Astrophys Space Sci (2011) 332: 481–490
DOI 10.1007/s10509-010-0541-5

ORIGINAL ARTICLE

A family of well behaved charge analogues of a well behaved neutral solution in general relativity

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Received: 11 October 2010 / Accepted: 7 November 2010 / Published online: 17 November 2010

Abstract A family of charge analogues of a neutral solution with $g_{44} = (1 + Cr^2)^6$ has been obtained by using a specific electric intensity, which involves a parameter K. Both neutral and charged solutions are analysed physically subject to the surface density 2×10^{14} gm/cm^3 (neutron star). The neutral solution is well behaved for $0.0 < Ca^2 \leq 0.10477$ while its charge analogues are well behaved for a wide range of a parameter K $(0 \leq K \leq 72)$ i.e. pressure, density, pressure-density ratio, velocity of sound is monotonically decreasing and the electric intensity is monotonically increasing in nature for the given range of the parameter K. The maximum mass and radius occupied by the neutral solution are $3.4126 M_\Theta$ and 18.9227 km for $Ca^2 = 0.10447$ respectively. While the red shift at centre $Z_0 = 0.9686$ and red shift at the surface $Z_a = 0.4612$. For the charged solution, the maximum mass and radius are $5.6111 M_\Theta$ and 17.2992 km respectively for $K = 3.0130$ and $Ca^2 = 0.2500$, with the red shift $Z_0 = 3.0113$ and $Z_a = 1.0538$.

Keywords Canonical coordinates · Charged fluids · Superdense star · General relativity

1 Introduction

Inception of the Schwarzchild's exterior solution representing gravitational field of neutral spherical material distribution, persuaded the research workers to derive interior solutions which can join smoothly to the aforesaid solution at the pressure free interface. The solutions composed of perfect fluid in this context are counted roughly as one hundred and sixty two (Delgaty and Lake 1998). This does not include the anisotropic fluid distributions. Then entry of the Nordstrom's metric describing the gravitational field of charged fluid, provided an open way to the flood of charge analogues of the neutral solutions obtained so far (Ivanov 2002; Gupta and Gupta 1986; Gupta and Kumar 2005a, 2005b; Bijalwan and Gupta 2008; Gupta and Maurya 2010a; Gupta et al. 2010; Pant et al. 2010b; Lemos and Zanchin 2010a, 2010b; Sharif and Abbas 2010; Maharaj and Komathiraj 2007a, 2007b, 2008; Maharaj and Thirukkanesh 2006a, 2006b, 2009; Maharaj and Hansraj 2006). The utility of charge fluids is mainly because of the fact that the presence of charge averts the gravitational collapse of the material ball to a point singularity (Krasinski 1997). Moreover the charged dust model (pressure free charged fluid) and model with electromagnetic mass increase hopes towards the formation of model of an electron (Bonnor 1960).

The present research article contains derivation of charge analogues of a neutral solution with the metric potential $g_{44} = (1 + Cr^2)^6$, which is a member of the family of neutral solutions with $g_{44} = (1 + Cr^2)^n$, $n = 1, 2, 3, 4, 5, 6, 7$. The members of this family were derived time to time by various authors i.e. Tolman (1939) for $n = 1$, Kuchowicz (1975), Adler (1974) and Adams and Cohen (1975) for $n = 2$, Heintzmann (1969) for $n = 3$ and Durgapal (1982) for $n = 4, 5$ and Pant et al. (2010a) for $n = 6, 7$. The detailed analyses of the solutions are available in the literature Durgapal (1982) and Pant et al. (2010a). Some of them are already charged by Gupta and Maurya (2010b) for $n = 5$, Pant et al. (2010b) for $n = 3$. The charged solutions obtained in this article are analysed physically and the results are tabulated and graphed for various values of the parameters involved.

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2 Field equations

Let us consider a spherically symmetric metric curvature co-ordinates as

$$ds^2 = -e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu}dt^2 \tag{1}$$

where the functions $\lambda(r)$ and $\nu(r)$ satisfy the Einstein-Maxwell equations

$$\begin{aligned}-\kappa T^i_j &= R^i_j - \frac{1}{2}R\delta^i_j \\ &= -\kappa\left[(c^2\rho + p)v^i v_j - p\delta^i_j \right. \\ &\quad \left. + \frac{1}{4\pi}\left(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F_{mn}F^{mn}\right)\right]\end{aligned} \tag{2}$$

with $\kappa = \frac{8\pi G}{c^4}$ while ρ, p, v^i, F_{ij} denote energy density, fluid pressure, flow vector and skew-symmetric electromagnetic field tensor respectively. The resulting field equations are

$$\frac{\lambda'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \kappa c^2\rho + \frac{q^2}{r^4} \equiv \kappa T^0_0 \tag{3}$$

$$\frac{\nu'}{r}e^{-\lambda} - \frac{(1-e^{-\lambda})}{r^2} = \kappa p - \frac{q^2}{r^4} \tag{4}$$

$$\left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right]e^{-\lambda} = \kappa p + \frac{q^2}{r^4}. \tag{5}$$

Equations (5) and (6) give

$$e^{-\lambda}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} + \frac{\nu'}{2r} - \frac{1}{r^2}\right] - e^{-\lambda}\lambda'\left[\frac{\nu'}{4} + \frac{1}{2r}\right] = \kappa p + \frac{q^2}{r^4}. \tag{6}$$

Let us now assume that the value of ν is given by a general expression

$$y^2 = e^{\nu} = B(1 + Cr^2)^n, \quad C > 0 \tag{6a}$$

where B is an arbitrary constant.

Substitution of (6a) in to (3)–(5) leads to

$$\frac{2nZ}{(1+x)} - \frac{(1-Z)}{x} + \frac{Cq^2}{x^2} = \frac{\kappa p}{C}, \tag{7}$$

$$\frac{(1-Z)}{x} - 2\frac{dZ}{dx} - \frac{Cq^2}{x^2} = \frac{\kappa c^2\rho}{C} \tag{8}$$

and

$$\frac{dZ}{dx} + P(x)Z = f(x) \tag{9}$$

where

$$x = Cr^2, \qquad e^{-\lambda} = Z,$$

$$P(x) = \frac{-[1 + 2x + (1 - 2n - n^2)x^2]}{x(1+x)[1 + (n+1)x]},$$

$$f(x) = \frac{[(2q^2C/x) - 1]}{x[1 + (n+1)x]}$$

3 New class of solutions

In order to solve the differential equation (9) for obtaining the new solution, we consider $n = 6$ and the electric intensity E of the form

$$\frac{E^2}{C} = \frac{Cq^2}{x^2} = \frac{K}{2}x(1+7x)^{5/7}, \tag{10}$$

where $K \geq 0$ is constant. The electric field intensity given by (10) is physically palatable since E^2 remains regular and positive throughout the sphere. In addition, the electric field given by (10) vanishes at the centre of the star.

With reference to (10) the differential equation (9) yields following solution

$$\begin{aligned}Z &= \frac{Kx(1+x)^2}{6(1+7x)^{2/7}} + \frac{1}{(1+x)^4}\left[1 - \frac{xg(x)}{71001}\right] \\ &\quad + \frac{Ax}{(1+x)^4(1+7x)^{2/7}}\end{aligned} \tag{11}$$

where $g(x) = x(3087x^3 + 21609x^2 + 74088x + 317961)$ and A is an arbitrary constant of integration.

Using (11), into (7) and (8), we get the following expressions for pressure and energy density

$$\begin{aligned}\frac{\kappa p}{C} &= \frac{K}{12}\left[\frac{(68x^2 + 34x + 2)}{(1+7x)^{2/7}}\right] \\ &\quad + \frac{1}{(1+x)^5}\left[\frac{h(x)}{71001} + \frac{A(1+13x)}{(1+7x)^{2/7}}\right]\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\kappa c^2\rho}{C} &= -\frac{K}{12}\left[\frac{l(x)}{(1+7x)^{2/7}}\right] \\ &\quad + \frac{1}{(1+x)^5}\left[\frac{L(x)}{71001} - \frac{A(3 + 12x - 39x^2)}{(1+7x)^{9/7}}\right]\end{aligned} \tag{13}$$

with

$$\begin{aligned}h(x) &= 250047 - 4917591x - 1694763x^2 \\ &\quad - 639009x^3 - 111132x^4,\end{aligned}$$

$$l(x) = 6 + 60x + 222x^2 + 384x^3,$$

$$L(x) = 1805895 - 509355x + 639009x^2$$

$$+361179x^3 + 74088x^4.$$

Consequently the expressions for pressure and density gradients read as

$$\frac{\kappa}{C}\frac{dp}{dx} = \frac{K}{12}\left[\frac{(30+306x+816x^2)}{(1+7x)^{9/7}}\right] + \frac{Q(x)}{71001(1+x)^6} + \frac{AR(x)}{(1+x)^6(1+7x)^{9/7}} \tag{14}$$

$$\frac{\kappa c^2}{C}\frac{d\rho}{dx} = -\frac{K}{12}\left[\frac{(6+324x+2262x^2+4608x^3)}{(1+7x)^{16/7}}\right] + \frac{H(x)}{71001(1+x)^6} + \frac{AT(x)}{(1+x)^6(1+6x)^{16/7}} \tag{15}$$

and hence the velocity of sound v is given by the following expression

$$\frac{v^2}{c^2} = \frac{dp}{c^2 d\rho} = \frac{(1+7x)\cdot\{71001\cdot[K\cdot U(x)\cdot(1+x)^6 + 12\cdot A\cdot R(x)] + 12\cdot Q(x)\cdot(1+7x)^{9/7}\}}{71001\cdot[K\cdot V(x)(1+x)^6 + 12\cdot A\cdot T(x)] + 12\cdot H(x)\cdot(1+7x)^{16/7}} \tag{16}$$

where

$$Q(x) = (-6167826 + 16280838x + 3167262x^2 + 833490x^3 + 111132x^4),$$

$$H(x) = (-9538830 + 3315438x - 833490x^2 - 426006x^3 - 74088x^4),$$

$$U(x) = (30 + 306x + 816x^2),$$

$$T(x) = (30 + 282x + 522x^2 - 1170x^3),$$

$$R(x) = (6 - 24x - 390x^2),$$

$$V(x) = -(6 + 324x + 2262x^2 + 4608x^3).$$

4 Conditions for regular and well behaved model

(i) Pressure p should be zero at boundary $r = a$

(ii) $c^2\rho \geq p > 0$ or $c^2\rho \geq 3p > 0$, $0 \leq r \leq a$, where former inequality denotes weak energy condition (WEC), while the later inequality implies strong energy condition (SEC)

(iii) $(dp/dr)_{r=0} = 0$ and $(d^2p/dr^2)_{r=0} < 0$ so that pressure gradient dp/dr is negative for $0 < r \leq a$.

(iv) $(d\rho/dr)_{r=0} = 0$ and $(d^2\rho/dr^2)_{r=0} < 0$ so that density gradient $d\rho/dr$ is negative for $0 < r \leq a$

The condition (iii) and (iv) imply that pressure and density should be maximum at the centre and monotonically decreasing towards the surface.

(v) The casualty condition $(dp/c^2d\rho)^{1/2}$ i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e. $\frac{d}{dr}(\frac{dp}{d\rho}) < 0$ or $(\frac{d^2p}{d\rho^2}) > 0$ for $0 \leq r \leq a$ i.e. the velocity of sound is increasing with the increase of density.

(vi) The ratio of pressure to the density $(p/c^2\rho)$ should be monotonically decreasing with the increase of r i.e. $\frac{d}{dr}(\frac{p}{c^2\rho})_{r=0} = 0$ and $\frac{d^2}{dr^2}(\frac{p}{c^2\rho})_{r=0} < 0$ and $\frac{d}{dr}(\frac{p}{c^2\rho})$ is negative valued function for $r > 0$. Which implies that the temperature of the model decreases towards the surface?

(vii) The central red shift Z_0 and surface red shift Z_a should be positive and finite i.e. $Z_0 = [(e^{-\nu/2} - 1)_{r=0}] > 0$ and $Z_a = [e^{\lambda(a)/2} - 1] > 0$ and both should be bounded.

(viii) The solution should be free from physical and geometric singularities. Including $e^{\lambda} = 1$ at $r = 0$

(ix) Electric intensity E, such that $E(0) = 0$, is taken to be monotonically increasing i.e. $(dE/dr) > 0$ for $0 < r < a$.

5 Properties of new class of solution

$$\left[\frac{\kappa p}{C}\right]_{r=0} = \frac{250047}{71001} + A + \frac{K}{6} \tag{17a}$$

$$\left[\frac{\kappa c^2\rho}{C}\right]_{r=0} = \frac{1805895}{71001} - 3A - \frac{K}{2} \tag{17b}$$

$$\left[\frac{\kappa}{C}\frac{dp}{dr}\right]_{r=0} = 2Cr\left[\frac{\kappa}{C}\frac{dp}{dx}\right]_{x=0} = 0 \tag{18a}$$

$$\left[\frac{\kappa c^2}{C}\frac{d\rho}{dr}\right]_{r=0} = 2Cr\left[\frac{\kappa}{C}\frac{d\rho}{dx}\right]_{x=0} = 0 \tag{18b}$$

$$\left[\frac{\kappa}{C}\frac{d^2p}{dr^2}\right]_{r=0} = 2C\left[-\frac{6167826}{71001} + 6A + \frac{5K}{2}\right] \tag{19a}$$

$$\left[\frac{\kappa c^2}{C}\frac{d^2\rho}{dr^2}\right]_{r=0} = 2C\left[-\frac{9538830}{71001} + 30A + \frac{K}{2}\right] \tag{19b}$$

For $p_{r=0}$ and $\rho_{r=0}$ must be positive, $\frac{p_{r=0}}{\rho_{r=0}} \le 1$ and $(d^2 p/dr^2)_{r=0} < 0$, $(d^2\rho/dr^2)_{r=0} < 0$. Consequently we have

$$-\frac{K}{6} - \frac{250047}{71001} < A < -\frac{5K}{12} + \frac{6167826}{426006}, \quad K \ge 0. \tag{20}$$

Hence velocity of sound at the centre given by

$$\left[\frac{dp}{c^2 d\rho}\right]_{r=0} = \left[\frac{6167826}{71001} - 6A - \frac{5K}{2}\right] \Big/ \left[\frac{9538830}{71001} - 30A - \frac{K}{2}\right] \tag{21}$$

which should be $[\frac{dp}{c^2 d\rho}]_{r=0} \le 1$, for all values of $K \ge 0$ and A.

Using (12) and (13)

$$\left[\frac{p}{c^2\rho}\right] = \frac{I(x)}{J(x)} \tag{21a}$$

where

$$I(x) = \frac{K}{12}\left[\frac{(68x^2+34x+2)}{(1+7x)^{2/7}}\right] + \frac{1}{(1+x)^5}\left[\frac{h(x)}{71001} + \frac{A(1+13x)}{(1+7x)^{2/7}}\right],$$

$$J(x) = -\frac{K}{12}\left[\frac{l(x)}{(1+7x)^{2/7}}\right] + \frac{1}{(1+x)^5}\left[\frac{L(x)}{71001} - \frac{A(3+12x-39x^2)}{(1+7x)^{9/7}}\right].$$

Differentiating (21a) w.r.t. x

$$\frac{d}{dx}\left[\frac{p}{c^2\rho}\right] = \frac{J(x)\frac{dI(x)}{dx} - I(x)\frac{dJ(x)}{dx}}{\{J(x)\}^2} \tag{21b}$$

$$\left[\frac{d^2}{dr^2}\left(\frac{p}{c^2\rho}\right)\right]_{r=0} = 2C \cdot \frac{3\alpha\cdot\beta - \gamma\cdot\delta}{3\alpha^2}, \tag{21c}$$

where

$\alpha = (-7334712 + 355005 \cdot A + 852012 \cdot K)$,

$\beta = (3611790 - 426006 \cdot A - 71001 \cdot K)$,

$\gamma = (1500282 + 426006 \cdot A + 71001 \cdot K)$,

$\delta = (-19077660 + 4260060 \cdot A + 71001 \cdot K)$.

The expression of right hand site of (21c) is negative for all values of $K \ge 0$ and A. Then pressure-density ratio $\frac{p}{c^2\rho}$ is maximum at the centre.

The expression for gravitational red-shift Z_1 is given by

$$Z_1 = \frac{(1+x)^{-3}}{\sqrt{B}} - 1 \tag{22}$$

The central value of gravitational red-shift to be non zero positive finite, we have

$1 > \sqrt{B} > 0$

Differentiating (22) w.r.t. x, we get,

$$\left[\frac{d^2 Z_1}{dr^2}\right]_{r=0} = -\frac{6C}{\sqrt{B}} < 0. \tag{22a}$$

The expression of right hand site of (22a) is negative, and then the gravitational red shift is the maximum at centre and monotonically decreasing towards the surface.

Differentiating (10) w.r.t. x, we get,

$$\frac{d}{dx}\left(\frac{E^2}{C}\right) = \frac{K}{2}\left[\frac{(1+12x)}{(1+7x)^{2/7}}\right] \tag{23}$$

$$\left[\frac{d}{dr}\left(\frac{E^2}{C}\right)\right] = Cr\left[\frac{(1+12x)}{(1+7x)^{2/7}}\right]\cdot K = (+ve) \quad \text{for } 0 < r < a. \tag{23a}$$

Thus the electric intensity is zero at the centre and monotonically increasing towards the pressure free interface for all values of $K > 0$.

6 Boundary conditions

Besides the above, the charged fluid spheres is expected to join smoothly with the Reissner-Nordstrom metric at the pressure free boundary $r = a$

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) dt^2 \tag{24}$$

which requires the continuity of e^λ, e^ν and q across the boundary

$$e^{-\lambda(a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}, \tag{25}$$

$$e^{\nu(a)} = y^2_{(r=a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}, \tag{26}$$

$$q(a) = e, \tag{27}$$

$$p_{(r=a)} = 0. \tag{28}$$

The condition (28) can be utilized to compute the values of arbitrary constants A as follows: On setting $x_{r=a} = X =$

Ca^2 (a being the radius of the charged sphere), pressure at $p_{(r=a)} = 0$ gives

$$A = -\frac{F(X)\cdot(1+7X)^{2/7}}{71001(1+13X)} - \frac{K}{10}\frac{(2+34X+68X^2)(1+X)^5}{(1+13X)} \tag{29}$$

where

$$F(X) = 250047 - 4917591X - 1694763X^2 - 639009X^3 - 111132X^4$$

The expression for mass can be written as

$$m(a) = \frac{aX}{2(1+X)^4}\left[N(X) + \frac{g(X)}{71001} - \frac{A}{(1+7X)^{2/7}}\right] - \frac{KX}{10}\left[\frac{M(X)}{(1+7X)^{2/7}}\right] \tag{30}$$

such that $e^{-\lambda(a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}$, where $M = m(a)$ with

$$N(X) = (4 + 6X + 4X^2 + X^3),$$

$$M(X) = (1 - X - 20X^2)$$

$$g(X) = X(3087X^3 + 21609X^2 + 74088X + 317961)$$

and $y^2_{(r=a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}$ gives

$$B = \frac{KX}{5(1+X)^5(1+7x)^{2/7}} + \frac{1}{(1+X)^{10}}\left[1 - \frac{Xg(X)}{71001}\right] + \frac{AX}{(1+X)^{10}(1+7X)^{2/7}} \tag{31}$$

Also, if the surface density ρ_a is prescribed as 2×10^{14} g cm^{-3} (super dense star case) then value of constant C can be calculated for a given X ($= Ca^2$), using the following expression

$$\frac{\kappa c^2\rho}{C} = -\frac{K}{10}\left[\frac{l(X)}{(1+7X)^{2/7}}\right] + \frac{1}{(1+X)^5}\left[\frac{L(X)}{71001} - \frac{A(3+12x-39X^2)}{(1+7X)^{9/7}}\right]$$

With

$$l(X) = 6 + 60X + 222X^2 + 384X^3,$$

$$L(X) = 1805895 - 509355X + 639009X^2 + 361179X^3 + 74088X^4.$$

7 Conclusions

Owing to the various conditions mentioned in the preceding section we arrived at the following conclusions:

** The constants K and A satisfy the inequalities $0 \le K \le 72$ and $-(K/6) - 3.5217 < A < -(5\cdot K/12) + 14.4783$ respectively.

The maximum mass and radius occupied by the neutral solution are $3.4035 M_\Theta$ and 18.9227 km respectively for $Ca^2 = 0.10447$ with the red shift at centre $Z_0 = 0.9686$ and red shift at the surface $Z_a = 0.4612$ for (SEC) and (WEC) both (Table 5). Also in neutral case the solution is well behaved in the range $0 < Ca^2 \le 0.10447$ i.e. velocity of sound is monotonically decreasing in the given range of Ca^2. For the charged solution, the maximum mass and corresponding radius for (WEC) are found to be $5.6111 M_\Theta$ and 17.2992 km respectively for $K = 3.013$ and $Ca^2 = 0.2500$ with the red shift $Z_0 = 3.0113$ and $Z_a = 1.0538$ (Table 6). While for the (SEC), the maximum mass and radius are $5.4794 M_\Theta$ and 17.3553 km respectively for $K = 3.490$ and $Ca^2 = 0.2300$ with red shift $Z_0 = 2.7084$ and $Z_a = 0.9928$ (Table 7). All the charged superdense star models for the given range of K and A in section (**) are regular and well behaved in the sense of the conditions displayed in Sect. 4.

Detailed physically behaviour of the models for various K are displayed by means of graphs and tables below, where Figs. 1–5 are given with reference of Tables 2, 4, 6, 7, 9, 11.

8 Tables for numerical values of physical quantities

In Tables 1–4: Z_0 = red shift at the centre, Z_a = red shift at the surface, Solar mass $M_\Theta = 1.475$ km, $G = 6.673\times 10^{-8}$ cm^3/g s^2, $c = 2.997\times 10^{10}$ cm/s, $D = (8\pi G/c^2)\rho a^2$, $P = (8\pi G/c^4) p a^2$, $\gamma = \frac{p + c^2\rho}{p}\frac{dp}{c^2 d\rho}$, $R = \frac{p}{c^2\rho}$, $V = (dp/c^2 d\rho)^{1/2}$.

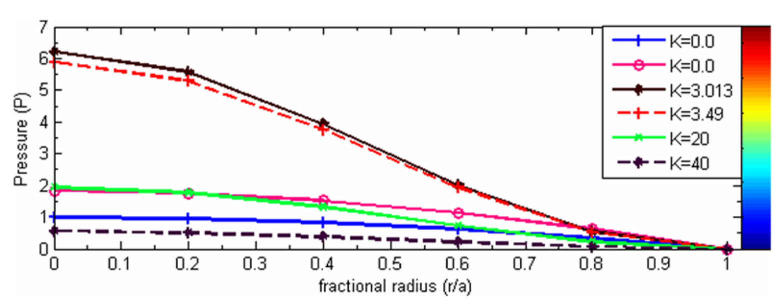

Fig. 1 Behaviour of pressure versus radius

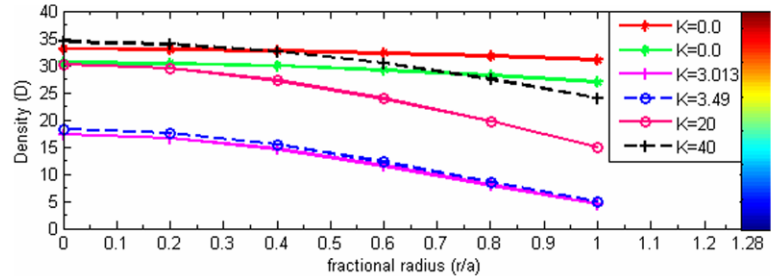

Fig. 2 Behaviour of density versus radius

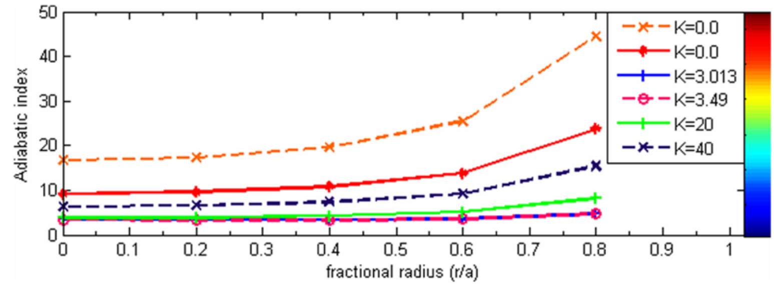

Fig. 3 Behaviour of adiabatic index (γ) versus radius

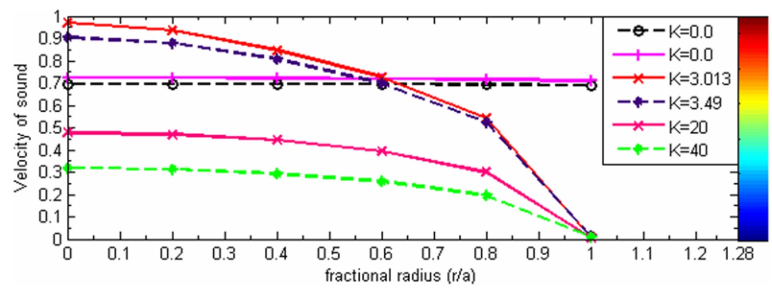

Fig. 4 Behaviour of velocity of sound versus radius

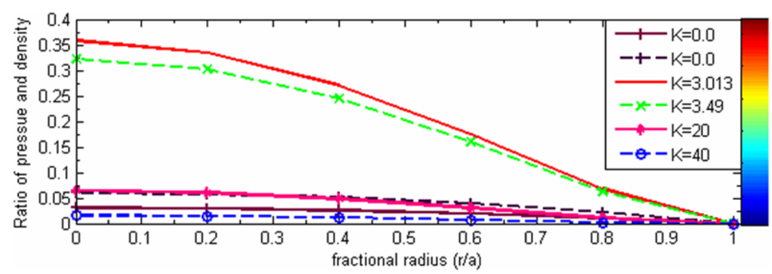

Fig. 5 Behaviour of ratio of pressure and density versus radius

Table 1 Maximum mass (M/M_Θ) and Radius (a) for different K and A for WEC and SEC

K	A	$0 \le p \le c^2\rho$ (WEC)			$0 \le 3p \le c^2\rho$ (SEC)		
		Ca^2	Radius (km)	Max M/M_Θ	Ca^2	Radius (km)	Max M/M_Θ
0.0	−2.5285	0.01025	9.2208	0.3393	0.01025	9.2208	0.3393
0.0	−2.0558	0.01598	11.0789	0.5963	0.01598	11.0789	0.5963
0.0	−1.6880	0.02089	12.2789	0.8206	0.02089	12.2789	0.8206
0.0	1.9768	0.10447	18.9364	3.4426	0.10447	18.9364	3.4426
3.013	2.1886	0.25000	17.2992	5.6111	–	–	–

Table 1 (*Continued*)

K	A	$0 \le p \le c^2\rho$ (WEC)			$0 \le 3p \le c^2\rho$ (SEC)		
		Ca^2	Radius (km)	Max M/M_Θ	Ca^2	Radius (km)	Max M/M_Θ
3.490	1.7910	0.23000	17.3553	5.4794	1.7547	0.25203	19.0816
10	−1.6542	0.11704	17.2459	4.1422	0.11704	17.2459	4.1422
20	−4.9124	0.06513	16.1634	2.7981	0.06513	16.1634	2.7981
30	−7.4440	0.04085	14.6806	1.8438	0.04085	14.6806	1.8438
40	−9.6353	0.02589	12.8937	1.1326	0.02589	12.8937	1.1326
50	−11.6186	0.01539	10.7348	0.6028	0.01539	10.7348	0.6028
60	−13.4571	0.00742	7.9412	0.2274	0.00742	7.9412	0.2274
70	−15.1867	0.00111	3.2405	0.0145	0.00111	3.2405	0.0145

Table 2 $K = 0.0$, $A = -2.5285$, $Ca^2 = 0.01025$, radius $(a) = 9.2208$ km, $M = 0.3393 M_\Theta$, $Z_0 = 0.0920$, $Z_a = 0.0591$

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	0.9932	33.0204	16.6246	0.0000	0.6967	0.0301
0.2	0.9514	32.9344	17.2776	0.0000	0.6965	0.0289
0.4	0.8272	32.6781	19.6098	0.0000	0.6958	0.0253
0.6	0.6237	32.2569	25.4390	0.0000	0.6946	0.0193
0.8	0.3457	31.6793	44.4779	0.0000	0.6929	0.0109
1.0	0.0000	30.9569	∞	0.0000	0.6905	0.0000

Table 3 $K = 0.0$, $A = -2.0558$, $Ca^2 = 0.01598$, radius $(a) = 11.0789$ km, $M = 0.5963 M_\Theta$, $Z_0 = 0.1434$, $Z_a = 0.0903$

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	1.4659	31.6023	11.4164	0.0000	0.7114	0.0464
0.2	1.4027	31.4773	11.8507	0.0000	0.7110	0.0446
0.4	1.2155	31.1064	13.4013	0.0000	0.7099	0.0391
0.6	0.9112	30.5010	17.2764	0.0000	0.7079	0.0299
0.8	0.5011	29.6794	29.9313	0.0000	0.7050	0.0169
1.0	0.0000	28.6653	∞	0.0000	0.7009	

Table 4 $K = 0.0$, $A = -1.6880$, $Ca^2 = 0.02089$, radius $(a) = 12.2789$ km, $M = 0.8206 M_\Theta$, $Z_0 = 0.1875$, $Z_a = 0.1160$

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	1.8337	30.4989	9.2455	0.0000	0.7241	0.0601
0.2	1.7530	30.3449	9.5876	0.0000	0.7236	0.0578
0.4	1.5147	29.8888	10.8090	0.0000	0.7220	0.0507
0.6	1.1303	29.1487	13.8611	0.0000	0.7193	0.0388
0.8	0.6176	28.1524	23.8280	0.0000	0.7152	0.0219
1.0	0.0000	26.9355	∞	0.0000	0.7095	0.0000

Table 5 $K = 0.0$, $A = 1.9768$, $Ca^2 = 0.10447$, radius $(a) = 18.9364$ km, $M = 3.4126M_\Theta$, $Z_0 = 0.9686$, $Z_a = 0.4612$

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	5.4849	19.5453	4.5416	0.0000	0.9998	0.2806
0.2	5.1771	19.2325	4.5884	0.0000	0.9885	0.2692
0.4	4.3061	18.3098	4.8122	0.0000	0.9588	0.2352
0.6	3.0171	16.8411	5.5390	0.0000	0.9186	0.1791
0.8	1.5118	14.9593	8.2796	0.0000	0.8726	0.1011
1.0	0.0000	12.8504	∞	0.0000	0.8214	0.0000

Table 6 $K = 3.013$, $A = 2.1886$, $Ca^2 = 0.25$, radius $(a) = 17.2992$ km, $M = 5.6111M_\Theta$, $Z_0 = 3.0113$, $Z_a = 1.0538$ (WEC)

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	6.2125	17.3624	3.5790	0.0000	0.9712	0.3578
0.2	5.5756	16.6603	3.4836	0.0435	0.9346	0.3347
0.4	3.9454	14.6009	3.3746	0.3710	0.8473	0.2702
0.6	2.0174	11.4986	3.5356	1.3652	0.7264	0.1754
0.8	0.5408	7.9442	4.6396	3.5544	0.5438	0.0681
1.0	0.0000	4.4674	∞	7.6182	0.0084	0.0000

Table 7 $K = 3.49$, $A = 1.7910$, $Ca^2 = 0.23$, radius $(a) = 17.3553$ km, $M = 5.4794M_\Theta$, $Z_0 = 2.7084$, $Z_a = 0.9928$ (SEC)

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	5.8944	18.3168	3.3612	0.0000	0.9046	0.3218
0.2	5.2975	17.5645	3.3210	0.0431	0.8772	0.3016
0.4	3.7654	15.4001	3.3065	0.3663	0.8060	0.2445
0.6	1.9402	12.1881	3.5431	1.3410	0.6975	0.1592
0.8	0.5254	8.5124	4.7360	3.4768	0.5247	0.0617
1.0	0.0000	4.8874	∞	7.4278	0.0138	0.0000

Table 8 $K = 10$, $A = -1.6542$, $Ca^2 = 0.11704$, radius $(a) = 17.2459$ km, $M = 4.1422M_\Theta$, $Z_0 = 1.2136$, $Z_a = 0.5881$

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	3.5342	25.3974	3.1101	0.0000	0.6164	0.1392
0.2	3.2085	24.5266	3.1798	0.0365	0.6065	0.1308
0.4	2.3505	22.0700	3.4244	0.3018	0.5741	0.1065
0.6	1.2716	18.4267	4.0262	1.0692	0.5098	0.0690
0.8	0.3666	14.0860	5.9933	2.6863	0.3899	0.0260
1.0	0.0000	9.4837	∞	5.5889	0.0061	0.0000

Table 9 $K = 20$, $A = -4.9124$, $Ca^2 = 0.06513$, radius $(a) = 16.1634$ km, $M = 2.7981M_\Theta$, $Z_0 = 0.6372$, $Z_a = 0.3548$

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0.0	1.9427	30.1720	3.7596	0.0000	0.4769	0.0644
0.2	1.7745	29.4198	3.8646	0.0268	0.4689	0.0603
0.4	1.3232	27.2526	4.2359	0.2185	0.4429	0.0486
0.6	0.7361	23.9092	5.1514	0.7592	0.3922	0.0308
0.8	0.2200	19.7076	8.1158	1.8676	0.2993	0.0112
1.0	0.0000	14.9701	∞	3.8069	0.0059	0.0000

Table 10 $K = 30$, $A = -7.4440$, $Ca^2 = 0.04085$, radius $(a) = 16.9281$ km, $M = 0.7982 M_{\Theta}$, $Z_0 = 0.3886$, $Z_a = 0.2314$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0.0	1.0778	32.7667	4.7637	0.0000	0.3895	0.0329
0.2	0.9876	32.1619	4.9111	0.0187	0.3825	0.0307
0.4	0.7435	30.3950	5.4353	0.1510	0.3603	0.0245
0.6	0.4200	27.5981	6.7369	0.5196	0.3178	0.0152
0.8	0.1283	23.9595	10.9628	1.2628	0.2416	0.0054
1.0	0.0000	19.6896	∞	2.5409	0.0126	0.0000

Table 11 $K = 40$, $A = -9.6353$, $Ca^2 = 0.02589$, radius $(a) = 12.8937$ km, $M = 1.1326 M_{\Theta}$, $Z_0 = 0.2418$, $Z_a = 0.1501$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0.0	0.5531	34.3407	6.3573	0.0000	0.3174	0.0161
0.2	0.5079	33.8835	6.5708	0.0120	0.3115	0.0150
0.4	0.3847	32.5353	7.3321	0.0965	0.2927	0.0118
0.6	0.2194	30.3627	9.2299	0.3298	0.2573	0.0072
0.8	0.0678	27.4657	15.4087	0.7949	0.1948	0.0025
1.0	0.0000	23.9644	∞	1.5844	0.0091	0.0000

Table 12 $K = 50$, $A = -11.6186$, $Ca^2 = 0.01539$, radius $(a) = 10.7348$ km, $M = 0.6028 M_{\Theta}$, $Z_0 = 0.2733$, $Z_a = 0.0906$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0.0	0.2365	35.2906	9.3416	0.0000	0.2494	0.0067
0.2	0.2175	34.9787	9.6792	0.0066	0.2445	0.0062
0.4	0.1654	34.0528	10.8843	0.0532	0.2294	0.0049
0.6	0.0950	32.5406	13.8940	0.1809	0.2011	0.0029
0.8	0.0296	30.4858	23.7143	0.4331	0.1517	9.7143×10^{-4}
1.0	0.0000	27.9440	∞	0.8568	0.0039	0.0000

Table 13 $K = 60$, $A = -13.4571$, $Ca^2 = 0.00768$, radius $(a) = 7.9459$ km, $M = 0.2279 M_{\Theta}$, $Z_0 = 0.0677$, $Z_a = 0.0442$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0.0	0.0646	35.8061	17.2262	0.0000	0.1761	0.0018
0.2	0.0595	35.6375	17.9162	0.0026	0.1726	0.0017
0.4	0.0454	35.1342	20.3971	0.0208	0.1617	0.0013
0.6	0.0262	34.3036	26.7194	0.0703	0.1415	7.6517×10^{-4}
0.8	0.0083	33.1578	48.7317	0.1675	0.1065	2.4928×10^{-4}
1.0	0.0000	31.7128	∞	0.3293	0.0060	0.0000

Table 14 $K = 70$, $A = -15.1867$, $Ca^2 = 0.111 \times 10^{-2}$, radius $(a) = 3.2405$ km, $M = 0.0145 M_{\Theta}$, $Z_0 = 0.0100$, $Z_a = 0.0067$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0.0	0.0017	35.9950	103.2922	0.0000	0.0692	4.632×10^{-5}
0.2	0.0015	35.9673	107.5141	1.7×10^{-4}	0.0678	4.273×10^{-5}
0.4	0.0012	35.8841	122.5732	0.0014	0.0635	3.284×10^{-5}
0.6	6.8712×10^{-4}	35.7457	160.0913	0.0046	0.0555	1.922×10^{-5}
0.8	2.2070×10^{-4}	35.5524	281.3904	0.0109	0.0418	6.207×10^{-6}
1.0	0.0000	35.3045	∞	0.0213	0.0057	0.0000

References

Adams, R.C., Cohen, J.M.: Astrophys. J. **198**, 507 (1975)
Adler, R.J.: J. Math. Phys. **15**, 727 (1974)
Bijalwan, N., Gupta, Y.K.: Astrophys. Space Sci. **317**, 251–260 (2008)
Bonnor, W.B.: Z. Phys. **160**, 59 (1960)
Delgaty, M.S.R., Lake, K.: Comput. Phys. Commun. **115**, 395 (1998)
Durgapal, M.C.: J. Phys. A **15**, 2637 (1982)
Gupta, Y.K., Gupta, R.S.: Acta Phys. Pol. B **17**, 855 (1986)
Gupta, Y.K., Kumar, M.: Gen. Relativ. Gravit. **37**(1), 575 (2005a)
Gupta, Y.K., Kumar, M.: Astrophys. Space Sci. **299**(1), 43–59 (2005b)
Gupta, Y.K., Maurya, S.K.: A class of charged analogues of Durgapal and Fuloria Superdense star. Astrophys. Space Sci. (2010a). doi:10.1007/s10509-010-4454
Gupta, Y.K., Maurya, S.K.: Astrophys. Space Sci. (2010b). doi:10.1007/s10509-010-0503-y
Gupta et al.: Int. J. Theor. Phys. **49**, 854–860 (2010). doi:10.1007/s10773-010-0267-8
Heintzmann, H.: Z. Phys. **228**, 489 (1969)
Ivanov, B.V.: Static charged perfect fluid sphere in general relativity. Phys. Rev. D **65**, 104001 (2002)
Krasinski, A.: Inhomogeneous Cosmological Models. Cambridge University Press, Cambridge (1997)
Kuchowicz, B.: Astrophys. Space Sci. **33**, L13 (1975)
Lemos, J.P.S., Zanchin, V.T.: arXiv:1004.3574v2 [gr-qc] (2010a)
Lemos, J.P.S., Zanchin, V.T.: Phys. Rev. D **81**, 124016 (2010b)
Maharaj, S.D., Hansraj, S.: Charged analogue of Finch-Skea stars. Int. J. Mod. Phys. D **15**, 1311–1327 (2006)
Maharaj, S.D., Komathiraj, K.: Classes of exact Einstein-Maxwell solutions. Gen. Relativ. Gravit. **39**, 2079–2093 (2007a)
Maharaj, S.D., Komathiraj, K.: Tikekar superdense stars in electric fields. J. Math. Phys. **48**, 042501 (2007b)
Maharaj, S.D., Komathiraj, K.: arXiv:0808.1998v1 [gr-qc] (2008)
Maharaj, S.D., Thirukkanesh, S.: Exact models for isotropic matter. Class. Quantum Gravity **23**, 2697–2709 (2006a)
Maharaj, S.D., Thirukkanesh, S.: Generating potentials via difference equations. Math. Methods Appl. Sci. **29**, 1943–1952 (2006b)
Maharaj, S.D., Thirukkanesh, S.: arXiv:0904.0781v1 [gr-qc] (2009)
Pant, N., et al.: Astrophys. Space Sci. (2010a). doi:10.1007/s10509-010-0453-4
Pant, N., et al.: Astrophys. Space Sci. (2010b). doi:10.1007/s10509-010-0509-5
Sharif, M., Abbas, G.: arXiv:1001.5316v1 [gr-qc] (2010)
Tolman, R.C.: Phys. Rev. **55**, 364 (1939)