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ORIGINAL ARTICLE

Cosmological evolution and statefinder diagnostic for new holographic dark energy model in non flat universe

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Abstract In this paper, the holographic dark energy model with new infrared cut-off proposed by Granda and Oliveros has been investigated in spatially non flat universe. The dependency of the evolution of equation of state, deceleration parameter and cosmological evolution of Hubble parameter on the parameters of new HDE model are calculated. Also, the statefinder parameters r and s in this model are derived and the evolutionary trajectories in $s-r$ plane are plotted. We show that the evolutionary trajectories are dependent on the model parameters of new HDE model. Eventually, in the light of SNe + BAO + OHD + CMB observational data, we plot the evolutionary trajectories in $s-r$ and $q-r$ planes for best fit values of the parameters of new HDE model.

Keywords Dark energy · New holographic model · Statefinder diagnostic · Cosmological evolution

1 Introduction

The observational evidences from distant Ia supernova, Large Scale Structure (LSS) and Cosmic Microwave Background (CMB) suggest that our universe is undergoing an accelerating expansion (Perlmutter et al. 1998; Riess et al. 1998, 1999; Spergel et al. 2003, 2007; Tegmark et al. 2004; Abazajian et al. 2004, 2005). Within the framework of general relativity this expansion may be driven by a component with negative pressure, namely, dark energy (DE) (Copeland et al. 2006; Frieman et al. 2008). The nature of DE is still unknown and many theoretical models have been provided to describe the cosmic behavior of DE. The simplest and important theoretical candidate for DE is the Einstein's cosmological constant, which can fit the observations well so far (Weinberg 1989; Sahni and Starobinsky 2000; Carroll 2001; Carroll et al. 2003; Padmanabhan 2003; Copeland et al. 2006; Bousso 2008). The cosmological constant suffers two well known problems namely "fine-tuning" and "cosmic coincidence" (Copeland et al. 2006). In order to alleviate or even solve these problems, many dynamical dark energy models have been proposed, whose equation of state is time-varying. Dynamical DE models can be classified into two categories (i) The scalar field dark energy models including quintessence (Wetterich 1988; Ratra and Peebles 1988), K-essence (Chiba et al. 2000; Armendáriz-Picón et al. 2000, 2001), phantoms (Caldwell 2002; Nojiri and Odintsov 2003a, 2003b), tachyon (Sen 1999; Bergshoeff et al. 2000; Sen 2002a, 2002b; Padmanabhan 2002; Padmanabhan and Choudhury 2002; Abramo and Finelli 2003), dilaton (Gasperini et al. 2002; Arkani-Hamed et al. 2004; Piazza and Tsujikawa 2004), quintom (Elizalde et al. 2004; Nojiri et al. 2005; Anisimov et al. 2005) and so forth. (ii) The interacting dark energy models, by considering the interaction between dark matter and dark energy, including Chaplygin gas (Kamenshchik et al. 2001; Bento et al. 2002), braneworld models (Deffayet et al. 2002; Sahni and Shtanov 2003), holographic DE (HDE) (Hsu 2004; Li 2004) and agegraphic DE (ADE) (Cai 2007; Wei and Cai 2008) models and so forth.

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On the other hand, a curvature driven acceleration model which is called, modified gravity, has been proposed by Starobinsky (1980), Kerner (1982), Duruisseau and Kerner (1986), for the first time, in 1980. Modified gravity approach suggests the gravitational alternative for unified description of inflation, dark energy and dark matter with no need of the hand insertion of extra dark components (Setare 2010; Khodam-Mohammadi et al. 2010).

The HDE model is constructed based on the holographic principle (Cohen et al. 1999; 't Hooft 2009; Susskind 1995; Horava and Minic 2000; Thomas 2002; Hsu 2004; Li 2004). In holographic principle, based on the validity of the effective local quantum field theory, a short distance (UV) cut-off Λ is related to the long distance (IR) cut-off L due to the limit set by the formation of a black hole (Cohen et al. 1999). The HDE model has been constrained by various astronomical observation (Huang and Gong 2004; Chang et al. 2006; Zhang and Wu 2005; Wu et al. 2008; Ma and Gong 2009; Enqvist et al. 2005; Shen et al. 2005; Kao et al. 2005) and also investigated widely in the literature (Huang and Li 2004, 2005; Ito 2005; Enqvist and Sloth 2004; Pavon and Zimdahl 2005; Wang et al. 2005; Kim et al. 2006; Nojiri and Odintsov 2006; Elizalde et al. 2005; Hu and Ling 2006; Li et al. 2006; Setare 2006a, 2006b, 2007a, 2007b, 2007c, 2007d, 2007e; Saridakis 2008a, 2008b, 2008c; Amendola 2000; Comelli et al. 2003, Jamil and Rashid 2008, 2009; Setare and Saridakis 2008). The HDE model with Hubble horizon or particle horizon as a length scale, can not derive the accelerated expansion of the universe (Hsu 2004; Li 2004). Although, in the case of event horizon, HDE model can derive the universe with accelerated expansion (Li 2004), but the arising problem with the event horizon is that it is a global concept of spacetime and existence of it depends on the future evolution of the universe only for a universe with forever accelerated expansion. Moreover, the HDE with the event horizon is not compatible with the age of some old high redshift objects (Wei and Zhang 2007). The above problems with HDE motivated us to follow the new HDE model proposed by Granda and Oliveros (GO, here after). GO proposed a new IR cut-off containing the local quantities of Hubble and time derivative Hubble scales (Granda and Oliveros 2008). The advantages of HDE with GO cutoff (new HDE, here after) is that it depends on local quantities and avoids the causality problem appearing with event horizon IR cutoff. The new HDE model can also obtain the accelerated expansion of the universe (Granda and Oliveros 2008). GO showed that in new HDE model, the transition redshift from deceleration phase ($q > 0$) to acceleration phase ($q < 0$) is consistent with current observational data (Granda and Oliveros 2008; Ma 2008; Daly et al. 2008).

Besides, the observational experiments such as CMB experiment (Sievers et al. 2003; Netterfield et al. 2002; Benoit et al. 2003a, 2003b) and luminosity—distance of supernova measurements (Caldwell and Kamionkowski 2004; Wang et al. 2005) imply that our universe is not perfectly flat and has a small positive curvature. Therefore, we are motivated to consider the new HDE in the non-flat universe.

Since many dynamical DE models have been proposed to interpret the cosmic acceleration, a sensitive and diagnostic tool is required to discriminate the various DE models. The various DE models have a degeneracy on the Hubble parameter H (first time derivative of scale factor) and the deceleration parameter q (second time derivative of scale factor). Therefore, these quantities cannot discriminate the DE models. For this aim, we need the higher order of the time derivative of scale factor. By using the third order time derivative, Sahni et al. (2003) and Alam et al. (2003) introduced the statefinder pair $\{r, s\}$ in order to remove the degeneracy of H_0 and q_0 of different DE models. The statefinder pair $\{r, s\}$ is defined as

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r - \Omega_{tot}}{3(q - \Omega_{tot}/2)}, \tag{1}$$

where Ω_{tot} is the dimensionless total energy density containing matter, DE and curvature. It is clear that the statefinder is a geometrical diagnostic, because it depends on the scale factor. The role of statefinder pair is to distinguish the behaviors of cosmological evolution of dark energy models with the same values of H_0 and q_0 at the present time. Up to now, the statefinder diagnostic tool has been used to study the various dark energy models. The statefinder has been used to diagnose different cases of the model, including different model parameters and various spatial curvature contributions. The various DE models have different evolutionary trajectories in $\{r, s\}$ plane. For example, the well-known ΛCDM model corresponds to a fixed point $\{r = 1, s = 0\}$ in $\{r, s\}$ plane (Sahni et al. 2003). Also, the quintessence DE model (Sahni et al. 2003; Alam et al. 2003), the interacting quintessence models (Zimdahl and Pavon 2004; Zhang 2005), the holographic dark energy models (Zhang 2005; Zhang et al. 2008), the holographic dark energy model in non-flat universe (Setare et al. 2007), the phantom model (Chang et al. 2007), the tachyon (Shao and Gui 2008), the agegraphic DE model with and without interaction in flat and non-flat universe (Wei and Cai 2007; Malekjani and Khodam-Mohammadi 2010) and the interacting new agegraphic DE model in flat and non-flat universe (Zhang et al. 2010; Khodam-Mohammadi and Malekjani 2010), are analyzed through the statefinder diagnostic tool.

In this paper, we study the cosmological treatment of new HDE model and investigate this model by means of statefinder diagnostic. The paper is organized as follows: In Sect. 2, we present the new HDE model and derive the statefinder parameters $\{r, s\}$ for this model. In Sect. 3, the numerical results are presented. We conclude in Sect. 4.

2 New HDE model

Following (Granda and Oliveros 2008), the energy density of new HDE is written as

$$\rho_\Lambda = 3M_P^2(\alpha H^2 + \beta \dot{H}), \tag{2}$$

where α and β are constants, M_p is the reduced Plank mass, H is the Hubble parameter and dot denotes the derivative with respect to the cosmic time.

Here we assume the Friedmann-Robertson-Walker (FRW) universe as follows:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{3}$$

where $a(t)$ is the scale factor, and $k = -1, 0, 1$ represent the open, flat, and closed universe, respectively. The observational evidence reveal a closed universe with small positive curvature ($\Omega_k \sim 0.02$) (Bennett et al. 2003; Spergel et al. 2003; Tegmark et al. 2004; Seljak et al. 2006; Spergel et al. 2007). The first Friedmann equation for a universe with curvature k is written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_m + \rho_\Lambda), \tag{4}$$

where ρ_Λ is the energy density of DE and ρ_m is the energy density of matter including the cold dark matter (CDM) and baryons. Substituting the energy density of new HDE, i.e. (2), in (4) and changing the variable from the cosmic time t to $x = \ln a$, yields

$$H^2(1-\alpha) + \frac{k}{a^2} - \frac{\beta}{2}\frac{dH^2}{dx} = \frac{\rho_{m0}}{3M_p^2}e^{-3x}, \tag{5}$$

where we assume the evolution of matter component as $\rho_m = \rho_{m0}e^{-3x}$. With the definition of normalized Hubble parameter as $E = H/H_0$, $\Omega_k = -k/H_0^2$, and $\Omega_{m0} = \rho_{m0}/3M_p^2 H_0^2$, we can rewrite (5) as follows

$$E^2(1-\alpha) = \Omega_k e^{-2x} + \frac{\beta}{2}\frac{dE^2}{dx} + \Omega_{m0}e^{-3x}. \tag{6}$$

Solving the above first order differential equation for E^2 and using the initial condition $E_0 = 1$, we obtain the dimensionless Hubble parameter, E, for a universe containing the new HDE and matter, as follows (Wang and Xu 2010)

$$E^2 = \Omega_k e^{-2x} + \Omega_{m0}e^{-3x} + \Omega_\Lambda(x), \tag{7}$$

where $\Omega_\Lambda(x)$ is the dimensionless energy density of new HDE model which is given as (Wang and Xu 2010)

$$\begin{aligned}\Omega_\Lambda &= \frac{\alpha-\beta}{-\alpha+\beta+1}\Omega_k e^{-2x} + \frac{2\alpha-3\beta}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x} \\ &\quad + \left(1 - \frac{1}{-\alpha+\beta+1}\Omega_k - \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}\right) \\ &\quad \times e^{-\frac{2(\alpha-1)x}{\beta}}.\end{aligned} \tag{8}$$

Inserting (8) in (7), the parameter E can be obtained as

$$\begin{aligned}E &= \left(\frac{1}{-\alpha+\beta+1}\Omega_k e^{-2x} + \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x}\right. \\ &\quad + \left(1 - \frac{1}{-\alpha+\beta+1}\Omega_k - \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}\right) \\ &\quad \left.\times e^{-\frac{2(\alpha-1)x}{\beta}}\right)^{1/2}.\end{aligned} \tag{9}$$

Here we consider the parameters $\beta \neq 0$ and $\alpha \neq 1$. The special cases when the denominators in (9) are equal zero have been discussed in Wang and Xu (2010).

Combining (8) with the conservation equation of new HDE model, we can obtain the EoS parameter of new HDE as follows

$$\begin{aligned}w_\Lambda &= -1 - \frac{1}{3}\frac{d\ln\Omega_\Lambda}{dx} = -1 + \frac{2}{3} \\ &\quad \times \left[\frac{\alpha-\beta}{-\alpha+\beta+1}\Omega_k e^{-2x} + \frac{3}{2}\frac{2\alpha-3\beta}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x}\right. \\ &\quad + \frac{(\alpha-1)}{\beta}\left(1 - \frac{1}{-\alpha+\beta+1}\Omega_k\right. \\ &\quad \left.\left. - \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}\right)e^{-\frac{2(\alpha-1)x}{\beta}}\right] \\ &\quad \Big/ \left[\frac{\alpha-\beta}{-\alpha+\beta+1}\Omega_k e^{-2x} + \frac{2\alpha-3\beta}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x}\right. \\ &\quad + \left(1 - \frac{1}{-\alpha+\beta+1}\Omega_k - \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}\right) \\ &\quad \left.\times e^{-\frac{2(\alpha-1)x}{\beta}}\right]\end{aligned} \tag{10}$$

which is time-dependent EoS parameter. The time-dependent of EoS parameter allows it to transit from $w_\Lambda > -1$ to $w_\Lambda < -1$ (Wang et al. 2006). Some recent observational evidences suggest DE models whose EoS parameter crosses -1 in the near past (Alam et al. 2004; Huterer and Cooray 2005; Wang and Tegmark 2005). In the limiting case of flat universe, by considering the DE dominated epoch (the matter contribution is negligible compare with the contribution of DE), (10) is reduced as

$$w_\Lambda = -1 + \frac{2(\alpha-1)}{3\beta} \tag{11}$$

which is same as (2.5) in Granda and Oliveros (2009), Karami and Fehri (2010).

Now we derive the deceleration parameter q for new HDE model. The deceleration parameter is given by

$$q = -\frac{\dot{H}}{H^2} - 1. \tag{12}$$

Re-witting q in terms of dimensionless Hubble parameter, E, we have

$$q(x) = -\frac{1}{E}\frac{dE}{dx} - 1. \tag{13}$$

Substituting E from (9) in (13), the parameter q can be obtained as

$$\begin{aligned} q = \Bigg[&\frac{2}{-\alpha+\beta+1}\Omega_k e^{-2x} + \frac{3}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x} \\ &+ (\alpha-1)\bigg(1 - \frac{1}{-\alpha+\beta+1}\Omega_k \\ &- \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}\bigg)e^{-\frac{2(\alpha-1)x}{\beta}}\Bigg] \\ &\Big/\Bigg[\frac{1}{-\alpha+\beta+1}\Omega_k e^{-2x} + \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x} \\ &+ \bigg(1 - \frac{1}{-\alpha+\beta+1}\Omega_k - \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}\bigg) \\ &\times e^{-\frac{2(\alpha-1)x}{\beta}}\Bigg]. \end{aligned} \tag{14}$$

Here, one can explicitly see the dependence of deceleration parameter q on the model parameters α and β. Finally, we derive the statefinder pair $\{r, s\}$ for new HDE model. Using the definition of statfinder parameters in (1), we have

$$r = \frac{\dddot{a}}{aH^3} = \frac{\ddot{H}}{H^3} - 3q - 2. \tag{15}$$

Similar to q, the parameter r is re-written in terms of dimensionless Hubble parameter, E, as

$$r = \frac{1}{E}\frac{d^2E}{dx^2} + \frac{1}{E^2}\left(\frac{dE}{dx}\right)^2 + \frac{3}{E}\frac{dE}{dx} + 1. \tag{16}$$

Using the definition of statefinder parameter s in (1) and also $\Omega_{tot} = 1 + \Omega_k$, the parameter s can be obtained in terms of E as

$$s = -\frac{\frac{1}{E}\frac{d^2E}{dx^2} + \frac{1}{E^2}(\frac{dE}{dx})^2 + \frac{3}{E}\frac{dE}{dx} + \Omega_k}{\frac{3}{E}\frac{dE}{dx} + \frac{3}{2}\Omega_k + \frac{9}{2}}. \tag{17}$$

Substituting E from (9) in (16) and (17), we obtain the parameters r and s for new HDE model

$$\begin{aligned} r = 1 + \frac{-2\alpha+3\beta+2}{\beta^2}&\Bigg[\bigg(\frac{1}{-\alpha+\beta+1}\Omega_k \\ &+ \frac{2}{-2\alpha+3\beta+2}\Omega_{m0} - 1\bigg)(\alpha-1)e^{-\frac{2(\alpha-1)x}{\beta}} \\ &- \frac{\beta^2}{(-\alpha+\beta+1)(-2\alpha+3\beta+2)}\Omega_k e^{-2x}\Bigg] \\ &\Big/\Bigg[-\bigg(\frac{1}{-\alpha+\beta+1}\Omega_k + \frac{2}{-2\alpha+3\beta+2}\Omega_{m0} - 1\bigg) \\ &\times(\alpha-1)e^{-\frac{2(\alpha-1)x}{\beta}} + \frac{1}{-\alpha+\beta+1}\Omega_k e^{-2x} \\ &+ \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x}\Bigg], \end{aligned} \tag{18}$$

$$\begin{aligned} s = \frac{2}{3} \times &\Bigg[\bigg(\frac{1}{-\alpha+\beta+1}\Omega_k + \frac{2}{-2\alpha+3\beta+2}\Omega_{m0} - 1\bigg) \\ &\times\bigg(\frac{\alpha-1}{\beta}(-2\alpha+3\beta+2) + \beta\Omega_k\bigg)e^{-\frac{2(\alpha-1)x}{\beta}} \\ &- \beta\Omega_k\bigg(\frac{1+\Omega_k}{-\alpha+\beta+1}e^{-2x} \\ &+ \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x}\bigg)\Bigg] \\ &\Big/\Bigg[\bigg(\frac{1}{-\alpha+\beta+1}\Omega_k + \frac{2}{-2\alpha+3\beta+2}\Omega_{m0} - 1\bigg) \\ &\times(-2\alpha+3\beta+2+\beta\Omega_k)e^{-\frac{2(\alpha-1)x}{\beta}} \\ &- \beta\Omega_k\bigg(\frac{1+\Omega_k}{-\alpha+\beta+1}e^{-2x} \\ &+ \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x}\bigg)\Bigg]. \end{aligned} \tag{19}$$

Equations (18,19) show the dependency of the statefinder pair $\{r, s\}$ on the model parameter of new HDE model α and β as well as the curvature parameter Ω_k.

Recently, Wang and Xu (2010), by applying the Markov Chain Monte Carlo method on the latest observational data, have constrained the new HDE model. The observational data that have been used are: the constitution dataset (Hicken et al. 2009) including 397 type supernova Ia (SNIa), the observational Hubble data (OHD) (Simon et al. 2005), the cluster X-ray gas mass fraction (Allen et al. 2008), the measurement results of baryon acoustic oscillation (BAO) from Sloan Digital Sky Survey (SDSS) (Eisenstein et al. 2005) and Two Degree Field Galaxy Redshift Survey (2dFGRS) (Percival et al. 2010), and the cosmic microwave background (CMB) data from five-year WMAP (Komatsu et al.

2009; Dunkley et al. 2009). In non flat universe, the best fit values of the new HDE model parameters (α, β) and the cosmological parameters $(\Omega_b h^2, H_0, \Omega_k, \Omega_{\Lambda 0}, \Omega_{m0})$ with their confidence level are obtained as: $\alpha = 0.8824^{+0.2180}_{-0.1163}$ (1σ) $^{+0.2213}_{-0.1378}$ (2σ), $\beta = 0.5016^{+0.0973}_{-0.0871}$ (1σ) $^{+0.1247}_{-0.1102}$ (2σ), $\Omega_b h^2 = 0.0228^{+0.0010}_{-0.0010}$ (1σ) $^{+0.0014}_{-0.0014}$ (2σ), $H_0 = 70.20^{+3.03}_{-3.17}$ (1σ) $^{+3.58}_{-4.00}$ (2σ), $\Omega_k = 0.0305^{+0.0092}_{-0.0134}$ (1σ) $^{+0.0140}_{-0.0176}$ (2σ), $\Omega_{\Lambda 0} = 0.6934^{+0.0364}_{-0.0304}$ (1σ) $^{+0.0495}_{-0.0413}$ (2σ) and $\Omega_{m0} = 0.2762^{+0.0278}_{-0.0320}$ (1σ) $^{+0.0402}_{-0.0412}$ (2σ).

In the next section, we use the above best fit values of α and β for studying the cosmological behavior of new HDE and also the evolutionary trajectories of this model in $s - r$ plane.

3 Numerical results

In this section we give the numerically description of the cosmological evolution and the statefinder trajectories in $s - r$ plane for new HDE model in spatially non flat universe. Here we use the best fit values of the model parameters of new HDE as well as the best fit values of cosmological parameters discussed in previous section. The evolution of EoS parameter, deceleration parameter and dimensionless Hubble parameter, E of new HDE in non-flat universe is calculated. Also, the evolutionary behavior of new HDE in $s - r$ plane is performed. Solving (10), the evolution of EoS parameter w_Λ as a function of scale factor a is shown in Fig. 1. In both panels, we see that w_Λ starts from zero at the early time, representing the CDM dominated universe, and crosses the phantom divide ($w_\Lambda < -1$) later. In upper panel, by fixing α with the constrained observational value 0.8824, we vary the parameter β. The EoS parameter w_Λ becomes larger for smaller value of β. We also see that the new HDE model crosses the phantom divide earlier, for larger value of β. In lower panel, by fixing β with the constrained observational value 0.5016, the parameter α is varied. Unlike β, The EoS parameter w_Λ becomes larger, for larger value of α. The phantom divide is achieved earlier, for smaller value of α. Here we showed the dependency of the EoS of new HDE on the parameters of model.

In Fig. 2, using (14), the evolution of deceleration parameter q as a function of scale factor a is plotted. In upper panel, we fixed $\alpha = 0.8824$ and varied the parameter β. The transition from decelerated phase ($q > 0$) to accelerated phase ($q < 0$) takes place sooner, by increasing the parameter β. Also, at any cosmic scale factor, the parameter q is smaller by increasing the parameter β. In lower panel, the behavior of deceleration parameter is studied by fixing $\beta = 0.5016$ and varying α. We see that q becomes larger by increasing α. Here, we find the dependency of the deceleration parameter q on the parameters of new HDE model.

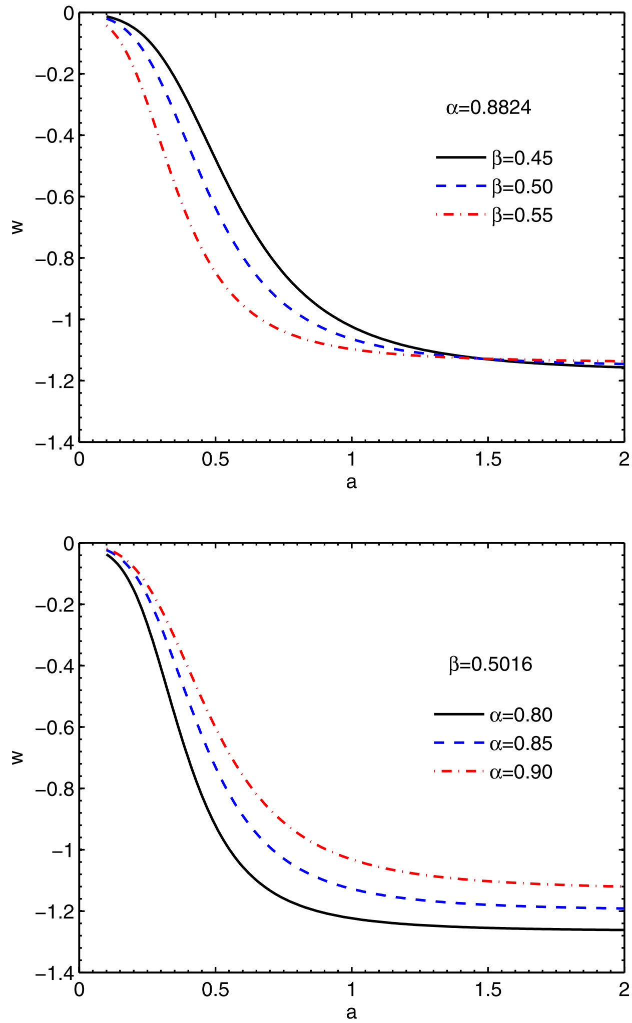

Fig. 1 The evolution of EoS parameter of new HDE model, w_Λ, versus scale factor a for different values of model parameters α and β in non flat universe with $\Omega_k = 0.0305$. In *upper panel*, by fixing α as a best fit value: $\alpha = 0.8824$, we vary β as 0.45, 0.50, 0.55 corresponding to *black solid line*, *blue dashed line* and *red dotted-dashed line*, respectively. In *lower panel*, by fixing β as a best fit value: $\beta = 0.5016$, α is varied as 0.80 (*black solid line*), 0.85 (*blue dashed line*), 0.90 (*red dotted-dashed line*)

Calling (9), we plot the evolution of dimensionless Hubble parameter, $E(a)$, for new HDE model in Fig. 3. In upper panel, we fix $\alpha = 0.8824$ and vary the parameter β. The smaller value the parameter β is taken, the bigger the Hubble parameter expansion rate $E(a)$ can reach. In lower panel, by fixing $\beta = 0.5016$, we vary the parameter α. The dimensionless Hubble parameter E is bigger for larger value of α at any scale factor $a < 1$. While for $a > 1$, E is bigger for smaller value of α. From this figure, we find that both the model parameters α and β can impact the cosmic expansion history in new HDE model.

Finally, we discuss the statefinder diagnostic for new HDE model. The statefinder pair $\{r, s\}$ in this model is given by (18) and (19). One can easily see the dependency of $\{r, s\}$

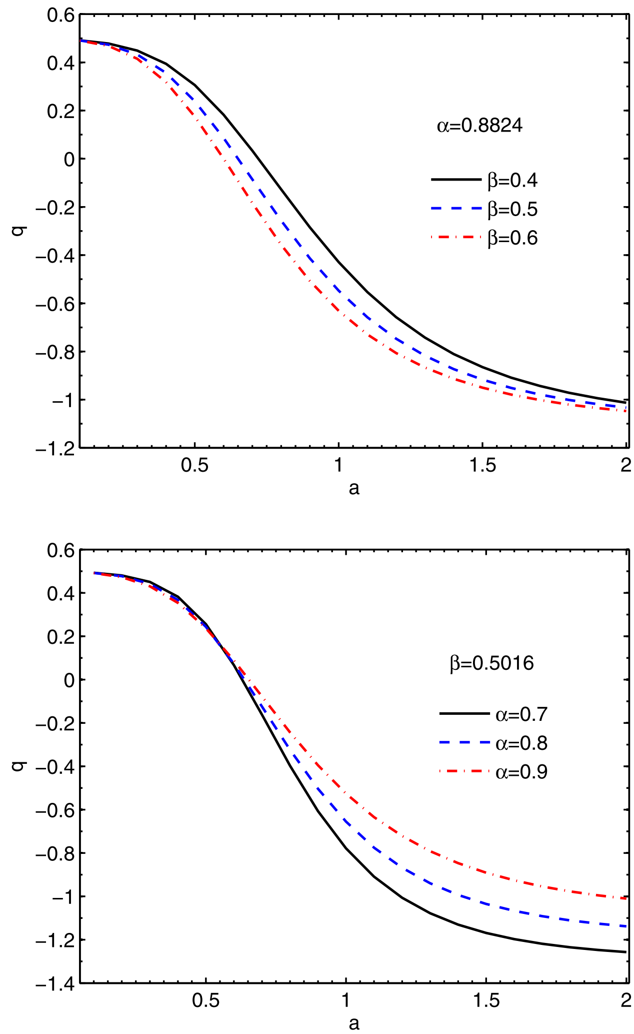

Fig. 2 The evolution of deceleration parameter q in new HDE model versus scale factor a for different values of model parameters α and β in non flat universe with $\Omega_k = 0.0305$. In *upper panel*, by fixing α, we vary β as 0.4, 0.5, 0.6 corresponding to *black solid line*, *blue dashed line* and *red dotted-dashed line*, respectively. In *lower panel*, by fixing β, α is varied as 0.7 (*black solid line*), 0.8 (*blue dashed line*), 0.9 (*red dotted-dashed line*)

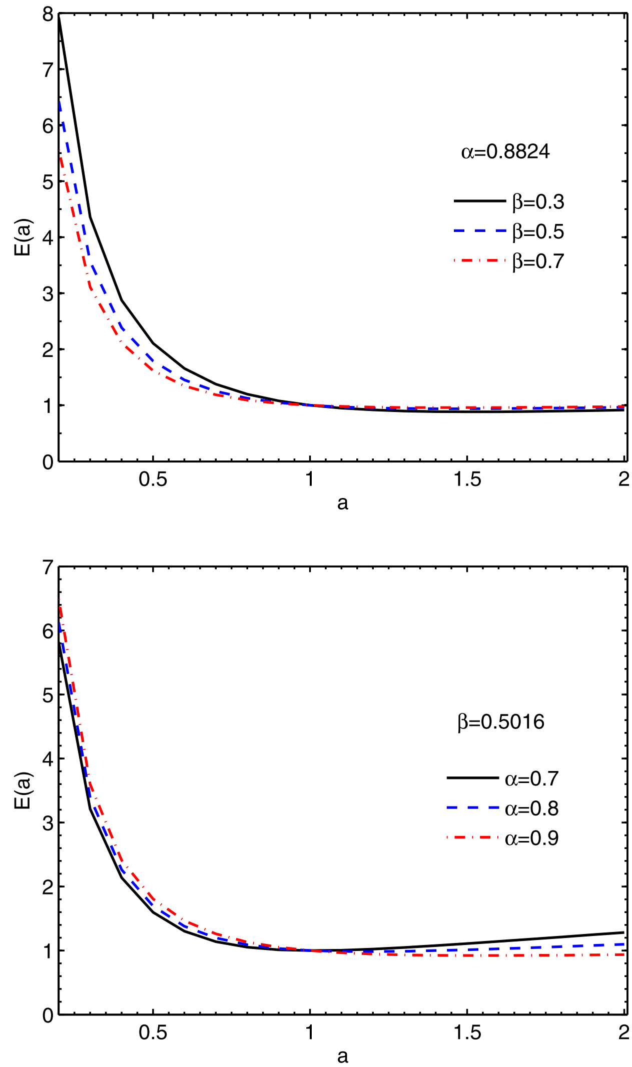

Fig. 3 Cosmological evolution of dimensionless Hubble parameter, E, as a function of scale factor in non flat universe for new HDE model. In *upper panel*, we choose the observational best fit values: $\alpha = 0.8824$ and $\Omega_k = 0.0305$ and vary the parameter β as 0.3, 0.5 and 0.7 corresponding to *black solid*, *blue dashed* and *red dotted-dashed lines*, respectively. In *lower panel*, by fixing $\beta = 0.5016$, α is varied as 0.7, 0.8, 0.9 corresponding to *black solid line*, *blue dashed line* and *red dotted-dashed line*, respectively

on the parameters of new HDE model as well as the curvature parameter Ω_k, in (18) and (19). In spatially flat universe, where $\Omega_k = 0.0$, the parameters r and s reduce as

$$r = 1 + \frac{-2\alpha + 3\beta + 2}{\beta^2} \times \left[\left(\frac{2}{-2\alpha+3\beta+2}\Omega_{m0} - 1\right)(\alpha-1)e^{-\frac{2(\alpha-1)x}{\beta}}\right] \Big/ \left[-\left(\frac{2}{-2\alpha+3\beta+2}\Omega_{m0} - 1\right)(\alpha-1)e^{-\frac{2(\alpha-1)x}{\beta}} + \frac{2}{-2\alpha+3\beta+2}\Omega_{m0}e^{-3x}\right], \tag{20}$$

$$s = \frac{2(\alpha-1)}{3\beta}. \tag{21}$$

From (21), we see that the parameter s is independent of cosmic scale factor in spatially flat universe. By Choosing $\alpha = 1, \beta \neq 0$, we get $\{r = 1, s = 0\}$ which is coincide to the location of spatially flat ΛCDM model in statefinder plane.

In Fig. 4, we show the evolutionary trajectories of new HDE model in statefinder plane for non flat universe with $\Omega_k = 0.0305$. In this diagram, the standard ΛCDM model in spatially flat universe corresponds to a fixed point $\{r = 1, s = 0\}$ indicated by star symbol. In upper panel, by fixing $\alpha = 0.8824$, we choose the illustrative values 0.4, 0.5 and

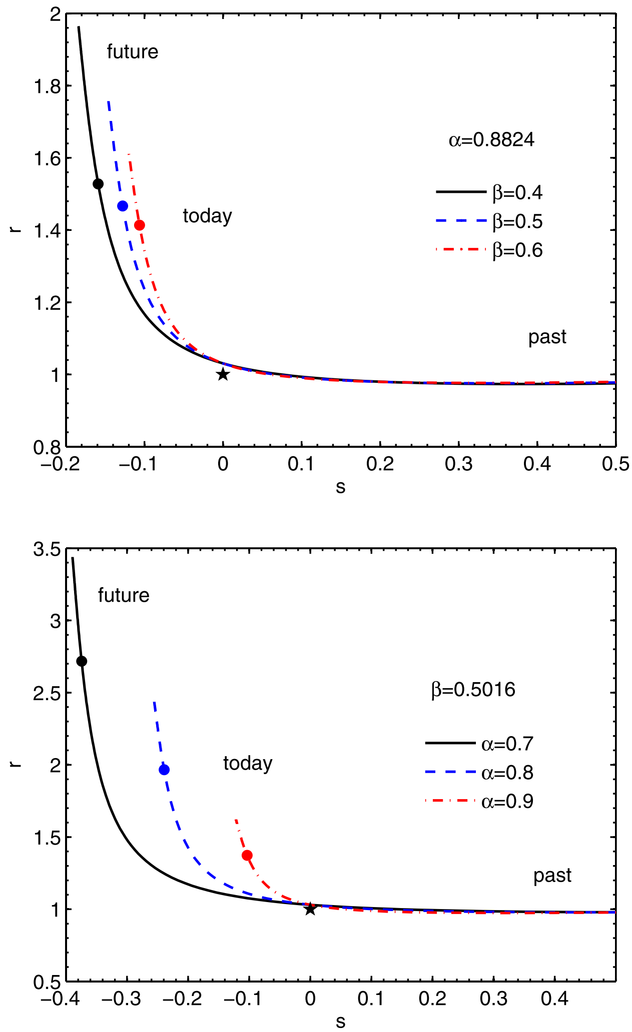

Fig. 4 An illustrative example for the statefinder diagnostic of new HDE model in non flat universe. In *upper panel*, the evolutionary trajectories in $s-r$ plane are plotted, by fixing $\alpha = 0.8824$ and varying β as 0.4, 0.5 and 0.6 corresponding to *black solid line*, *blue dashed line* and *red dotted dashed line*, respectively. The *circle point* on the curves show the today's value of statefinder parameters (s_0, r_0). The *star symbol* indicates the location of standard flat ΛCDM model in $s-r$ plane: $\{s=0, r=1\}$. In *lower panel*, the evolutionary trajectories are plotted for different illustrative values of α, by fixing $\beta = 0.5016$. The evolutionary trajectories of illustrative cases $\alpha = 0.7$, $\alpha = 0.8$ and $\alpha = 0.9$ have been shown by *black solid line*, *blue dashed line* and *red dotted-dashed line*, respectively. *Circle point* on the curves denotes the today's value (s_0, r_0) in $s-r$ plane. Same as upper panel, the *star symbol* indicates the location of standard flat ΛCDM model

0.6 for β. While the universe expands, the trajectories of the statefinder start from right to left. The parameter r increases while the parameter s decreases. The color circles on the curves represent the today's value of the statefinder parameter (s_0, r_0). The current data of s and r are valuable, if they can be extracted from coming data of SNAP (SuperNova Acceleration Probe) experiments. Hence, the statefinder diagnostic combined with future SNAP observation can be useful to discriminate between various dark energy models. Here, we can easily see that the statefinder trajectories are dependent on the parameter β of new HDE model. Different values of β give the different evolutionary trajectories in $\{r, s\}$ plane. Also, distance of the point (s_0, r_0) to ΛCDM fixed point becomes shorter for larger value of β. We can also see that for larger value of β, the present value s_0 increases and the present value r_0 decreases.

In lower panel, by fixing $\beta = 0.5016$, the evolutionary trajectories in $s-r$ diagram is plotted for different values of the parameter α. Same as upper panel, by expanding the universe, the trajectories start from right to left. The parameter r increases while the parameter s decreases. Also, it can be seen that the statefinder trajectories are dependent on the parameter α of new HDE model. Different values of α give the different evolutionary trajectories in this plane. The distance of the point (s_0, r_0) to ΛCDM fixed point becomes shorter for larger value of α. Like the effect of β, the present value s_0 increases while the present value r_0 decreases, by taking the larger value of α.

In Fig. 5, by using the best fit values for cosmological parameters: ($\Omega_b h^2 = 0.0228$, $H_0 = 0.7020$, $\Omega_k = 0.0305$, $\Omega_{\Lambda 0} = 0.6934$, $\Omega_{m0} = 0.2762$), we plot the evolutionary trajectory in $s-r$ plane (upper panel) and $q-r$ plane (lower panel) for fixed observational values: $\alpha = 0.8824$ and $\beta = 0.5016$. In upper panel the evolutionary trajectory starts from right at the past time, reaches to $(s_0 = -0.13, r_0 = 1.46)$ at the present time (circle point). In lower panel the evolutionary trajectory in $q-r$ plane starts from $(q = 0.5, r = 1)$ at the past, representing the CDM dominated universe at the early time, reaches to $(q = -0.55, r = 1.46)$ at the present time.

4 Conclusion

In this work, we investigated the holographic dark energy model with new infrared cut-off proposed by Granda and Oliveros in spatially non-flat universe. Contrary to the HDE model based on event horizon, this model depends on the local quantities and avoids the causality problem. Therefore, The new HDE model can be assumed as a phenomenological model for holographic energy density. Here, we calculated some relevant cosmological parameters and their evolution. Also, the statefinder diagnostic is performed for new HDE model in non-flat universe. In summery,

(i) The EoS parameter, w_Λ, starts from $w_\Lambda > -1$ at the early time and crosses the phantom divide $w_\Lambda < -1$ at the late time. This behavior of w_Λ is dependent on the model parameters. The larger value of α and smaller value of β give the larger EoS parameter, w_Λ. Also for smaller value of α or larger value of β, the phantom divide is achieved earlier.

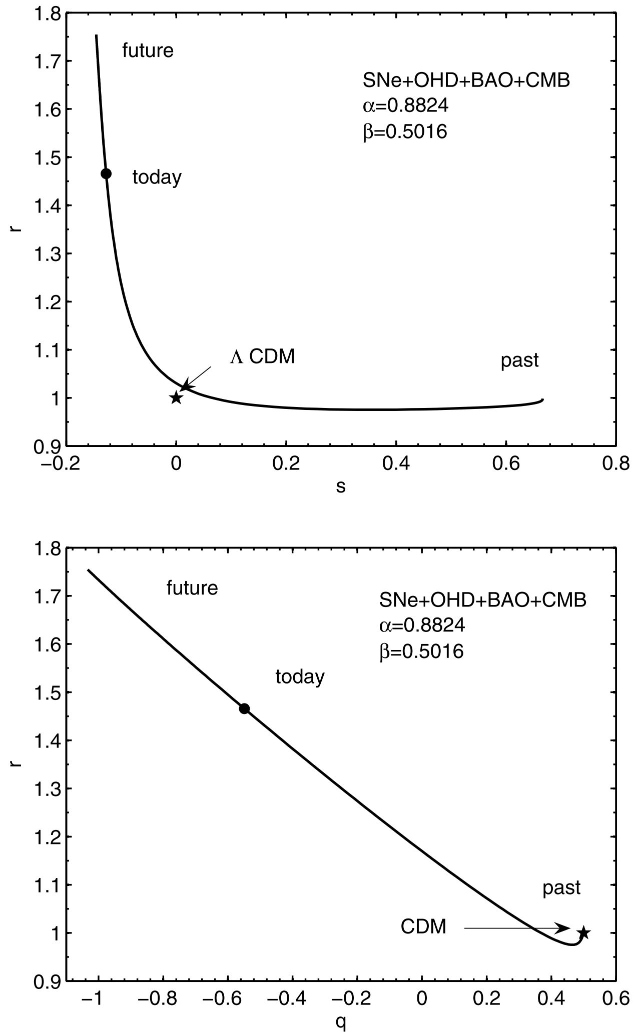

Fig. 5 The statefinder diagrams $r(s)$ (*upper panel*) and $r(q)$ (*lower panel*) for new HDE model in the light of best fit results of SNe + OHD + BAO + CMB experiments. The *circles* on the curves indicates the present value (s_0, r_0) in *upper panel*, and (q_0, r_0) in *lower panel*. The *star symbol* on the *upper panel* indicates the location of standard ΛCDM and in the *lower panel* represents the CDM dominated universe

In new HDE model, the universe undergoes decelerated expansion at the early time ($q > 0$) and then starts accelerated expansion ($q < 0$) at the later time. The transition epoch from decelerated phase to accelerated phase occurs sooner by increasing β or α.

The cosmic expansion history in new HDE model is dependent on the model parameters α and β. The smaller value of β is taken, the bigger Hubble parameter can reach. Also, the Hubble parameter become larger by increasing α at $a < 1$ and decreasing α at $a > 1$.

(ii) We studied the new HDE model from the viewpoint of statefinder diagnostic. The statefinder diagnostic is a crucial tool for discriminating different DE models. Also, the present value of $\{r, s\}$ can be viewed as a discriminator for testing different DE models if it can be extracted from precise observational data in a model-independent way. We calculate the evolution of new HDE model in the statefinder plane for different values of the model parameter α and β. The statefinder trajectories are dependent on the model parameters. Different values of α and β are taken, different evolutionary trajectories are achieved. By expanding the universe, the trajectories start from right to left in $s - r$ plane, the parameter s decreases and r increases. Distance of the present value (s_0, r_0) from the ΛCDM fixed point $(s = 0, r = 1)$ becomes shorter for larger values of β and α.

We also performed the statefinder diagnostic in $s - r$ and $q - r$ planes for new HDE model in the light of best fit results of SNe + BAO + OHD + CMB experiments. These trajectories yield $(s_0 = -0.13, r_0 = 1.46)$ and $(q = -0.55, r = 1.46)$ at present time. The evolutionary trajectory in $q - r$ plane starts from $(q = 1/2, r = 1.0)$ which is coincidence on the location of CDM model in $s - r$ plane.

Finally, it is of interest to compare the new HDE model and holographic DE (HDE) model from the viewpoint of statefinder diagnostic. The statefinder diagnostic for HDE model in non-flat universe is preformed in Setare et al. (2007). In the light of best fit result of the SN + CMB data analysis, the evolutionary trajectories in $s - r$ and $q - r$ planes gives the present values: $(s_0 = -0.102, r_0 = 1.357)$ and $(q_0 = -0.590, r_0 = 1.357)$ for HDE model in non-flat universe (Setare et al. 2007). Therefore the distance from the ΛCDM fixed point $(s = 0, r = 1.0)$ is shorter for new HDE model compare with HDE model. As a similarity, for both HDE and new HDE models, the trajectories in $q - r$ plane starts from $q = 1/2, r = 1$ at the early time which is denoting the CDM-dominated universe. WE hope that the future high-precision SNAP-type observations can determine the statefinder parameters and consequently single out the right cosmological DE models.

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