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ORIGINAL ARTICLE

Cylindrical and spherical dust acoustic solitary waves in dusty plasma with superthermal ions and electrons

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Abstract Nonlinear propagation of cylindrical and spherical dust-acoustic solitons in an unmagnetized dusty plasma consisting of cold dust grains, superthermal ions and electrons are investigated. For this purpose, the standard reductive perturbation method is employed to derive the cylindrical/spherical Korteweg-de-Vries equation which governs the dynamics of dust-acoustic solitons. The effects of non-planar geometry and superthermal distributions on the cylindrical and spherical dust acoustic solitons structures are also studied by numerical calculation of the cylindrical/spherical Korteweg-de-Vries equation.

Keywords Soliton · Dusty plasma · Superthermal electrons · Superthermal ions

1 Introduction

In recent years, there have been more investigations of the collective processes of nonlinear dust acoustic waves (DAWs) in space dusty plasmas and laboratory environments, such as solitons, shocks, vortices, and so on (Shukla and Mamun 2002, 2003, El-Labany and El-Shamy 2005). The linear and nonlinear DAWs have been investigated theoretically (Rao et al. 1990; Shukla and Rosenberg 1999; Mamun 1999, 2008; Shukla and Mamun 2001; Shukla and Stenflo 2002; Popel and Gisko 2006) as well as experimentally (Chu et al. 1994; Barkan et al. 1995; Prabhakara and Tanna 1996; Thomas and Watson 1999; Pieper and Goree 1996) during the last few years. Rao et al. (1990) have first reported theoretically the existence of extremely low phase velocity (in comparison with the electron and ion thermal velocities) dust-acoustic waves in an unmagnetized dusty plasma whose constituents are an inertial charged dust fluid and Boltzmann distributed ions and electrons. Thus, in the dust-acoustic waves the dust particle mass provides the inertia and the pressures of electrons and ions gives rise to the restoring force. The nonlinear waves associated with the DA waves, particularly the DA solitary waves (Shukla and Mamun 2002), have received a great deal of attention for understanding the basic properties of localized electrostatic perturbations in space and laboratory dusty plasmas (Pieper and Goree 1996; Shukla 2001; Mendis and Rosenberg 1994; Goertz 1989; Shukla et al. 1996; Mamun and Shukla 2002). Recently in a very nice work, Shukla and Eliasson (2009) have updated the basic concepts of collective dust-plasma interactions in dusty plasmas, and also presented several novel phenomena that have been observed in laboratories and in space dusty plasmas. The DA solitary waves have been investigated by a number of authors by assuming different unmagnetized dusty plasma models (Mamun et al. 1996; Ma and Liu 1997; Mamun 1999). However, most of these studies are limited to one-dimensional (planar) geometry which may not be a realistic situation in space and laboratory devices, since the waves observed in space (laboratory devices) are certainly not infinite (unbounded) in one dimension. Both space and laboratory plasma environments may have such an excess superthermal electron population due to velocity space diffusion, which may lead to an inverse power-law distribution at a velocity much higher

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than the electron and ion thermal speed (Leubner 2004; Schippers et al. 2008; Hellberg et al. 2000). Such behavior is effectively modeled by a Kappa superthermal distribution function (Hellberg et al. 2000; Hasegawa et al. 1985; Baluku and Hellberg 2008), which appears to be more appropriate than a thermal Maxwellian distribution in a wide range of plasma situations. Numerous observations of space plasmas (Feldman et al. 1973; Formisano et al. 1973; Scudder et al. 1981; Marsch et al. 1982) indicate clearly the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. The latter may arise due to the effect of external forces acting on the natural space environment plasmas or to the wave-particle interaction which ultimately leads to kappa-like distributions. As a consequence, a high-energy tail appears in the distribution function of the particles. To study the DASW in the nonplanar geometry with radial symmetry we consider an unmagnetized dusty plasma whose constituents are negatively charged inertial dust particles, superthermal distributed electrons and ions. We show here how the DA solitary waves in cylindrical and spherical geometries differ qualitatively from that in one-dimensional planar geometry and how superthermal ions and electrons affect on them. This paper is organized as follows: In Sect. 2, we give a short definition of the model. In Sect. 3 we shall derive the cylindrical and spherical modified KdV equations. The numerical solutions of cylindrical and spherical modified KdV equations and the effect of superthermality on the characteristics of these solutions will be discussed in Sect. 4 and finally the calculation results are interoperated in Sect. 5.

2 Formulation

We focus on cylindrical and spherical DA solitary waves in an unmagnetized dusty plasma whose constituents are negatively charged cold dust fluid and superthermal electrons and ions. In equilibrium, the charge neutrality condition is $n_{i0} = Z_d n_{d0} + n_{e0}$, where n_{i0}, n_{d0} and n_{e0} are the unperturbed number densities of the ion, dust and electron, respectively. The nonlinear dynamics of DA waves in cylindrical and spherical geometries is governed by

$$\frac{\partial n_d}{\partial t} + \frac{1}{r^m}\frac{\partial}{\partial r}(r^m n_d u_d) = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial r} = \frac{\partial \phi}{\partial r}, \tag{2}$$

$$\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial \phi}{\partial r}\right) = n_d + n_e - n_i \tag{3}$$

where $m = 0$, for one-dimensional geometry and $m = 1$, 2 for cylindrical and spherical geometry, respectively. n_d is the dust particle number density normalized by its equilibrium value n_{d0}, u_d is the dust fluid velocity normalized by the dust acoustic speed $C_d = (\frac{Z_d k_B T_i}{m_d})^{1/2}$, ϕ is the electrostatic wave potential normalized by $\frac{k_B T_i}{e}$, and T_i is the temperature of ions. The time and space variables are in units of the dust plasma period $\omega_{pd}^{-1} = (\frac{m_d}{4\pi n_{d0} Z_d^2})^{1/2}$ and the Debye length $\lambda_{Dm} = (\frac{k_B T_i}{4\pi n_{d0} Z_d e^2})^{1/2}$, respectively. To model the effects of superthermal electrons and ions we have:

$$n_e = \frac{\mu}{1-\mu}\left(1 - \frac{\sigma\phi}{k_e - 3/2}\right)^{-(k_e-1/2)}, \tag{4}$$

$$n_i = \frac{1}{1-\mu}\left(1 + \frac{\phi}{k_i - 3/2}\right)^{-(k_i-1/2)} \tag{5}$$

where the real parameters k_e and k_i measure the deviation from Maxwellian equilibrium (recall that k_i and $k_e > 3/2$ in order for a physically meaningful thermal speed to be defined) (Boubakour et al. 2009; Tribeche and Boubakour 2009). We have denoted $\mu = \frac{n_{e0}}{n_{i0}}$, where n_{e0} and n_{i0} are the unperturbed number densities of electrons and ions, respectively, and $\sigma = T_i/T_e$, where T_e is the temperature of electrons.

3 Derivation of the cylindrical/spherical KdV equation

In order to investigate the nonlinear solutions of (1)–(5), we have employed the standard reductive perturbation technique to derive the modified KdV equation. According to this method we introduce the stretched coordinates ξ and τ as follows (Maxon and Viecelli 1974a),

$$\tau = \varepsilon^{\frac{3}{2}} t, \qquad \xi = -\varepsilon^{\frac{1}{2}}(r + \lambda t) \tag{6}$$

where ε is an expansion parameter and λ is the wave phase velocity normalized by C_d. Now we expand n_d, u_d and ϕ in a power series of ε as,

$$\begin{aligned} n_d &= 1 + \varepsilon n_d^{(1)} + \varepsilon^2 n_d^{(2)} + \cdots, \\ u_d &= \varepsilon u_d^{(1)} + \varepsilon^2 u_d^{(2)} + \cdots, \\ \phi &= \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \cdots, \end{aligned} \tag{7}$$

substituting the stretching (6) and the expansions (7) into the basic equations (1)–(5), we obtain to the lowest-order in ε as,

$$\lambda n_d^{(1)} = -u_d^{(1)}, \tag{8}$$

$$\lambda u_d^{(1)} = \phi^{(1)} \tag{9}$$

where

$$\lambda = 1 \Bigg/ \sqrt{\frac{\mu\sigma(k_e - 1/2)}{(1-\mu)(k_e - 3/2)} + \frac{(k_i - 1/2)}{(1-\mu)(k_i - 3/2)}}. \tag{10}$$

To next higher order in ε, we obtain a set of the following equations,

$$\lambda\frac{\partial n_d^{(2)}}{\partial\xi}-\frac{\partial n_d^{(1)}}{\partial\tau}+\frac{\partial}{\partial\xi}(n_d^{(1)}u_d^{(1)}) +\frac{\partial u_d^{(2)}}{\partial\xi}+\frac{mu_d^{(1)}}{\lambda\tau}=0, \tag{11}$$

$$\lambda\frac{\partial u_d^{(2)}}{\partial\xi}-\frac{\partial u_d^{(1)}}{\partial\tau}+u_d^{(1)}\frac{\partial u_d^{(1)}}{\partial\xi}=\frac{\partial\phi^{(2)}}{\partial\xi}, \tag{12}$$

$$\frac{\partial^2\phi^{(1)}}{\partial\xi^2}=n_d^{(2)}+\left(\frac{\mu\sigma(k_e-1/2)}{(1-\mu)(k_e-3/2)}+\frac{(k_i-1/2)}{(1-\mu)(k_i-3/2)}\right)\phi^{(2)} +\left(\frac{\mu\sigma^2(k_e-1/2)(k_e-3/2)}{(1-\mu)(k_e-3/2)^2}-\frac{(k_i-1/2)(k_i-3/2)}{(1-\mu)(k_i-3/2)^2}\right)\frac{(\phi^{(1)})^2}{2}, \tag{13}$$

combining (11)–(13) and making use of the first-order results, we deduce a modified KdV equation

$$\frac{\partial\phi^{(1)}}{\partial\tau}+\frac{m}{2\tau}\phi^{(1)}-A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}=0 \tag{14}$$

where

$$A=\frac{3}{2\lambda}+\frac{\lambda^3}{2}\left[\frac{\mu\sigma^2(k_e-1/2)(k_e+1/2)}{(1-\mu)(k_e-3/2)^2}-\frac{(k_i-1/2)(k_i+1/2)}{(1-\mu)(k_i-3/2)^2}\right] \tag{15}$$

and

$$B=\frac{\lambda^3}{2}. \tag{16}$$

Equation (14) is the cylindrical/spherical KdV equation describing the nonlinear propagation of the dust acoustic solitary waves in a plasma with superthermal electrons and ions. In this equation A and B are the nonlinear coefficient and dispersive term. The above results in (14)–(16) are comparable with the results in Mamun and Shukla (2001) for dusty plasma with the Boltzmann distribution of electrons and ions.

4 Numerical results and discussion

When the geometrical effect is taken into account ($m\neq 0$), an exact analytical solution of the modified KdV equation (14) is not possible. Therefore, we have numerically solved (14) and have studied the effects of superthermal ions and electrons and cylindrical/spherical geometry on the propagation of dust acoustic solitary waves. In the numerical procedure the modified KdV equation was advanced in time with a standard fourth-order Runge-Kutta scheme (Press et al. 1992) with a time step of 10^{-4}. The spatial derivatives were approximated with centered finite difference approximations with a spatial grid spacing of 0.1 (Maxon and Viecelli 1974a, 1974b). The initial condition that we have used in all our numerical results is the form of the stationary solution of (14), assuming $m=0$, at $\tau=-6.66$ (at this stage the geometry effect is weaker, so we can take this stage as the initial stage of evolution) i.e.

$$\phi^{(1)}=\phi_\circ\sec h\left(\frac{\xi-u\tau}{w}\right), \tag{17}$$

where $\phi_\circ$, is the amplitude, and w is the width of the solitary waves, as $\phi_\circ=\frac{3u}{A}$ and $w=2\sqrt{\frac{B}{u}}$, here u is a constant velocity. Numerical solutions of (14) reveals that for a large value of $|\tau|$ (e.g., $\tau=-6.66$) the spherical and cylindrical solitary waves are similar to one dimensional solitary waves. This is due to a large value of $|\tau|$ which causes the term $\frac{m}{2\tau}\phi^{(1)}$, is no longer dominant. However, as the value of $|\tau|$ decreases, the term $\frac{m}{2\tau}\phi^{(1)}$ becomes dominant and both spherical and cylindrical solitary waves differ from one dimensional solitary wave.

The results are displayed in Figs. 1 to 5. In all our figures we focus only on the fixed values of $\mu=0.1$ and $\sigma=0.1$. From Figs. 1–4 it is seen that how the superthermal parameters (k_e and k_i) effect dust acoustic solitons in cylindrical and spherical geometries for fixed values of μ and σ. In Fig. 1 the solutions of dust acoustic solitary waves have been plotted at $\tau=-6.66$, -5.16 and -4.16, respectively and for $m=1$, $k_e=3$ and of course for different values of k_i. From Fig. 1 it is seen that amplitude and width increase with increasing in τ as well as the value of k_i. Furthermore, it can be seen that a small value of k_i and thus an increase in superthermality lowers the value of speed of dust acoustic solitary waves.

Figure 2 shows the effect of superthermal electrons on the cylindrical solitary waves for fixed value of $k_i=3$. It is clear that the amplitude of the pulses increases when $|\tau|$ decreases, but here is a most interesting point to be made, an increase in k_e (meaning an increase in superthermal-parameter of electrons) have not significantly effect on the characteristics of solitons.

Figures 3 and 4 show, respectively, the solutions of (14) for several values of τ ranging from $\tau=-6.66$ to $\tau=-4.16$ in spherical ($m=2$) geometry for $k_e=3$ and different values of k_i and $k_i=3$ and different value of k_e. The figures show that results for cylindrical and spherical geometry are the same. But it is seen that for $m=2$, as the value

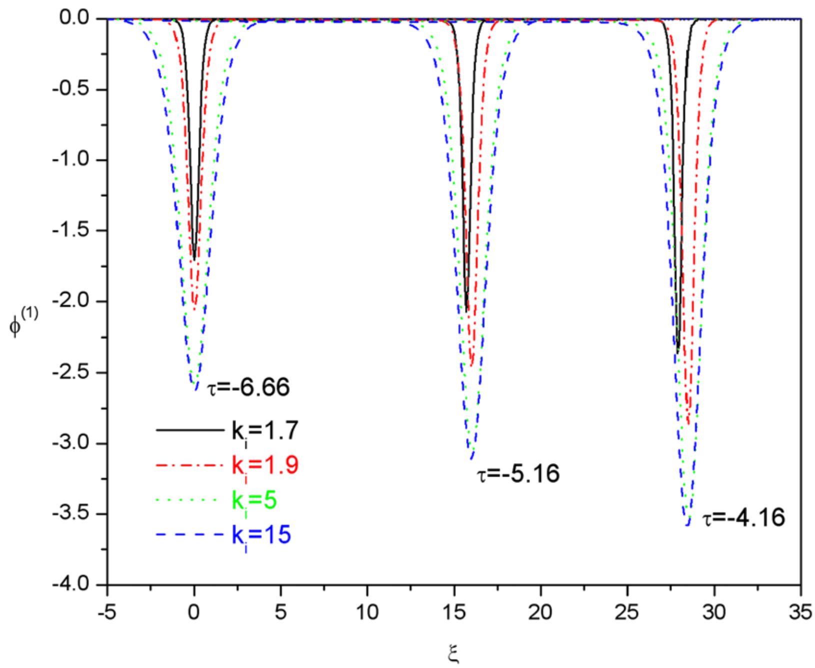

Fig. 1 Time evolution of cylindrical solitary waves: $\phi^{(1)}$ versus spatial coordinate ξ at times $\tau = -6.66$, $\tau = -5.16$, and $\tau = -4.16$ for $\sigma = 0.1$, $\mu = 0.1$, $k_e = 3$ and different values of k_i

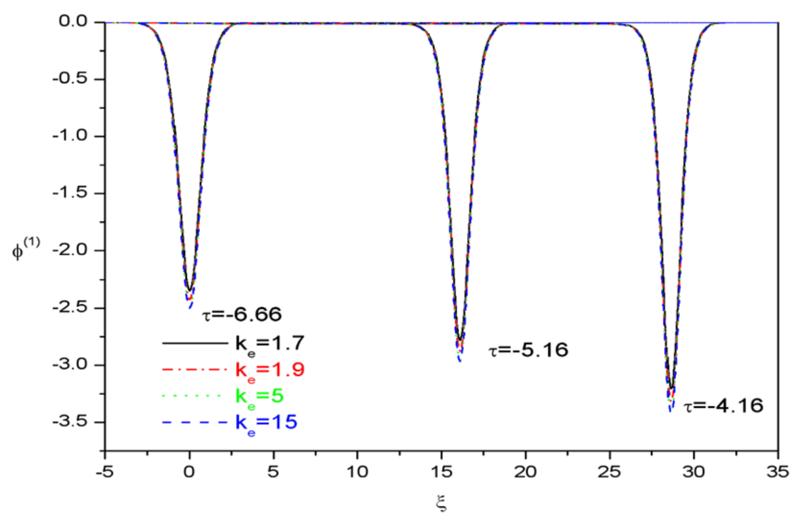

Fig. 2 Time evolution of cylindrical solitary waves, $\phi^{(1)}$ versus spatial coordinate ξ at times $\tau = -6.66$, $\tau = -5.16$, and $\tau = -4.16$ for $\sigma = 0.1$, $\mu = 0.1$, $k_i = 3$ and different values of k_e

of $|\tau|$ decreases, the soliton amplitude and profile steepness are increased in cooperation with $m = 1$.

In Fig. 5 we compare the profiles of spherical and cylindrical solitary waves for different values of τ with $k_e = 3$ and $k_i = 1.8$. It is clear that the profiles of the solitary waves are different in spherical and cylindrical geometry. It is also seen that the cylindrical solitary waves move slower than the spherical ones and that the amplitude of cylindrical solitary waves is slightly smaller than that of the spherical ones for small values of $|\tau|$ (Mamun and Shukla 2001). This happens due to the effect of the second term in (14) which become twice for the spherical geometry. The numerical solutions for both the geometries, i.e., $m = 1, 2$ at large values of τ seems to be similar for one-dimensional planar KdV soliton solutions because the term $m/2\tau$ become smaller and smaller in this case. However, the numerical solutions differ from one-dimensional planar KdV soliton solutions at smaller values of τ. It is seen that when the parameters k_i and k_e are increased, both the amplitude and the width of the solitary waves are also increased in cylindrical and spherical geometries and the solitary peaks are well separated with the passage of time. Figures 1–5 indicate that the kappa distribution has only a quantitative, not a qualitative effect on the existence domains and only negative potential solitons exist regardless of whether the electrons or the ions, or both, have a kappa distribution (Baluku and Hellberg 2008; Mamun and Shukla 2001). On the other hand, Rao et al. (1990) showed, in a planner model of a dusty plasma, that the waves can propagate linearly as a normal mode, and nonlinearly as supersonic solitons of either positive or negative

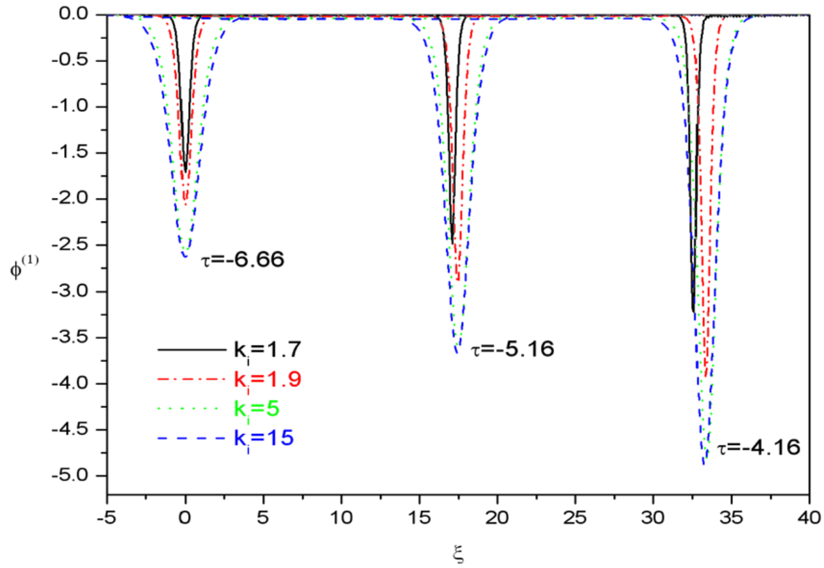

Fig. 3 Time evolution of spherical solitary waves, $\phi^{(1)}$ versus spatial coordinate ξ at times $\tau = -6.66$, $\tau = -5.16$, and $\tau = -4.16$ for $\sigma = 0.1$, $\mu = 0.1$, $k_e = 3$ and different values of k_i

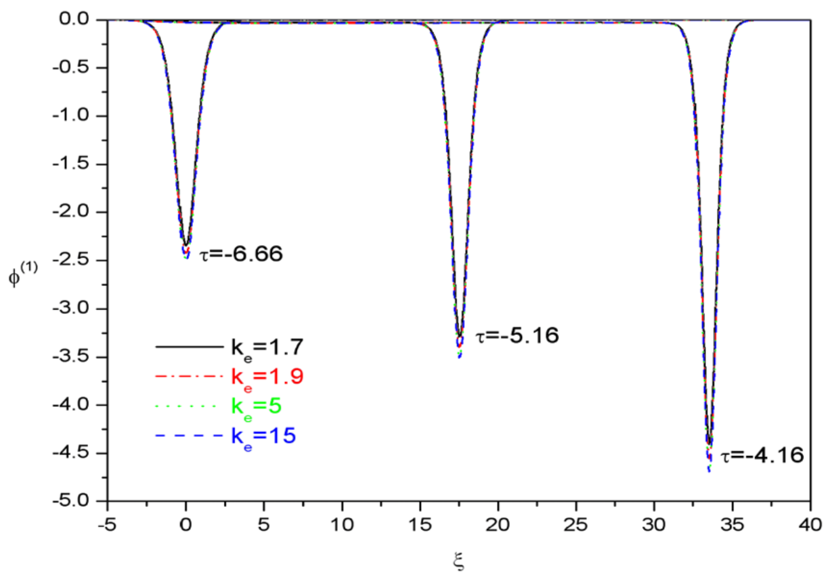

Fig. 4 Time evolution of spherical solitary waves, $\phi^{(1)}$ versus spatial coordinate ξ at times $\tau = -6.66$, $\tau = -5.16$, and $\tau = -4.16$ for $\sigma = 0.1$, $\mu = 0.1$, $k_i = 3$ and different values of k_e

electrostatic potential. Therefore, it has been found that the properties of the DASWs in a nonplanar cylindrical or spherical geometry differ from those in a planar one-dimensional geometry.

5 Conclusion

Nonlinear analysis has been carried out to derive the appropriate Korteweg-de Vries equation for cylindrical and spherical dust-acoustic waves in an unmagnetized three species plasma system, comprised of cold dust and superthermal ions and electrons. The effects of nonplanar geometry and superthermal distributions on the amplitude and width of the dust-acoustic solitary waves are described using relevant graphs. The results found from this investigation can be summarized as follows,

(i) The cylindrical solitary waves move slower than the spherical ones and that the amplitude of cylindrical solitary waves is slightly smaller than that of the spherical ones.
(ii) The cylindrical solitary waves with bigger values of k_i move faster than that of ones with smaller values of k_i.
(iii) Superthermality effects on plasma may become significant when the superthermal ions are varied.

In conclusion, we mention that the present results may help to understand the salient features of multi-dimensional DA solitary waves when data for space and laboratory observations become available. In view of the observations of both kappa-distributed ions and electrons in Saturn's mag-

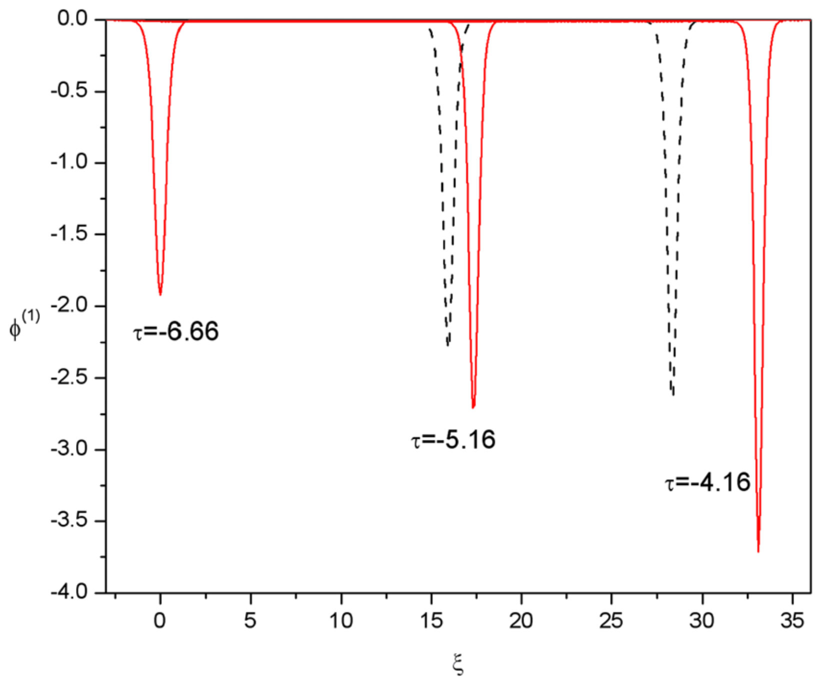

Fig. 5 Time evolution of spherical and cylindrical solitary waves, $\phi^{(1)}$ versus spatial coordinate ξ at times $\tau=-6.66$, $\tau=-5.16$, and $\tau=-4.16$ for $\sigma=0.1$, $\mu=0.1$, $k_i=1.8$ and $k_e=3$. The *dashed curves* correspond to $m=1$; *solid curves* to $m=2$

netosphere (Baluku and Hellberg 2008) as well as inertial dust grains, the results of this paper should assist in the interpretation of properties of cylindrical and spherical dust acoustic solitary waves that may be observed in that region.

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