

Dark energy model in anisotropic Bianchi type-III space-time with variable EoS parameter

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Abstract A new dark energy model in anisotropic Bianchi type-III space-time with variable equation of state (EoS) parameter has been investigated in the present paper. To get the deterministic model, we consider that the expansion θ in the model is proportional to the eigen value σ_2^2 of the shear tensor σ_i^j . The EoS parameter ω is found to be time dependent and its existing range for this model is in good agreement with the recent observations of SNe Ia data (Knop et al. in *Astrophys. J.* 598:102, 2003) and SNe Ia data with CMBR anisotropy and galaxy clustering statistics (Tegmark et al. in *Astrophys. J.* 606:702, 2004). It has been suggested that the dark energy that explains the observed accelerating expansion of the universe may arise due to the contribution to the vacuum energy of the EoS in a time dependent background. Some physical aspects of dark energy model are also discussed.

Keywords Bianchi-III universe · Dark energy · Variable EoS parameter

1 Introduction

Recent cosmological observations obtained by SNe Ia (Garavich et al. 1998a, 1998b; Perlmutter et al. 1997, 1998,

1999; Riess et al. 1998; Schmidt et al. 1998; Knop et al. 2003), WMAP (Bennett et al. 2003; Spergel et al. 2003), SDSS (Tegmark et al. 2004; Seljak et al. 2005; Adelman-McCarthy et al. 2006), Chandra X-ray observatory (Allen et al. 2004) indicate that the observable universe is undergoing an accelerating expansion. To explain the cosmic positive acceleration, mysterious dark energy has been proposed. There are several dark energy models which can be distinguished by, for instance, their variable equation of state (EoS) $\omega(t) = \frac{p}{\rho}$ (p is the fluid pressure and ρ its energy density) during the evolution of the universe. By now, methods allowing for restoration of the quantity $\omega(t)$ from expressional data have been developed (Sahni and Starobinsky 2006), and an analysis of the experimental data has been conducted to determine this parameter as a function of cosmological time (see Sahni et al. 2008 and references therein). DE has been conventionally characterized by EoS parameter mentioned above which is not necessarily constant. Recently, the parameter $\omega(t)$ is calculated with some reasoning which reduced to some simple parametrization of the dependences by some authors (Huterer and Turner 2001; Weller and Albrecht 2002; Chevallier and Polarski 2001; Krauss et al. 2007; Usmani et al. 2008; Chen et al. 2009). The simplest DE candidate is the vacuum energy ($\omega = -1$), which is mathematically equivalent to the cosmological constant Λ (Allen et al. 2004; Sahni and Starobinsky 2000; Sola and Stefancic 2005; Shapiro and Sola 2009). The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence ($\omega > -1$) (Ratra and Peebles 1988; Wetterich 1988; Liddle and Scherrer 1999), phantom energy ($\omega < -1$) (Caldwell 2002; Caldwell et al. 2003) and the combination of quintessence and phantom in a unified model, namely quintom (Feng et al. 2005; Guo et al. 2005) as evolved and have time dependent EoS parameter. Some other limits obtained from observational

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results coming from SNe Ia data (Knop et al. 2003) and SNe Ia data collaborated with CMBR anisotropy and galaxy clustering statistics (Tegmark et al. 2004) are $-1.67 < \omega < -0.62$ and $-1.33 < \omega < -0.79$ respectively. The latest results, obtained after a combination of cosmological datasets coming from CMB anisotropies, luminosity distances of high redshift type Ia supernovae and galaxy clustering, constrain the dark energy EoS to $-1.44 < \omega < -0.92$ at 68% confidence level (Hinshaw et al. 2009; Komatsu et al. 2009). However, it is not at all obligatory to use a constant value of ω . Due to lack of the observational evidence in making a distinction between constant and variable ω , usually the equation of state parameter is considered as a constant (Kujat et al. 2002; Bartelmann et al. 2005) with phase wise value $-1, 0, -\frac{1}{3}$ and $+1$ for vacuum fluid, dust fluid, radiation and stiff dominated universe, respectively. But in general, ω is a function of time or redshift (Jimenez 2003; Das et al. 2005; Ratra and Peebles 1988). For instance, quintessence models involving scalar fields give rise to time dependent EoS parameter ω (Turner and White 1997; Caldwell et al. 1998; Liddle and Scherrer 1999; Steinhardt et al. 1999). Some literature are also available on models with varying fields, such as cosmological models with variable EoS parameter in Kaluza-Klein metric and wormholes (Rahaman et al. 2006, 2009). In recent years various form of time dependent ω have been used for variable Λ models (Mukhopadhyay et al. 2008, 2009; Usmani et al. 2008). Recently Ray et al. (2010), Mukhopadhyay et al. (2010), Akarsu and Kilinc (2010), Yadav (2010), Yadav and Yadav (2010), Pradhan et al. (2010c) and Kumar (2010) have obtained dark energy models with variable EoS parameter in different contexts.

Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on. The major difference among these models are that they predict different equation of state of the dark energy and different history of the cosmos expansion. The simplest dark energy (DE) candidate is the cosmological constant Λ , but it needs some fine-tuning to satisfy the current value of the DE. Overduin and Cooperstock (1998), Sahni and Starobinsky (2000), Komatsu et al. (2009) have suggested some dynamic models, where Λ varies slowly with cosmic time (t). Srivastava (2005), Jackiw (2000), Bertolami et al. (2004), Bento et al. (2002); Bilic et al. (2002) and Avelino et al. (2003) have considered Chaplygin gas and generalized Chaplygin gas as possible dark energy sources due to negative pressure. Other than these approaches, some authors have considered modified gravitational action by adding a function $f(R)$ (R being the Ricci scalar curvature) to Einstein-Hilbert Lagrangian, where $f(R)$ provides a gravitational alternative for DE causing late-time acceleration of the universe (Capozziello 2002; Carroll et al. 2004; Dolgov and Kawasaki 2003; Nojiri and Odintsov 2003, 2004; Abdalaa et al. 2005; Mena et al.

2006). Recently Gupta and Pradhan (2010) have presented an entirely new approach as cosmological nuclear energy is a possible candidate for DE. For detail informations regarding the dynamics of dark energy, the readers are advised to see the reviews by Copeland et al. 2006 and Nojiri and Odintsov (2007). The aforementioned models offer satisfactory description of dark-energy behaviour and its observable features. In spite of the success of these attempts, the nature of DE is one of the greatest challenge of modern cosmology.

Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behaviour of universe and such models have been widely studied in framework of General Relativity in search of a realistic picture of the universe in its early stages. Yadav et al. (2007), Pradhan et al. (2010a, 2010b) have recently studied homogeneous and anisotropic Bianchi type-III space-time in context of massive strings. Recently Yadav (2010) has obtained Bianchi type-III anisotropic DE models with constant deceleration parameter. In this paper, we have investigated a new anisotropic Bianchi type-III DE model with variable ω without assuming constant deceleration parameter. The out line of the paper is as follows: In Sect. 2, the metric and the field equations are described. Section 3 deals with the solution of the field equations. In Sect. 4, some physical aspects of the derived DE model are given. Finally, conclusions are summarized in the last Sect. 5.

2 The Metric and Field Equations

We consider the space-time of general Bianchi-III type with the metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2ax}dy^2 + C^2(t)dz^2, \quad (1)$$

where a is constant.

The simplest generalization of EoS parameter of perfect fluid may be to determine the EoS parameter separately on each spatial axis by preserving the diagonal form of the energy momentum tensor in a consistent way with the considered metric. Therefore, the energy momentum tensor of fluid is taken as

$$T_i^j = \text{diag}[T_0^0, T_1^1, T_2^2, T_3^3]. \quad (2)$$

Thus, one may parameterize it as follows,

$$\begin{aligned} T_i^j &= \text{diag}[\rho, -p_x, -p_y, -p_z] \\ &= \text{diag}[1, -\omega_x, -\omega_y, -\omega_z]\rho \\ &= \text{diag}[1, -\omega, -(\omega + \delta), -(\omega + \gamma)]\rho. \end{aligned} \quad (3)$$

Here ρ is the energy density of fluid; p_x , p_y and p_z are the pressures and ω_x , ω_y and ω_z are the directional EoS parameters along the x -, y - and z -axes respectively. ω is the

deviation-free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameter δ and γ that are the deviations from ω along y - and z -axis respectively.

The Einstein's field equations (in gravitational units $c = 1, 8\pi G = 1$) read as

$$R_i^j - \frac{1}{2}Rg_i^j = T_i^j, \quad (4)$$

where R_i^j is the Ricci tensor; $R = g^{ij}R_{ij}$ is the Ricci scalar. In a co-moving co-ordinate system, the Einstein's field equation (4) with (3) for the metric (1) subsequently lead to the following system of equations:

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = -(\omega + \gamma)\rho, \quad (5)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\omega\rho, \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -(\omega + \delta)\rho, \quad (7)$$

$$\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} - \frac{a^2}{A^2} = \rho, \quad (8)$$

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0. \quad (9)$$

Here and in what follows an over dot denotes ordinary differentiation with respect to t .

The spatial volume for the model (1) is given by

$$V^3 = ABCe^{-ax}. \quad (10)$$

We define $a = (ABCe^{-ax})^{\frac{1}{3}}$ as the average scale factor so that the Hubble's parameter in anisotropic models may be defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right). \quad (11)$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (12)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of x , y and z respectively.

An important observational quantity is the deceleration parameter q , which is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (13)$$

The scalar expansion θ , components of shear σ_{ij} and the average anisotropy parameter A_m are defined by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (14)$$

$$\sigma_{11} = \frac{A^2}{3} \left[\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right], \quad (15)$$

$$\sigma_{22} = \frac{B^2 e^{-2ax}}{3} \left[\frac{2\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right], \quad (16)$$

$$\sigma_{33} = \frac{C^2}{3} \left[\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right], \quad (17)$$

$$\sigma_{44} = 0. \quad (18)$$

Therefore

$$\sigma^2 = \frac{1}{3} \left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA} \right]. \quad (19)$$

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2, \quad (20)$$

where $\Delta H_i = H_i - H$ ($i = 1, 2, 3$).

3 Solutions of the field equations

The field equations (5)–(9) are a system of five equations with seven unknown parameters $A, B, C, \rho, \omega, \delta$ and γ . Two additional constraints relating these parameters are required to obtain explicit solutions of the system. We assume that the expansion (θ) in the model is proportional to the eigen value σ^2 of the shear tensor σ_i^j . This condition leads to

$$B = \ell_1(AC)^{m_1}, \quad (21)$$

where ℓ_1 and m_1 are arbitrary constants. The motive behind assuming this condition is explained with reference to Thorne (1967), the observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 per cent (Kantowski and Sachs 1966; Kristian and Sachs 1966). To put more precisely, red-shift studies place the limit

$$\frac{\sigma}{H} \leq 0.3,$$

on the ratio of shear σ to Hubble constant H in the neighbourhood of our Galaxy today. Collins et al. (1980) have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

Equations (9) leads to

$$A = mB, \quad (22)$$

where m is a positive integrating constant. We have revisited the solution recently obtained by Pradhan et al. (2010a).

Using the value of (22) in (7) and subtract the result from (6), we obtain the skewness parameter on y -axis is null i.e. $\delta = 0$. In this case (6) and (7) are reduced to

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} = 0. \quad (23)$$

Using (22) in (23), we obtain

$$(1 - m) \left(\frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC} \right) = 0. \quad (24)$$

As $m \neq 0$, (24) gives

$$\left(\frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC} \right) = 0, \quad (25)$$

which on integration reduces to

$$\dot{B}C = k_1, \quad (26)$$

where k_1 is an integrating constant.

From (21) and (22), we obtain

$$B = \ell_2 C^\ell, \quad (27)$$

where $\ell_2 = \ell_1^{\frac{1}{1-m}} m^\ell$, $\ell = \frac{m_1}{1-m_1}$. Using (27) in (26), we get

$$C^\ell \dot{C} = \frac{k_1}{\ell \ell_2}, \quad (28)$$

which on integration gives

$$C = (\ell + 1)^{\frac{1}{\ell+1}} \left[\frac{k_1}{\ell \ell_2} t + k_2 \right]^{\frac{1}{\ell+1}}, \quad (29)$$

where k_2 is an integrating constant. Using (29) in (27) and (22) we obtain

$$B = \ell_2 (\ell + 1)^{\frac{\ell}{\ell+1}} \left[\frac{k_1}{\ell \ell_2} t + k_2 \right]^{\frac{\ell}{\ell+1}}, \quad (30)$$

and

$$A = m \ell_2 (\ell + 1)^{\frac{\ell}{\ell+1}} \left[\frac{k_1}{\ell \ell_2} t + k_2 \right]^{\frac{\ell}{\ell+1}}, \quad (31)$$

respectively.

Hence the metric (1) reduces to the form

$$\begin{aligned} ds^2 = & -dt^2 + \left[m \ell_2 (\ell + 1)^{\frac{\ell}{\ell+1}} \left(\frac{k_1}{\ell \ell_2} t + k_2 \right)^{\frac{\ell}{\ell+1}} \right]^2 dx^2 \\ & + \left[\ell_2 (\ell + 1)^{\frac{\ell}{\ell+1}} e^{-ax} \left(\frac{k_1}{\ell \ell_2} t + k_2 \right)^{\frac{\ell}{\ell+1}} \right]^2 dy^2 \\ & + \left[(\ell + 1)^{\frac{1}{\ell+1}} \left(\frac{k_1}{\ell \ell_2} t + k_2 \right)^{\frac{1}{\ell+1}} \right]^2 dz^2. \end{aligned} \quad (32)$$

Using the suitable transformation

$$m \ell_2 (\ell + 1)^{\frac{\ell}{\ell+1}} x = X, \quad (33)$$

$$\ell_2 (\ell + 1)^{\frac{\ell}{\ell+1}} y = Y, \quad (33)$$

$$(\ell + 1)^{\frac{1}{\ell+1}} z = Z,$$

$$\frac{k_1}{\ell \ell_2} t = T,$$

the metric (32) is reduced to

$$\begin{aligned} ds^2 = & -\beta^2 dT^2 + T^{2L} dX^2 + T^{2L} e^{-\frac{2a}{N} X} dY^2 \\ & + T^{\frac{2L}{\ell}} dZ^2, \end{aligned} \quad (34)$$

where

$$\beta = \frac{\ell \ell_2}{k_1},$$

$$M = (\ell + 1)^{\frac{1}{\ell+1}}, \quad (35)$$

$$N = m \ell_2 M,$$

$$L = \frac{\ell}{\ell + 1}.$$

4 Some physical and geometric properties of the model

The expressions for the scalar of expansion θ , magnitude of shear σ^2 , the average anisotropy parameter A_m , deceleration parameter q and proper volume V for the DE model (34) are given by

$$\theta = \frac{(2\ell + 1)L}{\ell \beta T}, \quad (36)$$

$$\sigma^2 = \frac{1}{3} \left(\frac{(\ell - 1)L}{\ell \beta T} \right)^2, \quad (37)$$

$$A_m = 2 \left(\frac{\ell - 1}{2\ell + 1} \right)^2, \quad (38)$$

$$q = -\frac{\ell \beta}{(2\ell + 1)}, \quad (39)$$

$$V = \frac{N^2}{m} M^{\frac{1}{\ell}} T^{\frac{L(2\ell+1)}{\ell}}. \quad (40)$$

The rate of expansion H_i in the direction of x , y and z are given by

$$H_1 = H_2 = \frac{L}{\beta T}, \quad (41)$$

$$H_3 = \frac{L}{\ell \beta T}. \quad (42)$$

Hence the average generalized Hubble’s parameter is given by

$$H = \frac{L(2\ell + 1)}{3\ell\beta T}. \tag{43}$$

Using (29)–(31) in (8), the energy density of the fluid is obtained as

$$\rho = \frac{L^2(\ell + 2)}{\ell\beta^2 T^2} - \frac{a^2}{N^2 T^{2L}}. \tag{44}$$

Now, by using (29), (30) and (44) in (6), the equation of state parameter ω is obtained as

$$\omega = \frac{L(\ell^2 + \ell + 1) - \ell(\ell + 1)}{\frac{\ell^2 a^2 \beta^2}{N^2 L T^{2(L-1)}} - L\ell(\ell + 2)}. \tag{45}$$

Using (30), (31), (44) and (45) in (5), the skewness parameter γ (i.e. deviation from ω along z -axis) is derived as

$$\gamma = \frac{\frac{a^2}{N^2 T^{2L}} + \frac{L}{\ell^2 \beta^2 T^2} [\ell(\ell - 1) - L(2\ell^2 - \ell - 1)]}{\frac{L^2(\ell+2)}{\ell\beta^2 T^2} - \frac{a^2}{N^2 T^{2L}}}. \tag{46}$$

From (45), it is observed that the equation of state parameter ω is time dependent, it can be function of redshift z or scale factor a as well. The redshift dependence of ω can be linear like

$$\omega(z) = \omega_0 + \omega'z, \tag{47}$$

with $\omega' = (\frac{d\omega}{dz})_{z=0}$ (see Huterer and Turner 2001; Weller and Albrecht 2002) or nonlinear as

$$\omega(z) = \omega_0 + \frac{\omega_1 z}{1 + z}, \tag{48}$$

(Chevallier and Polarski 2001; Linder 2003). So, as for as the scale factor dependence of ω is concern, the parametrization is given by

$$\omega(a) = \omega_0 + \omega_a(1 - a), \tag{49}$$

where ω_0 is the present value ($a = 1$) and ω_a is the measure of the time variation ω' (Linder 2008).

The SNe Ia data suggests that $-1.67 < \omega < -0.62$ (Knop et al. 2003) while the limit imposed on ω by a combination of SNe Ia data with CMB anisotropy and galaxy clustering statistics is $-1.33 < \omega < -0.79$ (Tegmark et al. 2004). So, if the present work is compared with experimental results mentioned above, then one can conclude that the limit of ω provided by (45) may accommodated with the acceptable range of EoS parameter. Also it is observed that either for $T = 0$ or for $m_1 = 0$, the ω vanishes and our model represents a dusty universe.

For the value of ω to be in consistent with observation (Knop et al. 2003), we have the following general condition

$$T_1 < T < T_2, \tag{50}$$

where

$$T_1 = \left[\frac{0.79\ell a\beta}{N\sqrt{L\{\ell(\ell + 1) - L(0.38\ell^2 - 0.24\ell + 1)\}}} \right]^{\frac{1}{(L-1)}}, \tag{51}$$

and

$$T_2 = \left[\frac{1.39\ell a\beta}{N\sqrt{L\{\ell(\ell + 1) - L(0.67\ell^2 - 2.34\ell + 1)\}}} \right]^{\frac{1}{(L-1)}}. \tag{52}$$

For this constrain, we obtain $-1.67 < \omega < -0.62$, which is in good agreement with the limit obtained from observational results coming from SNe Ia data (Knop et al. 2003). For a special case for which $\ell = 1, a = 0.5, \beta = 2, L = 0.5, N = 1$, where $0.899700 < T < 1.153226$, we obtain the same limit $-1.67 < \omega < -0.62$.

From (45), we have observed that, at cosmic time

$$T = \left[\frac{\ell a\beta}{N\sqrt{L\{L\ell + \ell(\ell + 1) - 1\}}} \right]^{\frac{1}{(L-1)}}, \tag{53}$$

$\omega = -1$ (i.e. cosmological constant dominated universe) and when

$$T < \left[\frac{\ell a\beta}{N\sqrt{L\{L\ell + \ell(\ell + 1) - 1\}}} \right]^{\frac{1}{(L-1)}}, \tag{54}$$

$\omega > -1$ (i.e. quintessence) and when

$$T > \left[\frac{\ell a\beta}{N\sqrt{L\{L\ell + \ell(\ell + 1) - 1\}}} \right]^{\frac{1}{(L-1)}}, \tag{55}$$

$\omega < -1$ (i.e. super quintessence or phantom fluid dominated universe) (Caldwell 2002).

The variation of EoS parameter ω with cosmic time T is clearly shown in Fig. 1, as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well known situations. From Fig. 1, we conclude that in early stage of evolution of the universe, the EoS parameter ω was very small but positive (i.e. the universe was matter dominated) and at late time it is evolving with negative value (i.e. at the present time). The earlier real matter later on converted to the dark energy dominated phase of the universe.

From (44), we note that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. This behaviour is clearly depicted in Fig. 2 as a representative case with appropriate

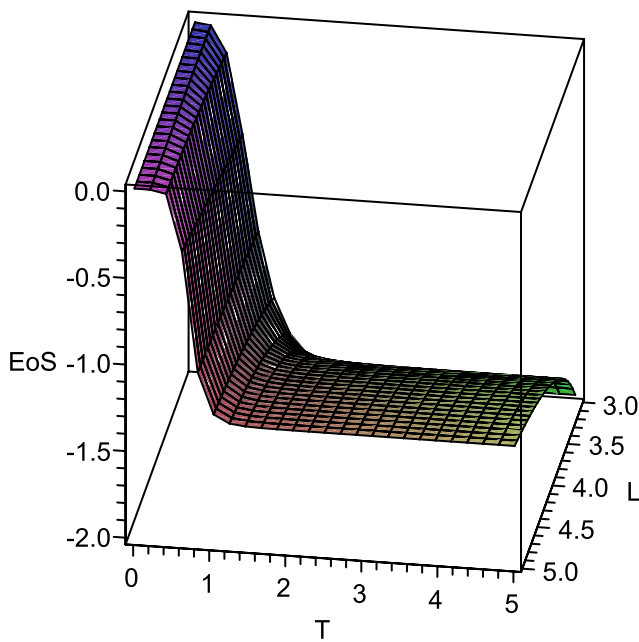


Fig. 1 The plot of EoS parameter ω versus T and L

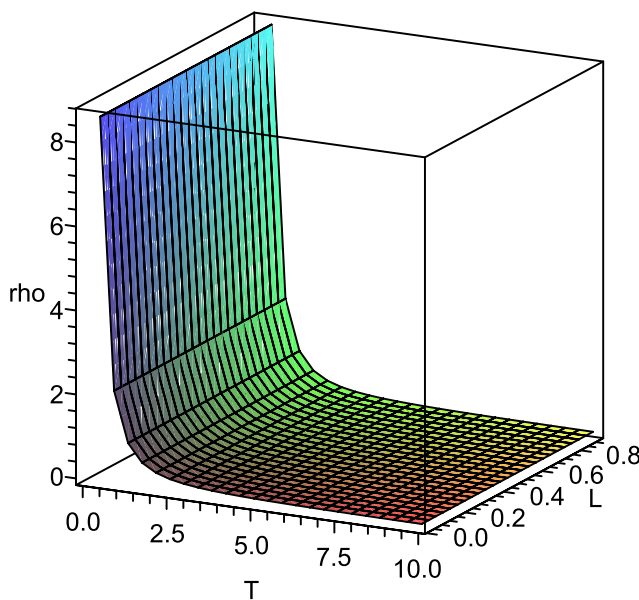


Fig. 2 The plot of energy density ρ versus T and L

choice of constants of integration and other physical parameters using reasonably well known situations.

In absence of any curvature, matter energy density (Ω_m) and dark energy (Ω_Λ) are related by the equation

$$\Omega_m + \Omega_\Lambda = 1, \tag{56}$$

where $\Omega_m = \frac{\rho}{3H^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H^2}$. Thus, (56) reduces to

$$\frac{\rho}{3H^2} + \frac{\Lambda}{3H^2} = 1. \tag{57}$$

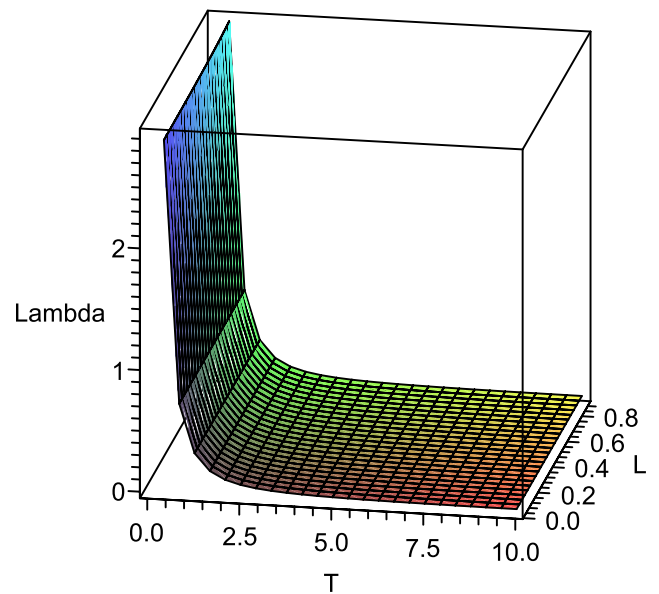


Fig. 3 The plot of cosmological term Λ versus T and L

Using (43) and (44), in (57), the cosmological constant is obtained as

$$\Lambda = \frac{L^2(2\ell + 1)^2}{3\ell^2\beta^2T^2} - \frac{L^2(\ell + 2)}{\ell\beta^2T^2} + \frac{a^2}{N^2T^2L}. \tag{58}$$

In recent time the Λ -term has interested theoreticians and observers for various reasons. The nontrivial role of the vacuum in the early universe generate a Λ -term that leads to inflationary phase. Observationally, this term provides an additional parameter to accommodate conflicting data on the values of the Hubble constant, the deceleration parameter, the density parameter and the age of the universe (for example, see the references Gunn and Tinsley 1975; Wampler and Burke 1988). The behaviour of the universe in this model will be determined by the cosmological term Λ ; this term has the same effect as a uniform mass density $\rho_{eff} = -\Lambda$, which is constant in time. A positive value of Λ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of Λ , the expansion will tend to accelerate; whereas in the universe with negative value of Λ , the expansion will slow down, stop and reverse. From (58), we see that the cosmological term Λ is a decreasing function of time and it approaches a small positive value at late time. From Fig. 3, we note this behaviour of cosmological term Λ in the model. Recent cosmological observations suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ -term. It is remarkable to mention here that the dark energy that explains the observed

accelerating expansion of the universe may arise due to the contribution to the vacuum energy of the EoS in a time dependent background. Thus, our DE model is consistent with the results of recent observations.

From the above results, it can be seen that the spatial volume is zero at $T = 0$ and it increases with the increase of T . This shows that the universe starts evolving with zero volume at $T = 0$ and expands with cosmic time T . From (41) and (42), we observe that all the three directional Hubble parameters are zero at $T \rightarrow \infty$. In derived model, the energy density tend to infinity at $T = 0$. The model has the point-type singularity at $T = 0$ (MacCallum 1971). The shear scalar diverges at $T = 0$. As $T \rightarrow \infty$, the scale factors $A(t)$, $B(t)$ and $C(t)$ tend to infinity. The energy density becomes zero as $T \rightarrow \infty$. The expansion scalar and shear scalar all tend to zero as $T \rightarrow \infty$. The mean anisotropy parameter are uniform throughout whole expansion of the universe when $\ell \neq -\frac{1}{2}$ but for $\ell = -\frac{1}{2}$ it tends to infinity. This shows that the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to isotropic. At the initial stage of expansion, when ρ is large, the Hubble parameter is also large and with the expansion of the universe H , θ decrease as does ρ . Since $\frac{\sigma^2}{\theta^2} = \text{constant}$ provided $\ell \neq -\frac{1}{2}$, the model does not approach isotropy at any time. The cosmological evolution of Bianchi type-III space-time is expansionary, with all the three scale factors monotonically increasing function of time. The dynamics of the mean anisotropy parameter depends on the value of ℓ .

From (39) we observe that

$$(i) \quad \text{for } \ell < -\frac{1}{2}, q > 0$$

i.e., the model is decelerating and

$$(ii) \quad \text{for } \ell > -\frac{1}{2}, q < 0$$

i.e., the model is accelerating. Thus this case implies an accelerating model of the universe. Recent observations of type Ia supernovae (see Perlmutter et al. 1999; Riess et al. 1998 and references therein) reveal that the present universe is in accelerating phase and deceleration parameter lies somewhere in the range $-1 < q \leq 0$. It follows that our DE model of the universe is consistent with the recent observations.

5 Concluding remarks

An anisotropic Bianchi type-III DE model with variable EoS parameter ω has been investigated which is new and different from the other author's solutions. In the derived model,

ω is obtained as time varying which is consistent with recent observations (Knop et al. 2003; Tegmark et al. 2004). It is observed that, in early stage, the equation of state parameter ω is positive i.e. the universe was matter dominated in early stage but in late time, the universe is evolving with negative values i.e the present epoch (see, Fig. 1). Our DE model is in accelerating phase which is consistent with the recent observations. Thus the model (34) represents a realistic model.

In the derived DE model of the universe, the cosmological term is a decreasing function of time and it approaches a small positive value at late time (i.e., the present epoch). The values of cosmological “constant” for the model is found to be small and positive, which is supported by the recent observations (Garnavich et al. 1998a, 1998b; Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 2000, 2004; Schmidt et al. 1998).

The DE model is based on exact solution of Einstein's field equations for the anisotropic Bianchi-III space-time filled with perfect fluid. To my knowledge, the literature has hardly witnessed this sort of exact solution for anisotropic Bianchi-III space-time. So the derived DE model adds one more feather to the literature.

The DE model presents the dynamics of EoS parameter ω provided by (45) may accommodated with the acceptable range $-1.67 < \omega < -0.62$ of SNe Ia data (Knop et al. 2003). It is already observed and shown in previous section that for different cosmic times, we obtain cosmological constant dominated universe, quintessence and phantom fluid dominated universe (Caldwell 2002), representing the different phases of the universe through out the evolving process. Therefore, we can not rule out the possibility of anisotropic nature of DE at least in the framework of Bianchi-III space-time.

Though there are many suspects (candidates) such as cosmological constant, vacuum energy, scalar field, brane world, cosmological nuclear-energy, etc. as reported in the vast literature for DE, the proposed model in this paper at least presents a new candidate (EoS parameter) as a possible suspect for the DE.

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