c_o^2 e^{-6\kappa t}} \tag{34}$$

In this case, we observed that universe evolves with finite size in the past and grows exponentially. The increasing temperature mode observed here in FIS is relevant for the later solution. Here the temperature grows with time with or without massless scalar field ϕ. Thus an inflationary universe with temperature evolution may be determined in this theory, which will be taken up elsewhere.

4 Conclusions

Interacting bulk viscous fluid and zero-mass scalar fields play a vital role in understanding the early stages of evolution of the universe. In this paper, we investigate the power law and exponential cosmology separately of the field equations generated by perfect fluid and imperfect fluid coupled with zero-mass scalar field for FRW space time in higher derivative theory with or without source density χ. In the case of power law cosmology, the perfect fluid model follows an expanding and isotropic model when$\chi = 0$. The scalar field ϕ exist for $n \neq \frac{1}{3}$. When $\chi \neq 0$, we obtain ϕ by considering $n = \frac{1}{2}$. Thus scalar field ϕ exist for $\gamma > \frac{4}{3}$ and power law expansion can take place as we observed in Einstein's gravity. In the case of exponential cosmology where $\chi = 0$, scalar field exist for $\kappa \neq 0$ in this perfect fluid model. However, we can't obtain the significant solution when $\chi \neq 0$. We also discussed a viscous universe described by the full Israel-Stewart theories. In the case of power law cosmology, the thermodynamic temperature T follows a power law function of time and increases with time in the absence of zero-mass scalar field when $s = 1/2$. It is interesting to observe that temperature increase exponentially with time in the presence of scalar field.This model shows an isotropic and expanding universe. This model also shows that the present universe is accelerating and matter dominated. The model is empty at initial time. The increasing temperature mode with expansion of universe will be useful later. In the case of exponential cosmology, we find the exact solution when $s = 1$ and $\gamma = 1$ as the relevant equations are highly non linear. The thermodynamic temperature increase with the expansion of universe which will be useful later. This model also shows an accelerating universe in accordance with the present observation.

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