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ORIGINAL ARTICLE

Ion acoustic solitons of KdV and modified KdV equations in weakly relativistic plasma containing nonthermal electron, positron and warm ion

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Abstract In this paper, the ion-acoustic solitons in a weakly relativistic electron-positron-ion plasma have been investigated. Relativistic ions, Maxwell-Boltzmann distributed positrons and nonthermal electrons are considered in collisionless warm plasma. Using a reductive perturbation theory, a Korteweg-de Vries (KdV) equation is derived, and the relativistic effect on the solitons is studied. It is found that the amplitude of solitary waves of the KdV equation diverges at the critical values of plasma parameters. Finally, in this situation, the solitons of a modified KdV (mKdV) equation with finite amplitude is derived.

Keywords Soliton · Relativistic ions · Nonthermal

1 Introduction

Solitary waves have been extensively studied both theoretically and experimentally as an important nonlinear topic (Shukla 2003; Misra and Bhowmik 2007; Verheest 1996; Tian and Gao 2007). The ion acoustic soliton (IAS) is one of the most aspects of nonlinear phenomena in modern plasma physics researches. They arise due to delicate balance of nonlinearity and dispersion. Investigation of nonlinear structures is carried out usually by adopting some forms of perturbation method. In small amplitude approximation, one ends up deriving some forms of nonlinear differential equations like Korteweg-de Vries (KdV) or modified Korteweg-de Vries (m-KdV) or nonlinear Schrodinger equation, etc. which have solitary or solitonic solutions. Using the reductive perturbation technique, ion-acoustic solitons have been studied by the numbers of authors (Bharuthram and Shukla 1986; Yadav and Sharma 1991). In contrast to the usual plasmas that consisting of electrons and positive ions, it has been observed that the nonlinear waves in plasmas which containing additional components such as positrons behave differently (Rizzato 1988). The behavior of the electron-positron-ion plasmas have an important role for explanation of the early universe which assumes to be a kind of plasma (Rees 1983; Misner et al. 1973), describing the active galactic nuclei (Miller and Witta 1987), pulsar magnetospheres (Michel 1982) and also the solar atmosphere (Goldreich and Julian 1969). The positrons can be used to probe particle transport in tokamaks and since they have sufficient lifetime, the two-component (e-i) plasma becomes a three-component (e-i-p) one (Surko et al. 1986; Surko and Murphy 1990). During the last decade, e-p-i plasmas have attracted the attention of several authors (Mushtaq and Shah 2005; Mahmood and Sallem 2002; Nejoh 1997). We know that when the ion velocity approaches the velocity of light, relativistic effects may significantly modify the behavior of the solitary waves. Relativistic plasmas occur in a variety of situations, such as, space–plasma phenomena (Grabbe 1989), laser–plasma interaction (Arons 1979), plasma sheet boundary layer of earth's magnetosphere (Vette 1970) and describing the Van Allen radiation belts (Ikezi 1973). Das and Paul (1985) have investigated the weakly relativistic effects on ion-acoustic wave propagation in one dimension using the KdV equation for cold plasma and without electron inertia. Nejoh (1987) has investigated the same results in the warm plasmas. Kalita et al. (1996) have investigated the existence of solitons considering the complete fluid equation of electrons. EL-

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Labany and Shaaban (1995) have considered nonlinear ion-acoustic waves in weakly relativistic plasma consisting of warm ion-fluid with nonisothermal electrons through the modified equations. EL-Labany et al. (1996) have investigated the same problem by considering Boltzmann distribution for electrons and third order perturbation. Nejoh (1987) have considered large amplitude Langmuir and ion-acoustic waves in a relativistic two fluid plasmas deriving the pseudopotential. Understanding the behavior of multi species plasma of cold or warm ions with Boltzmann's distribution has been of considerable interest for the last few years. However, it has been found that the electron and ion distributions play the crucial role in characterizing the physics of the nonlinear wave structures. They further offer a considerable increase in richness and variety of the wave motion, which can exist in the plasma. Moreover, they significantly influence the conditions required for the formation of the waves. Based on observations of solitary wave structures with density depletion made by Freja and Viking satellites (Dovner et al. 1994), Carins et al. (1995a, 1995b) have considered a plasma model consisting of nonthermally distributed electrons and ions and emphasizing the role of this distribution on the characterization of waves structures (Mamun 2000; Bandyopadhyay 2001; Carins et al. 1996). It may be noted that plasmas with different temperatures and masses frequently occur in space environment, particularly, two temperature electrons are very common in laboratory and space plasmas. Nonthermal distributions are common feature of the auroral zone (Lundin et al. 1987; Hall et al. 1991). In this paper, the ion acoustic solitary waves in weakly relativistic plasma consisting of nonthermal electrons, positrons with Boltzmann distribution and warm ion have been studied. In Sect. 2, we have derived the KdV-Burger equation using the reductive perturbation method. Then, relativistic effects on solitons have investigated in Sect. 3. Based on appropriate conditions for inclusion of higher effects Sect. 3 is devoted to derive the mKdV equation.

2 Derivation of KdV equation

In this medium, an unmagnetized and collisionless plasma consisting of a mixed fluid with Boltzmann distributed positrons, nonthermal electrons and warm ions is considered. Moreover, it is assumed that ion velocity has weak relativistic effect, and the ion acoustic wave propagate in the x direction. Such plasmas are described by the following normalized equations

$$
\begin{aligned}
&\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0,\\
&\frac{\partial (\gamma u)}{\partial t}+u\frac{\partial (\gamma u)}{\partial x}+\frac{\sigma}{n}\frac{\partial p}{\partial x}+\frac{\partial \phi}{\partial x}=0,\\
&\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+3p\frac{\partial (\gamma u)}{\partial x}=0,\\
&\frac{\partial^2 \phi}{\partial x^2}=n_e-n-n_p.
\end{aligned}
\tag{1}
$$

γ is relativistic factor and for a weakly relativistic plasma it is approximated by its expansion up to second term ($u \ll c$)

$$
\gamma=\left(1-\frac{u^2}{c^2}\right)^{-\frac{1}{2}}\cong 1+\frac{u^2}{2c^2}. \tag{2}
$$

Positrons densities are governed by Boltzmann distribution and for the nonthermal distributed electrons; the number density of electron is given by Carins et al. (1995a)

$$
\begin{aligned}
&n_p=\frac{\mu}{1-\mu}e^{-s\phi},\\
&n_e=\frac{1}{1-\mu}(1-\beta\phi+\beta\phi^2)e^{\phi}.
\end{aligned}
\tag{3}
$$

In (1)–(3), the ion density (n), electron density (n_e) and positron density (n_p) are normalized by unperturbed ion density (n_0). $s=\frac{T_e}{T_p}$, in which T_e and $T_p$are the temperature of electrons and positrons, respectively. $\beta=\frac{4\alpha}{1+3\alpha}$ in which α is a parameter that determines the population of nonthermal (fast) electrons (Kalita et al. 1996). $\sigma=\frac{T_i}{T_{\mathrm{eff}}}$, in which T_i is the temperature of ion and T_{eff} is the effective temperature $T_{\mathrm{eff}}=n_0/(\frac{n_{e0}}{T_e}+\frac{n_{p0}}{T_P})$. $\mu=\frac{n_{0p}}{n_{0e}}$, where n_{0p} and n_{0e} are the number density of positron and electron in the equilibrium case, respectively. u, p and ϕ, are the ion fluid velocity, the pressure and electrical potential. These quantities are normalized by the sound velocity $\sqrt{\frac{kT_{\mathrm{eff}}}{m}}$, n_0kT_i and $(\frac{kT_{\mathrm{eff}}}{e})$, respectively, where k is Boltzmann's constant, m is the ion mass and e is the charge of electron. c is the velocity of light which is normalized by the sound velocity. The time (t) and the distance (x) are normalized by the ion-plasma period $\omega_p^{-1}=\sqrt{\frac{m}{4\pi n_0 e^2}}$ and electron Debye length $\lambda_D=\sqrt{\frac{kT_{\mathrm{eff}}}{4\pi n_0 e^2}}$, respectively. Now, to study the behavior of nonlinear ion acoustic wave, we use the reductive perturbation method and define the following stretched coordinates (El-Labany and El-Taibany 2003)

$$
\xi=\varepsilon^{1/2}(x-\lambda t),\quad \tau=\varepsilon^{3/2}t, \tag{4}
$$

where ε is a small parameter which characterizes the strength of the nonlinearity, and λ is the phase velocity of the wave. Dependent variables are expanded as follows

$$\begin{aligned} n &= 1+\varepsilon n_1+\varepsilon^2 n_2+\varepsilon^3 n_3+\cdots, \\ u &= u_0+\varepsilon u_1+\varepsilon^2 u_2+\varepsilon^3 u_3+\cdots, \\ p &= 1+\varepsilon p_1+\varepsilon^2 p_2+\varepsilon^3 p_3+\cdots, \\ \phi &= \varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+\cdots. \end{aligned} \tag{5}$$

On substituting (5) into (1), using (4) and collecting the terms in the different powers of ε, we obtain the following equations at the lowest order of ε

$$\begin{aligned} & n_1=\frac{u_1}{\lambda-u_0}, \qquad p_1=\frac{3\gamma_1}{\lambda-u_0}u_1, \\ & n_1=\frac{1-\beta+\mu s}{1-\mu}\phi_1, \\ & [(\lambda-u_0)^2-3\sigma]\gamma_1=\left(\frac{1-\mu}{1-\beta+\mu s}\right), \end{aligned} \tag{6}$$

and for the higher orders of ε, we have

$$\begin{aligned} & -(\lambda-u_0)\frac{\partial n_2}{\partial\xi}+\frac{\partial n_1}{\partial\tau}+\frac{2}{\lambda-u_0}u_1\frac{\partial u_1}{\partial\xi}+\frac{\partial u_2}{\partial\xi}=0, \\ & -(\lambda-u_0)\gamma_1\frac{\partial u_2}{\partial\xi}+\gamma_1\frac{\partial u_1}{\partial\tau}+\frac{\partial\phi_2}{\partial\xi}+\Bigg[\gamma_1-2\gamma_2(\lambda-u_0) \\ & \quad -\frac{3\sigma\gamma_1}{(\lambda-u_0)^2}\Bigg]u_1\frac{\partial u_1}{\partial\xi}+\sigma\frac{\partial p_2}{\partial\xi}=0, \\ & -(\lambda-u_0)\frac{\partial p_2}{\partial\xi}+\frac{\partial p_1}{\partial\tau}+\Bigg[6\gamma_2+\frac{9\gamma_1^2}{\lambda-u_0} \\ & \quad +\frac{3\gamma_1}{\lambda-u_0}\Bigg]u_1\frac{\partial u_1}{\partial\xi}+3\gamma_1\frac{\partial u_2}{\partial\xi}=0, \\ & \frac{\partial^2\phi_1}{\partial\xi^2}=\left(\frac{1-\beta+\mu s}{1-\mu}\right)\phi_2-\left(\frac{1-\mu s^2}{1-\mu}\right)\frac{1}{2}\phi_1^2+n_2, \end{aligned} \tag{7}$$

where $\gamma_1=1+\frac{3u_\circ^2}{2c^2}$ and $\gamma_2=\frac{3u_\circ}{2c^2}$.

Finally the KdV equation is derived from (6) and (7)

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{8}$$

where

$$\begin{aligned} A &= \left(\frac{(\lambda-u_0)(1-\beta+\mu s)^2}{2[3\sigma\gamma_1(1-\beta+\mu s)+1-\mu]}\right) \\ & \quad \times\left\{\frac{2(1-(\gamma_2/\gamma_1)(\lambda-u_0))+\gamma_1(\lambda-u_0)^2+9\sigma\gamma_1^2}{(1-\mu)}\right. \\ & \quad \left.-\frac{(1-\mu s^2)}{(1-\beta+\mu s)^2}\right\}, \\ B &= \frac{1}{2}\left\{\frac{(1-\mu)^2(\lambda-u_0)}{(1-\mu)(1-\beta+\mu s)+3\sigma\gamma_1(1-\beta+\mu s)^2}\right\}. \end{aligned} \tag{9}$$

The stationary solution of (8) is given by

$$\phi_1=\phi_0\sec h^2\frac{(\xi-U\tau)}{w}, \tag{10}$$

in which U is constant velocity of solitary wave. The amplitude ϕ_0 and width w of the ion acoustic waves are given by

$$\phi_0=\frac{3U}{A}, \qquad w=2\sqrt{\frac{B}{U}}. \tag{11}$$

Relativistic effect is represented as $\eta=\frac{u_0}{c}$. Apparently, coefficients of the nonlinear and dispersion terms depend upon the different parameters like the ion temperature (σ), relative density, relativistic factor (η) nonthermal parameter (β).

Some researchers have studied one-dimensional (Das and Paul 1985; Nejoh 1987; Tagare 1973; Washimi and Taniuti 1996; Kuehl and Zhang 1991; Gill et al. 2007) and two-dimensional (EL-Labany et al. 1996; Nejoh 1987; Singh and Honzawa 1993) ion acoustic solitary waves in weakly relativistic plasmas containing ions and electrons. It seems that the results of their studies are consistent with the results of this paper. More precisely, the above results are in agreement with the results of Nejoh (1987) for electron-ion plasma ($\mu=0$) with the Boltzmann distribution of electron ($\beta=0$). The mentioned results reduce to Tagare (1973) when $\beta=0$ and $\sigma=0$. Our results are turned to Washimi and Taniuti (1996) and Das and Paul (1985) for nonrelativistic e-i plasmas with cold and warm ions, respectively and Boltzmann distribution of electron. We use the numerical computation for doing quantitative analyzes on the results. In different parameter regimes, these coefficients take on different values. The relative values of these parameters are applied to characterize the existence of solitons. Now, we can investigate the behavior of the amplitude and width of solitons in the plasma. The ranges of different plasma parameters used in this investigations are very wide ($0\le\mu\le0.9$, $0\le\sigma\le0.2$, $0\le\eta\le0.3$, $s=1, 0\le\beta\le1$, $u=0.0075$) and are relevant to Nejoh (1987), Gill et al. (2007), Singh and Honzawa (1993), Kaur et al. (2009), Masood et al. (2008). Figure 1 shows the variation of nonlinear coefficient, "A", as a function of β. It is clear that with the increase of nonthermal parameter nonlinearity coefficient can have positive, zero and negative values. It implies that with the increase of the nonthermal parameter there is a transition from compressive to rarefactive solitons. Also, we see that only compressive soliton exists, when the nonthermal parameter is zero (Gill et al. 2007). Also, we can conclude that the amplitude of com-

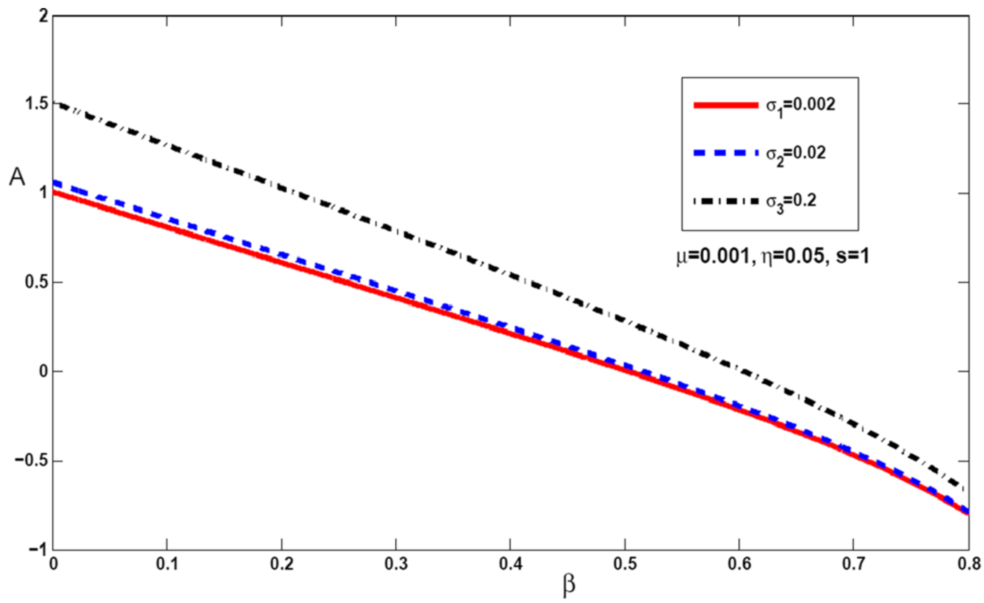


Fig. 1 Variation of nonlinear coefficient A with nonthermal parameter (β) for different values of σ with $\mu = 0.001$, $\eta = 0.05$, $s = 1$

pressive solitons decreases when the temperature of plasma increases.

To study the effect of relativistic parameter on peak amplitude, we plot it as function of μ for different values of η and fixed values of nonthermal parameter and relative temperatures (Figs. 2a and 2b). Since the soliton amplitude becomes diverges in $A = 0$, and relativistic effect is more pronounced for less relative density μ (Gill et al. 2007), we have studied the variation of amplitude in Figs. 2a and 2b for small values of μ when $A > 0$ and $A < 0$, separately.

It is obvious that the amplitude of compressive solitons decreases when μ increases. Also the increase in positron density results in an increase in the amplitude of rarefactive solitons. It is clear that amplitude is almost independent of η. However, the compressive solitons amplitude decreases when the relativistic effect appears. Further investigations show that the amplitude of soliton decreases when relative temperature (s) increases.

Figure 3 shows the variation of width with respect to μ for different values of η and fixed values of β, s, σ and U. We see that the width decreases when values of μ and η increase. Figure 3 shows that the width of solitons doesn't have critical behavior for any values of μ.

3 Derivation of modified KdV equation

The propagation of compressive and rarefactive solitons depends on the sign of the nonlinear coefficient of the KdV equation, A, (9). Thus, the ion acoustic waves are compressive (rarefactive) if $A > 0$ ($A < 0$). The singularity exists when the nonlinear coefficient is zero and the transition occurred through critical points. When the relative density μ reaches a critical value μ_c, the nonlinear coefficient of the KdV equation vanishes ($A = 0$) and soliton amplitude diverges. This leads to the failure of KdV model for ion acoustic solitary waves and we derive modified KdV (mKdV) equation. Figure 4 shows the critical behavior of the soliton amplitude corresponding to four cases. It is found that the values of critical densities are shifted toward higher values as η is decreased. Figures 5 show the dependence of μ_c on β variation in the different cases; these figure show μ_c increases as β increases. It is obvious that the low and high values of μ_c appears when s, σ and η change.

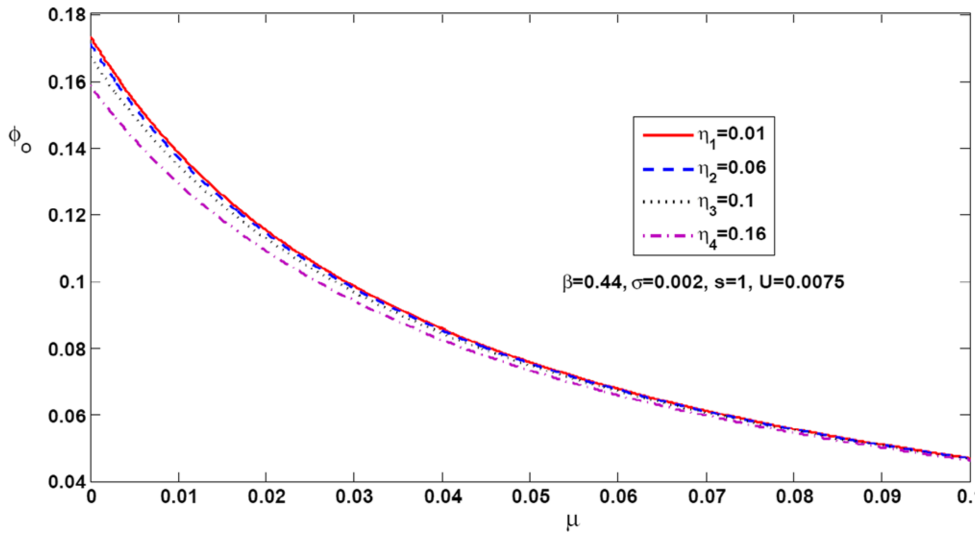
For describing the evolution of the system at these critical situations, one has to seek another evolution equation adequate for describing the system. This implies that the stretching coordinates mentioned before are not valid for this critical case, thus we will introduce a new stretching, namely (Kaur et al. 2009)

$$\xi = \varepsilon(x - \lambda t), \quad \tau = \varepsilon^3 t, \tag{12}$$

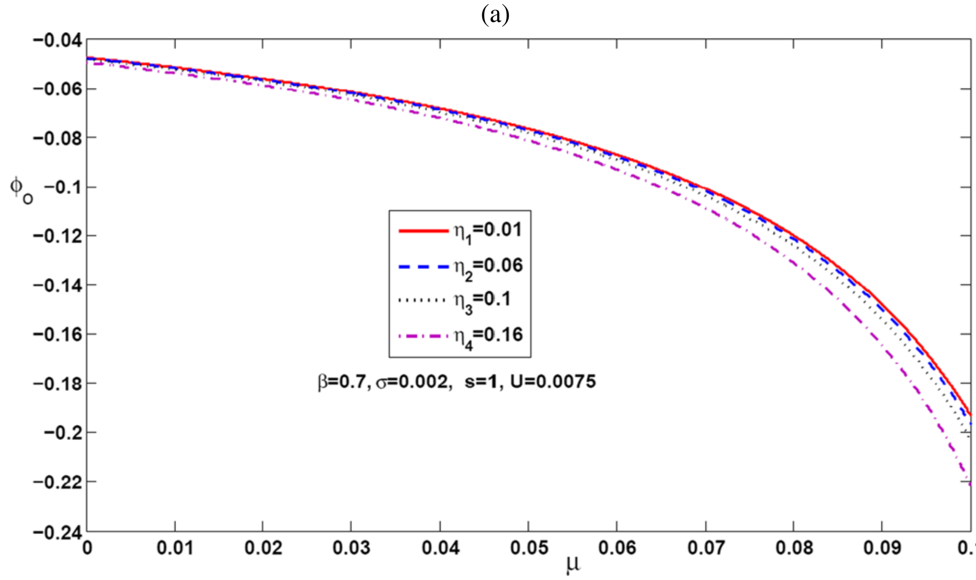
keeping the expansion of the remainder dependent variables as before. Using (5) and (12) in (1) and collecting the terms of different powers of ε, we obtain the following equations

$$\begin{aligned} n_2 &= \frac{u_1^2}{(\lambda - u_0)^2} - \frac{u_2}{(\lambda - u_0)}, \\ p_2 &= \left(\frac{3\gamma_1 + 9\gamma_1^2}{2(\lambda - u_0)^2} - \frac{3\gamma_2}{(\lambda - u_0)}\right)u_1^2 - \frac{3\gamma_1}{\lambda - u_0}u_2, \\ \phi_2 &= \left(\frac{1 - \beta + \mu s}{1 - \mu}\right)\phi_1^3 - \frac{3\gamma_1\phi_2}{(\lambda - u_0)}. \end{aligned} \tag{13}$$

Fig. 2 Variation of amplitude with respect to μ for different values of η with (**a**) $\beta = 0.44$, $s = 1$, $\sigma = 0.002$, $U = 0.0075$. (**b**) $\beta = 0.7$, $s = 1$, $\sigma = 0.002$, $U = 0.0075$



(a)



(b)

Fig. 3 Variation of width with respect to (μ) for different values of η with $\beta = 0.4$, $s = 1$, $\sigma = 0.002$, $U = 0.0075$

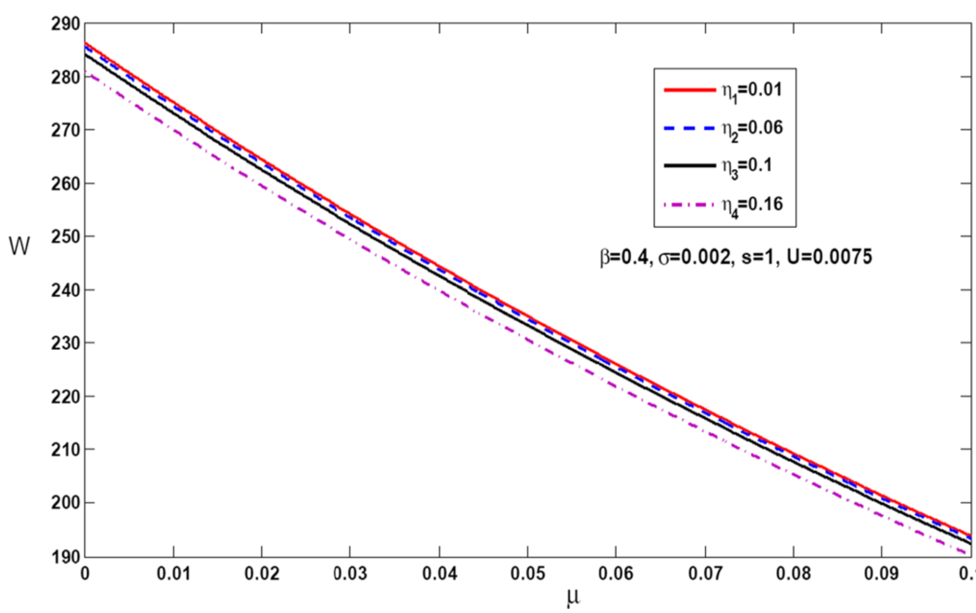


Fig. 4 Variation of soliton amplitude of KdV equation with respect to μ for $\beta = 0.8$, $s = 1$, $\sigma = 0.1$, $U = 0.0075$ and different values of η

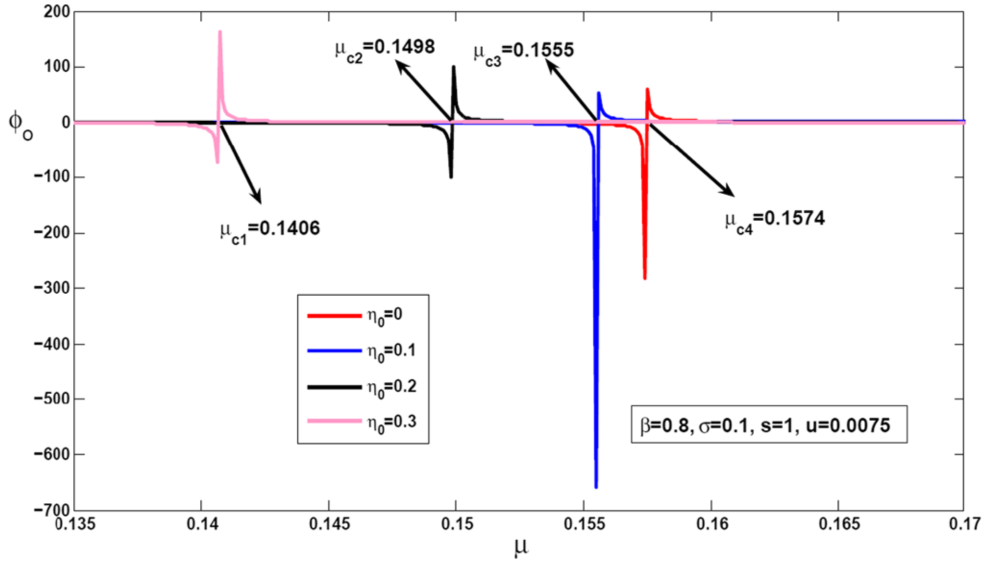
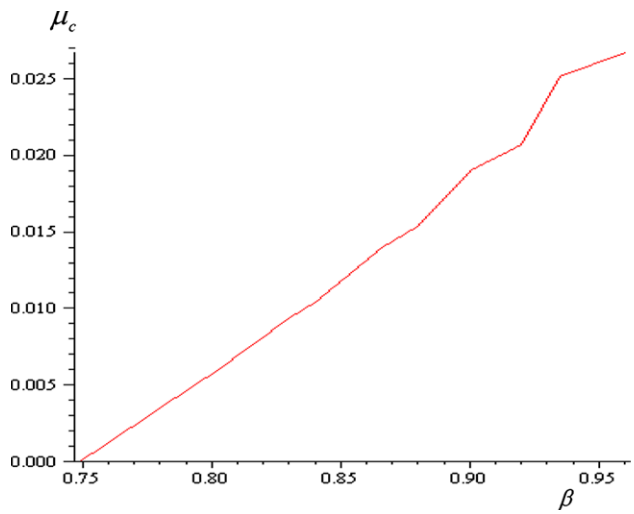
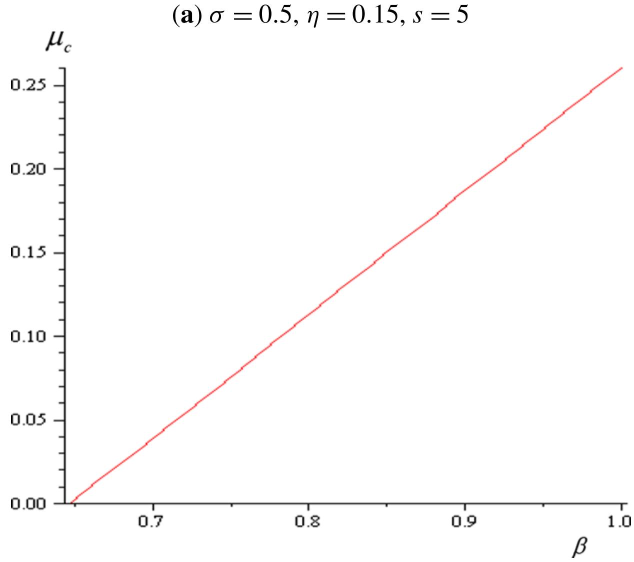


Fig. 5 (**a**, **b**, **c**, **d**) μ_c is plotted against β for different values of s, σ and η

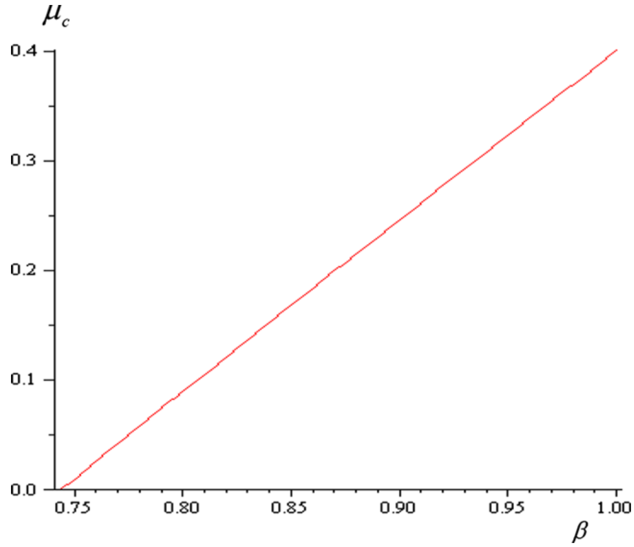


(**a**) $\sigma = 0.5$, $\eta = 0.15$, $s = 5$

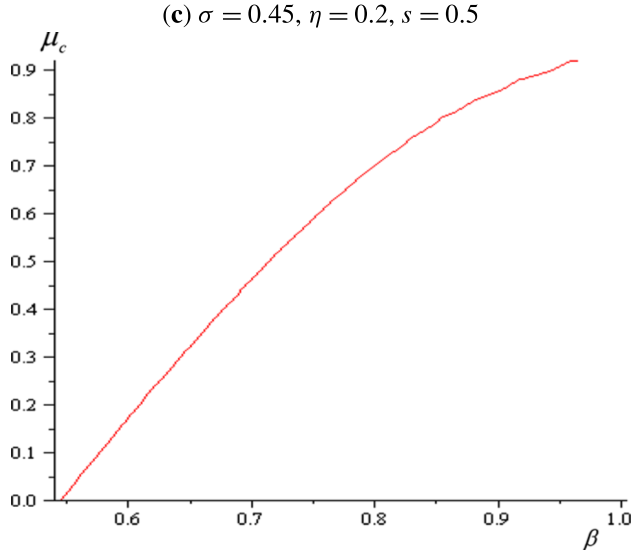


(**b**) $\sigma = 0.15$, $\eta = 0.15$, $s = 1$

Fig. 5 (*Continued*)



(**c**) $\sigma = 0.45$, $\eta = 0.2$, $s = 0.5$



(**d**) $\sigma = 0.025$, $\eta = 0.25$, $s = 0.25$

And for higher orders of ε we will find

$$\begin{aligned}
&-(\lambda - u_0)\frac{\partial n_3}{\partial \xi} + \frac{\partial n_1}{\partial \tau} - \frac{\partial}{\partial \xi}(n_1 u_2 + n_2 u_1) + \frac{\partial u_3}{\partial \xi} = 0,\\
&-(\lambda - u_0)\frac{\partial}{\partial \xi}(\gamma_1 u_3 + \gamma_3 u_1^3) + \gamma_1 \frac{\partial u_1}{\partial \tau} + \gamma_1 \frac{\partial}{\partial \tau}(u_1 u_2)\\
&\quad + \gamma_1 \frac{\partial}{\partial \tau}(u_1^3) + \frac{\partial \phi_3}{\partial \xi} + \sigma(n_1^2 - n_2)\frac{\partial p_1}{\partial \xi} + \sigma \frac{\partial p_3}{\partial \xi}\\
&\quad - \sigma n_1 \frac{\partial p_2}{\partial \xi} = 0,\\
&-(\lambda - u_0)\frac{\partial p_3}{\partial \xi} + \frac{\partial p_1}{\partial \tau} + u_1 \frac{\partial p_2}{\partial \xi} + u_2 \frac{\partial p_1}{\partial \xi}\\
&\quad + \frac{1}{2}\frac{\gamma_2}{\gamma_1}\frac{\partial}{\partial \xi}(u_1^3) + \frac{3\gamma_1}{\lambda - u_0}\frac{\partial u_3}{\partial \xi} + 9\gamma_2 \frac{\partial}{\partial \xi}(u_1 u_2)\\
&\quad + 3p_1\left(\gamma_1 \frac{\partial u_2}{\partial \xi} + 2\gamma_2 u_1 \frac{\partial u_1}{\partial \xi}\right) + 3p_2\gamma_1 \frac{\partial u_1}{\partial \xi} = 0,
\end{aligned} \tag{14}$$

$$\begin{aligned}
\frac{\partial^2 \phi_1}{\partial \xi^2} &= \left(\frac{1 - \beta + \mu s}{1 - \mu}\right)\phi_3 - \frac{1}{2}\left(\frac{1 - \mu s^2}{1 - \mu}\right)\phi_1 \phi_2\\
&\quad + \frac{1}{6}\left(\frac{1 - 3\beta + \mu s^3}{1 - \mu}\right)\phi_1^3 + n_3.
\end{aligned}$$

Finally we obtain the following equation

$$\frac{\partial \phi_1}{\partial \tau} + A\frac{\partial(\phi_1 \phi_2)}{\partial \xi} + C\phi_1^3 \frac{\partial \phi_1}{\partial \xi^3} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{15}$$

where

$$\begin{aligned}
C &= \frac{1}{(1 - \beta + \mu s)}\left\{\frac{3}{4(\lambda - u_0)^3 \gamma_1}\right.\\
&\quad \times \left\{3\sigma \gamma_1^3 (1 - \beta + \mu s) - \frac{1}{8}\right\}
\end{aligned}$$

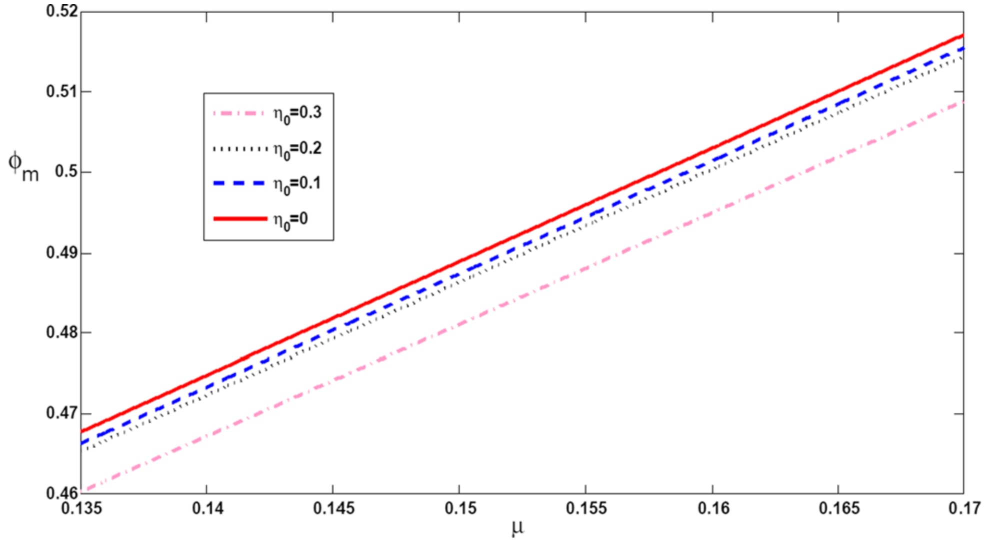


Fig. 6 Variation of soliton amplitude of modified KdV equation with respect to μ for $\beta = 0.8$, $s = 1$, $\sigma = 0.1$, $U_0 = 0.0075$ and different values of η

$$+\frac{27\sigma\gamma_2}{2(\lambda-u_0)^2}+\frac{3\gamma_2^2}{4(\lambda-u_0)\gamma_1^2}\bigg\}+\frac{\gamma_2}{\gamma_1}$$
$$-\left(\frac{1}{4\gamma_1(1-\beta+\mu s)^5(\lambda-u_0)^3}\right)\bigg\{\frac{(1-\mu s^2)\gamma_1^2}{(1-\beta+\mu s)^2}$$
$$-\frac{3\gamma_2^2}{\gamma_1}+\frac{2(\lambda-u_0)(1-\mu s^2)(1-3\beta+\mu s^3)}{(1-\beta+\mu s)}\bigg\}. \tag{16}$$

At $A = 0$, (15) reduces in to the modified KdV equation and the stationary solution of it can be written as

$$\phi_1 = \pm\phi_m \sec h\big[(\xi - U_0\tau)/W\big], \tag{17}$$

where U, $\phi_m = \sqrt{6U_0/C}$ and $W = \sqrt{B/U_0}$ are velocity, amplitude and width of the solitons, respectively. Obviously, the physically reasonable solitons correspond to the condition $C > 0$, and in this case both compressive and rarefactive solitons are also allowed to coexist. Figure 6 shows the validity of the soliton solution (17) for describing the system at the critical case. In comparison with Fig. 4, solitons of modified KdV equation in Fig. 6 are finite. It shows that the amplitude ϕ_m increases as μ increases. The effect of relativistic factor (η) will also decrease the soliton amplitude as for the original soliton (11). Equations (10) and (17) describe the same solitonic solutions in the plasma. The solitons of (8) are small amplitude and (15) describes the solitons with long amplitude.

4 Conclusions

The properties ion acoustic waves in weakly relativistic plasmas consisting of Maxwell-Boltzmann distributed positrons, nonthermal electrons, and warm relativistic ions are systematically and explicitly investigated with the help of the reductive perturbation method. The results found from this investigation can be summarized as follows:

(i) The compressive and rarefactive ion acoustic solitons will be created depending on the plasma parameters. Only compressive solitons are obtained in the absence of nonthermal electrons ($\beta = 0$).
(ii) The nonlinear coefficient of the KdV equation is equal to zero at critical values of μ, μ_c. In this case a solitonic solution of KdV equation can not be established. Hence, by inserting the high order nonlinear terms and new scales, the modified KdV equation is derived. The solitons of modified KdV equation are finite.
(iii) By the increase of relativistic factor amplitude of compressive solitons decreases while the amplitude of rarefactive solitons increases.
(iv) The amplitude of soliton for warm plasma is less than that of cold plasma.

Such plasmas occur in plasma sheet boundary layer of earth's magnetosphere, Van-Allen belts. Our results also help understand nonlinear structures in plasma containing nonthermal electrons, as in certain Auroral zone.

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