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ORIGINAL ARTICLE

A class of regular and well behaved charge analogue of Kuchowicz's relativistic super-dense star model

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Abstract We obtain a well behaved class of charge analogues of neutral superdense star model due to Kuchowicz, by using a particular electric field, which involves a parameter K and vanishes when $K = 0$. The members of this class are seen to satisfy the various physical conditions e.g. $c^2\rho \geq 3p \geq 0$, $dp/dr < 0$, $d\rho/dr < 0$, along with the velocity of sound, $dp/c^2d\rho < 1$ and the adiabatic index $((p + c^2\rho)/p)(dp/(c^2d\rho)) > 1$, for the interval $0 < K < 1$ with the maximum mass $6.8374M_\Theta$ and the radius 23.4679 km with the central red shift $Z_c = 0.75364$. In the interval, $0 < K \leq 0.1179$, the velocity of sound and the ratio $p/c^2\rho$ are found monotonically decreasing towards the pressure free interface, which presents a relevant model for massive star like Neutron star or pulsar with the maximum mass as $4.1474M_\Theta$ and the radius 20.5481 km with the central red shift $Z_c = 0.6654$.

Keywords Charged fluids · Reissener-Nordstom metric · Superdence star · General relativity

1 Introduction

Exact interior solutions of the Einstein-Maxwell field equations joining smoothly to the Nordstrom solution at the pressure free interface are gathering big applause due to some of the following reasons:

(i) Gravitational collapse of a charged fluid sphere to a point singularity may be avoided.

(ii) Solutions of Einstein-Maxwell equations are useful in the study of cosmic sensership.

(iii) Charged-dust models and electromagnetic mass models are expected to provide some clue about structure of an electron.

In making of charged fluid spheres, the most desirable condition demands its reduction to neutral fluid sphere in the absence of charge. Moreover both the solutions should satisfy the physical requirements e.g., energy density $(c^2\rho)$ should dominate the non-zero pressure (p) throughout the sphere $0 \leq r < a$ such that

$$(dp/dr) < 0, \qquad \left(\frac{d\rho}{dr}\right) < 0, \qquad \left(\frac{dp}{c^2 d\rho}\right) < 1,$$
$$\frac{d}{dr}\left(\frac{p}{\rho}\right) < 0 \quad \text{and} \quad \frac{d}{dr}\left(\frac{dp}{d\rho}\right) < 0.$$

The said conditions are necessary for a well behaved fluid distribution. However if we drop the last two conditions, the corresponding fluid sphere is taken to be sensible up to some extent. In fact well behaved conditions may or may not be satisfied by both the solutions (neutral or its charged analogue). The charged fluids obtained by Gupta and Kumar (2005), Bijalwan and Gupta (2008) and Neeraj Pant et al. (2010a) do not admit their well behaved neutral counterpart. While the charged spheres by Gupta and Maurya's (2010a, 2010b) are well behaved along with their neutral analogues.

In the present problem we have picked up a well behaved neutral solution due to Kuchowicz (1967) and charged the same by applying a specific form of electric intensity. The charged solution so obtained is well behaved. If we go the back history Delgaty and Lake (1998) have tabled around 116 neutral interior solutions but only 9 of them are well behaved. One more solution in this context is published recently by Neeraj Pant et al. (2010a, 2010b). So obviously

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who wishes to construct a well behaved charge fluid sphere is likely to pick up a well behaved solution out of the above 9 tabled as seed solution. However there is no guarantee that the resulting solution will also be well behaved. It all depends upon the form of electric intensity and the metric potential used.

2 Field equations

The following static spherically symmetric metric is taken to describe the charged fluid spheres

$$ds^2 = -e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu}dt^2 \tag{1}$$

where the functions $\lambda(r)$ and $\nu(r)$ satisfy the Einstein-Maxwell equations

$$\begin{aligned}-\kappa T^i_j &= R^i_j - \frac{1}{2}R\delta^i_j \\ &= -\kappa\left[(c^2\rho + p)v^i v_j - p\delta^i_j \right.\\ &\quad \left.+ \frac{1}{4\pi}\left(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F_{mn}F^{mn}\right)\right]\end{aligned} \tag{2}$$

with $\kappa = \frac{8\pi G}{c^4}$ while ρ, p, v^i, F_{ij} denote energy density, fluid pressure, flow vector and skew-symmetric electromagnetic field tensor respectively. F_{ij} further satisfies the Maxwell equations

$$F_{ik,j} + F_{kj,i} + F_{ji.k} = 0 \tag{3}$$

$$\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi\sqrt{-g}\,j^i \tag{4}$$

where $j^i = \sigma v^i$ represents the four-current vector of the charged fluid with σ as the charged density.

In view of the metric (1), the field equation (2) gives (Dionysiou 1982)

$$\frac{\nu'}{r}e^{-\lambda} - \frac{(1-e^{-\lambda})}{r^2} = \kappa p - \frac{q^2}{r^4} \tag{5}$$

$$\left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right]e^{-\lambda} = \kappa p + \frac{q^2}{r^4} \tag{6}$$

$$\frac{\lambda'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \kappa c^2\rho + \frac{q^2}{r^4} \tag{7}$$

where, prime denotes the differentiation with respect to r and

$$q(r) = 4\pi\int_0^r \sigma r^2 e^{\lambda/2}dr = r^2\sqrt{-F_{14}F^{14}} = r^2F^{41}e^{(\lambda+\nu)/2} \tag{8}$$

represents the total charge contained within the sphere of radius r in view of (1).

Equation (4) reduces to

$$\frac{\partial}{\partial r}(r^2F^{41}e^{(\lambda+\nu)/2}) = -4\pi r^2 e^{(\lambda+\nu)/2}j^4 \tag{9}$$

Beyond the pressure free interface $r = a$ the charged fluid sphere is expected to join smoothly with the Reissner-Nordstrom metric

$$\begin{aligned}ds^2 &= -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \\ &\quad + \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)dt^2\end{aligned} \tag{10}$$

where m is the gravitational mass of the distribution such that

$$m = \mu(a) + \varepsilon(a) \tag{11}$$

while

$$\begin{aligned}\mu(a) &= \frac{\kappa}{2}\int_0^a \rho r^2 dr, \\ \varepsilon(a) &= \frac{\kappa}{2}\int_0^a r\sigma q e^{\lambda/2}dr, \quad e = q(a)\end{aligned} \tag{12}$$

$\varepsilon(a)$ is the mass equivalence of the electromagnetic energy of distribution, $\mu(a)$ is the mass and e is the total charge inside the sphere (Florides 1983).

Now let us set

$$g_{44} = y^2 = Be^{x/2} \tag{13}$$

which is the same as that of the metric obtained by Kuchowicz (1967).

Insertion of (13) into (5)–(7) gives

$$Z - \frac{(1-Z)}{x} + \frac{Cq^2}{x^2} = \frac{\kappa p}{C}, \tag{14}$$

$$\frac{(1-Z)}{x} - 2\frac{dZ}{dx} - \frac{Cq^2}{x^2} = \frac{\kappa c^2\rho}{C} \tag{15}$$

and Z satisfying the equation

$$\frac{dZ}{dx} + \frac{(2-x)}{2x}Z = \frac{2\{(2q^2C/x) - 1\}}{x(x+2)} \tag{16}$$

where $x = Cr^2, e^{-\lambda} = Z$.

In order to solve the differential equation (16) let us consider the electric intensity E of the following form

$$\frac{E^2}{C} = \frac{Cq^2}{x^2} = \frac{Kx}{(x+2)^2} \tag{17}$$

where K is a positive constant. The electric intensity so assumed is physically palatable since E remains regular and positive throughout the sphere. In addition, E vanishes at the centre of the star.

In view of (17) differential equation (16) yields the following solution

$$Z = 1 + \frac{x}{e^{x/2}}\left[A + \frac{K-1}{2e}Ei\left(1+\frac{x}{2}\right)\right] - \frac{Kx(x+4)}{(x+2)^2} \quad (18)$$

where A is an arbitrary constant of integration and

$$Ei\left(1+\frac{x}{2}\right) = \log\left(1+\frac{x}{2}\right) + \sum_{n=1}^{\infty}\frac{(1+\frac{x}{2})^n}{n!n}$$

Using (18), into (14) and (15), we get the following expressions for pressure and energy density

$$\frac{\kappa p}{C} = (1-K) + \frac{(x+1)}{e^{x/2}}\left[A + \frac{K-1}{2e}Ei\left(1+\frac{x}{2}\right)\right] \quad (19)$$

$$\frac{\kappa c^2\rho}{C} = \frac{x}{(x+2)} + \frac{(x-3)}{e^{x/2}}\left[A + \frac{K-1}{2e}Ei\left(1+\frac{x}{2}\right)\right] + \frac{K(24-x^3-4x^2)}{(x+2)^3} \quad (20)$$

Consequently the expressions for pressure and density gradients read as

$$\frac{\kappa}{C}\frac{dp}{dx} = \frac{(1-x)}{2e^{x/2}}\left[A + \frac{K-1}{2e}Ei\left(1+\frac{x}{2}\right)\right] - \frac{(x+1)}{2(x+2)} + \frac{K(x^3+5x^2+8x+4)}{2(x+2)^3} \quad (21)$$

$$\frac{\kappa c^2}{C}\frac{d\rho}{dx} = \frac{(5-x)}{2e^{x/2}}\left[A + \frac{K-1}{2e}Ei\left(1+\frac{x}{2}\right)\right] + \frac{Kg(x)}{2(x+2)^4} - \frac{h(x)}{2(x+2)^2} \quad (22)$$

and hence the velocity of sound v is given by the following expression

$$v^2 = \frac{dp}{c^2 d\rho} = \frac{(1-x)(x+2)^4 f(x) - (x+1)(x+2)^3 + Km(x)}{(5-x)(x+2)^4 f(x) - h(x)(x+2)^2 + Kg(x)} \quad (22(a))$$

where

$$g(x) = (x^4 + 3x^3 - 10x^2 - 30x - 108),$$

$$h(x) = (x^2 - x - 10),$$

$$m(x) = (x^3 + 5x^2 + 8x + 4)(x+2),$$

$$f(x) = \left[A + \frac{K-1}{2e}Ei\left(1+\frac{x}{2}\right)\right].$$

3 Conditions for regular and well behaved model

(i) Pressure p should be zero at boundary $r = a$

(ii) $c^2\rho \geq p > 0$ or $c^2\rho \geq 3p > 0$, $0 \leq r \leq a$, where former inequality denotes weak energy condition (WEC), while the later inequality implies strong energy condition (SEC)

(iii) $(dp/dr)_{r=0} = 0$ and $(d^2p/dr^2)_{r=0} < 0$ so that pressure gradient dp/dr is negative for $0 < r \leq a$.

(iv) $(d\rho/dr)_{r=0} = 0$ and $(d^2\rho/dr^2)_{r=0} < 0$ so that density gradient $d\rho/dr$ is negative for $0 < r \leq a$

The condition (iii) and (iv) imply that pressure and density should be maximum at the centre and monotonically decreasing towards the surface.

(v) The casualty condition $(dp/c^2d\rho)^{1/2}$ i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e. $\frac{d}{dr}(\frac{dp}{d\rho}) < 0$ or $(\frac{d^2p}{d\rho^2}) > 0$ for $0 \leq r \leq a$ i.e. the velocity of sound is increasing with the increase of density.

(vi) The ratio of pressure to the density $(p/c^2\rho)$ should be monotonically decreasing with the increase of r i.e. $\frac{d}{dr}(\frac{p}{c^2\rho})_{r=0} = 0$ and $\frac{d^2}{dr^2}(\frac{p}{c^2\rho})_{r=0} < 0$ and $\frac{d}{dr}(\frac{p}{c^2\rho})$ is negative valued function for $r > 0$. Which implies that the temperature of the model decreases towards the surface.

(vii) The central red shift Z_c and surface red shift Z_s should be positive and finite i.e. $Z_c = [(e^{-\nu/2} - 1)_{r=0}] > 0$ and $Z_s = [e^{\lambda(a)/2} - 1] > 0$ and both should be bounded.

(viii) The solution should be free from physical and geometric singularities. Including

$$e^{\lambda} = 1 \quad \text{at } r = 0.$$

(ix) Electric intensity E, such that $E(0) = 0$, is taken to be monotonically increasing i.e. $(dE/dr) > 0$ for $0 < r < a$.

Besides the above, the charged fluid spheres is expected to join smoothly with the Reissner-Nordstrom metric

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)dt^2 \quad (23)$$

at the pressure free boundary $r = a$, which requires the continuity of e^{λ}, e^{ν} and q of either metric at $r = a$. Consequently we have,

$$e^{-\lambda(a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2} \quad (24)$$

$$e^{\nu(a)} = y^2_{(r=a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2} \tag{25}$$

$$q(a) = e \tag{26}$$

$$p_{(r=a)} = 0 \tag{27}$$

The condition (27) can be utilized to compute the values of arbitrary constant A as follows:

On setting $x_{r=a} = X = Ca^2$, (a being the radius of the charged sphere)

Pressure at $p_{(r=a)} = 0$ gives

$$A = \frac{e^{X/2}}{(1+X)}\left[(K-1) - \frac{(K-1)}{2e}(X+1)e^{-X/2}Ei\left(1+\frac{X}{2}\right)\right] \tag{28}$$

The expression for mass can be written as

$$m(a) = \frac{a}{2}\left[\frac{2KX}{(X+2)} - \frac{X}{e^{X/2}}\left(A + \frac{(K-1)}{2e}Ei\left(1+\frac{X}{2}\right)\right)\right] \tag{29}$$

such that $e^{-\lambda(a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}$, where $M = m(a)$ and $y^2_{(r=a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}$ gives

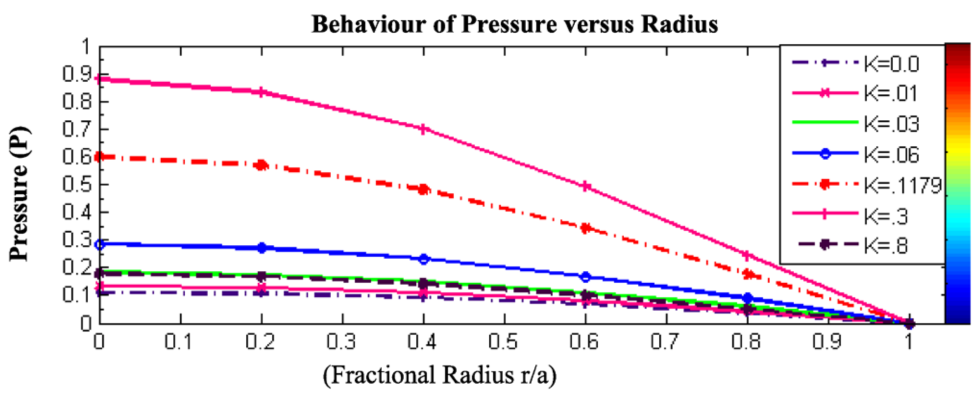
$$B = \frac{1}{e^{X/2}}\left[1 + \frac{X}{e^{X/2}}\left[A + \frac{K-1}{2e}Ei\left(1+\frac{x}{2}\right)\right] - \frac{KX(X+4)}{(X+2)^2}\right] \tag{30}$$

Also, if the surface density ρ_a is prescribed as 2×10^{14} g cm^{-3} (super dense star case) then value of constant C can be calculated for a given X $(= Ca^2)$, using the following expression

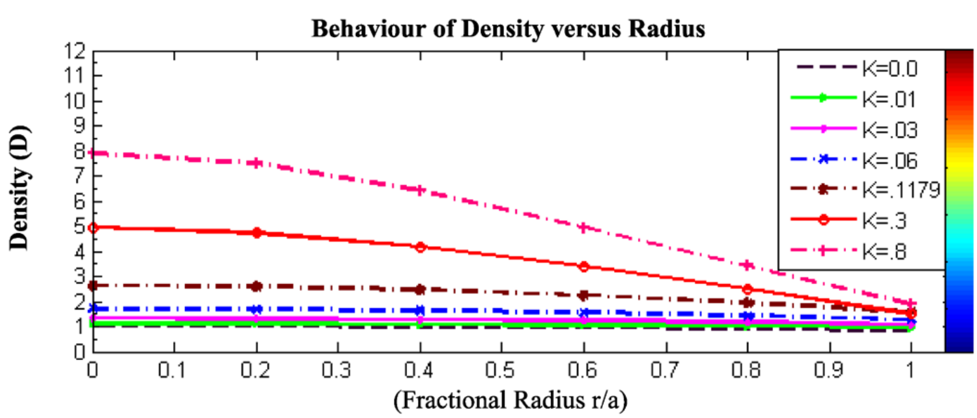
$$\kappa c^2 \rho_a = C\left[\frac{X}{(X+2)} + \frac{(X-3)}{e^{X/2}}\left[A + \frac{K-1}{2e}Ei\left(1+\frac{X}{2}\right)\right] + \frac{K(24 - X^3 - 4X^2)}{(X+2)^3}\right] \tag{31}$$

4 Physical analysis and conclusions

In the preceding sections, we have derived a class of charge analogues of neutral superdense star model due to Kuchowicz, by using a particular electric field, which depend upon a parameter K, vanishing of which leads to the neutral case.



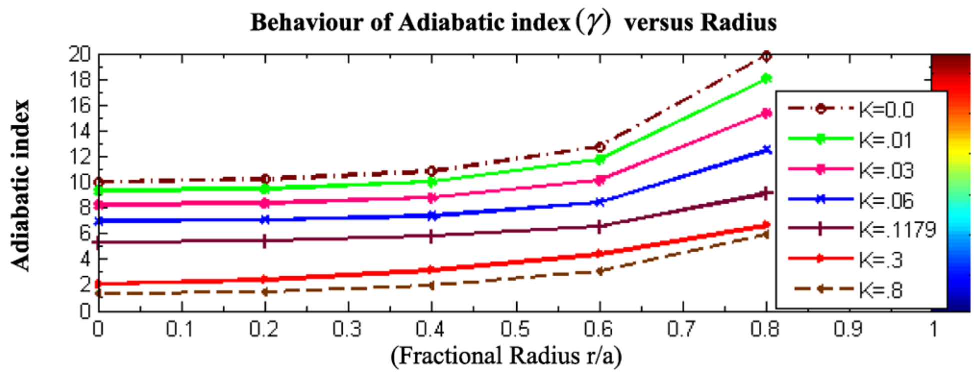
(a)



(b)

Fig. 1 (**a**): Behaviour of pressure (P) versus radius for $K = 0.0$, 0.01, 0.03, 0.06, 0.1179, 0.3 and 0.8 respectively. (**b**): Behaviour of density (D) versus radius for $K = 0.0$, 0.01, 0.03, 0.06, 0.1179, 0.3 and 0.8 respectively. (**c**): Behaviour of adiabatic index (γ) versus radius for $K = 0.0$, 0.01, 0.03, 0.06, 0.1179, 0.3 and 0.8 respectively. (**d**): Behaviour of charge (Q) versus radius for $K = 0.0$, 0.01, 0.03, 0.06, 0.1179, 0.3 and 0.8 respectively. (**e**): Behaviour of velocity of sound (v) versus radius for $K = 0.0$, 0.03, 0.1179 and 0.3 respectively. (**f**): Behaviour of ratio of pressure and density ($p/c^2\rho$) versus radius for $K = 0.0$, 0.01, 0.03, 0.06, 0.1179, 0.3 and 0.8 respectively

Fig. 1 (*Continued*)



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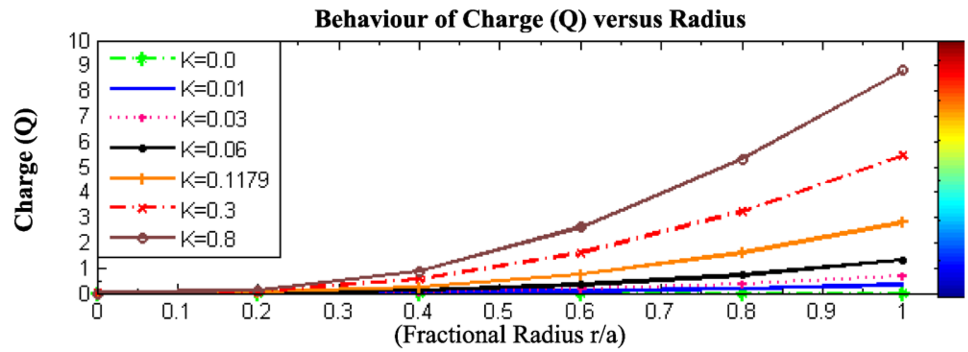

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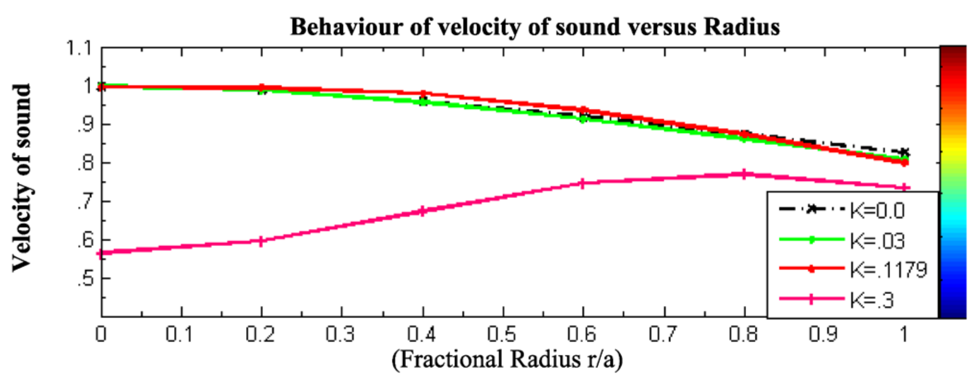
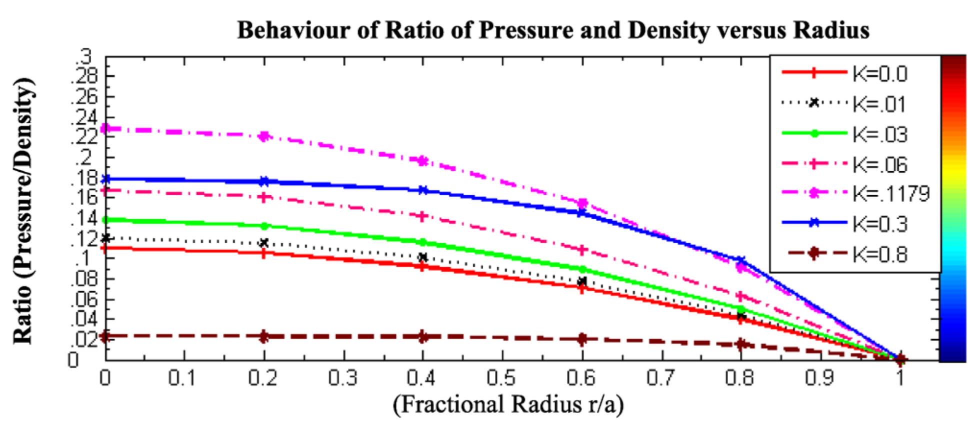


(e)



(f)

The charged superdence star models satisfy the energy conditions $c^2\rho \geq 3p > 0$, $dp/dr < 0$, $d\rho/dr < 0$, adiabatic index $\gamma = ((p + c^2\rho)/p)(dp/(c^2 d\rho)) > 1$, and the causality condition $dp/c^2 d\rho < 1$ for $0 < K < 1$. The velocity of sound and the ratio $p/c^2\rho$ are seen monotonically decreasing towards the surface for $0 < K \leq 0.1179$.

The red shift is also decreasing monotonically from the centre to the pressure free interface for $0 < K \leq 0.1179$. Further for the interval $0 < K \leq 0.1179$, the maximum mass and the corresponding radius of the model are computed to be $4.1474 M_\Theta$ and 20.5481 km respectively along with the central red shift $Z_c = 0.75364$. While for the interval $0 < K < 1$, the maximum mass of the superdence star model is found to be $6.8374 M_\Theta$ with the corresponding radius 23.4679 km with the central red shift $Z_c = 0.6654$ for the weak energy as well as strong energy conditions.

Detailed physically behaviour of the models for various K are displayed by means of graphs (Fig. 1) and tables.

5 Tables for numerical values of physical quantities

In Tables 1–9: Z_c = red shift at the centre, Z_s = red shift at the surface, Solar mass $M_\Theta = 1.475$ km, $G = 6.673 \times 10^{-8}$ cm^3/gs^2, $c = 2.997 \times 10^{10}$ cm/s, $D = (8\pi G/c^2)\rho a^2$, $P = (8\pi G/c^4) p a^2$, $\gamma = \frac{p+c^2\rho}{p} \frac{dp}{c^2 d\rho}$, $R = \frac{p}{c^2\rho}$, $V = (dp/c^2 d\rho)^{1/2}$.

Table 1 Maximum mass (M/M_Θ) and radius (a) for different K and Ca^2 for WEC and SEC

K	Ca^2	$0 \leq p \leq c^2\rho$(WEC)		Ca^2	$0 \leq 3p \leq c^2\rho$(SEC)	
		Radius (Km.)	Max (M/M_Θ)		Radius (Km.)	Max (M/M_Θ)
0.0	0.4456	15.2557	1.5941	0.4456	15.2557	1.5941
0.01	0.4984	15.8829	1.7944	0.4984	15.8829	1.7944
0.03	0.6058	16.9853	2.1873	0.6058	16.9853	2.1873
0.06	0.8032	18.5062	2.8424	0.8032	18.5062	2.8424
0.1179	**1.3190**	**20.5481**	**4.1474**	**1.3190**	**20.5481**	**4.1474**
0.3	1.9428	20.0794	5.1579	1.9428	20.0794	5.1579
0.8	1.5608	22.4984	6.2783	1.5608	22.4984	6.2783
0.9999	1.5073	23.4679	6.8374	1.5073	23.4679	6.8374

Table 2 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2 d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.0$, $Ca^2 = 0.4456$)

$K = 0.0$, $Ca^2 = 0.4456$, Radius (a) = 15.2557 Km, $M = 1.5941 M_\Theta$, $Z_c = 0.2264$, $Z_s = 0.2023$.

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0	0.1104	1.0055	10.0353	0.0000	0.9966	0.1098
.2	0.1055	1.0004	10.1976	0.0000	0.9862	0.1054
.4	0.0909	0.9850	10.8590	0.0000	0.9579	0.0923
.6	0.0675	0.9583	12.7938	0.0000	0.9175	0.0704
.8	0.0366	0.9196	19.8420	0.0000	0.8713	0.0398
1	0.0000	0.8686	∞	0.0000	0.8235	0.0000

Table 3 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2 d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.01$, $Ca^2 = 0.4984$)

$K = 0.01$, $Ca^2 = 0.4984$, Radius (a) = 15.8829 Km, $M = 1.7944 M_\Theta$, $Z_c = 0.2539$, $Z_s = 0.2243$.

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0	0.1330	1.1112	9.3351	0.0000	0.9989	0.1197
.2	0.1270	1.1050	9.4705	0.0029	0.9879	0.1149
.4	0.1092	1.0858	10.0386	0.0230	0.9580	0.1006
.6	0.0809	1.0526	11.7466	0.0751	0.9156	0.0768
.8	0.0437	1.0046	18.0590	0.1706	0.8674	0.0435
1	0.0000	0.9415	∞	0.3168	0.8178	0.0000

Table 4 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2 d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.03$, $Ca^2 = 0.6058$)

$K = 0.03$, $Ca^2 = 0.6058$, Radius (a) = 16.9853 Km, $M = 2.1873 M_\Theta$, $Z_c = 0.3103$, $Z_s = 0.2682$.

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0	0.1824	1.3247	8.2223	0.0000	0.9976	0.1377
.2	0.1740	1.3154	8.3250	0.0065	0.9862	0.1323
.4	0.1493	1.2871	8.7723	0.0509	0.9548	0.1160
.6	0.1100	1.2383	10.1636	0.1651	0.9105	0.0888
.8	0.0589	1.1680	15.4022	0.3717	0.8600	0.0504
1	0.0000	1.0767	∞	0.6839	0.8077	0.0000

Table 5 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2 d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.06$, $Ca^2 = 0.8032$)

$K = 0.06$, $Ca^2 = 0.8032$, Radius (a) = 18.5062 Km, $M = 2.8424 M_\Theta$, $Z_c = 0.4142$, $Z_s = 0.3462$.

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$
0	0.2845	1.7008	6.9569	0.0000	0.9984	0.1673
.2	0.2709	1.6847	7.0354	0.0130	0.9872	0.1608
.4	0.2313	1.6355	7.3729	0.1009	0.9559	0.1415
.6	0.1690	1.5512	8.4286	0.3231	0.9099	0.1089
.8	0.0893	1.4311	12.4708	0.7167	0.8559	0.0624
1	0.0000	1.2781	∞	1.2989	0.7984	0.0000

Table 6 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.1179$, $Ca^2 = 1.3190$)

$K = 0.1179$, $Ca^2 = 1.3190$, Radius (a) = 20.5481 Km, $M = 4.1474M_\Theta$, $Z_c = 0.6654$, $Z_s = 0.5372$.

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0	0.5991	2.6262	5.3318	0.0000	0.9952	0.2281
.2	0.5691	2.5833	5.4656	0.0321	0.9933	0.2203
.4	0.4815	2.4541	5.8241	0.2424	0.9773	0.1962
.6	0.3450	2.2381	6.5460	0.7501	0.9351	0.1542
.8	0.1767	1.9401	9.1015	1.6025	0.8716	0.0911
1	0.0000	1.5757	∞	2.8039	0.7977	0.0000

Table 7 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.3$, $Ca^2 = 1.9428$)

$K = 0.3$, $Ca^2 = 1.9428$, Radius (a) = 20.0794 Km, $M = 5.1579M_\Theta$, $Z_c = 0.8399$, $Z_s = 0.7816$.

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0	0.8782	4.9424	2.0972	0.0000	0.5625	0.1777
.2	0.8331	4.7382	2.3606	0.0741	0.5941	0.1758
.4	0.7003	4.1887	3.1526	0.5397	0.6720	0.1672
.6	0.4929	3.4129	4.4038	1.5927	0.7455	0.1444
.8	0.2438	2.4994	6.6141	3.2383	0.7667	0.0975
1	0.0000	1.5047	∞	5.4192	0.7318	0.0000

Table 8 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.8$, $Ca^2 = 1.5608$)

$K = 0.8$, $Ca^2 = 1.5608$, Radius (a) = 22.4984 Km, $M = 6.2783M_\Theta$, $Z_c = 0.7739$, $Z_s = 0.7395$.

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0	0.1776	7.8956	1.3373	0.0000	0.1715	0.0225
.2	0.1685	7.4848	1.4784	0.1191	0.1804	0.0225
.4	0.1422	6.4029	1.9658	0.8759	0.2066	0.0222
.6	0.1011	4.9637	3.0606	2.6075	0.2472	0.0204
.8	0.0511	3.4209	5.9023	5.3145	0.2946	0.0149
1	0.0000	1.8890	∞	8.8205	0.3360	0.0000

Table 9 Behaviour of pressure (P), density (D), adiabatic index (γ), charge (Q), velocity of sound ($\sqrt{dp/c^2d\rho}$) and ratio of pressure–density ($p/c^2\rho$) for ($K = 0.9999$, $Ca^2 = 1.5073$)

$K = 0.9999$, $Ca^2 = 1.5073$, Radius (a) = 23.4679 Km, $M = 6.8374M_\Theta$, $Z_c = 0.75364$, $Z_s = 0.75363$.

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$
0	8.4009×10^{-5}	9.0431	1.2685	0.0000	0.0034	9.2898×10^{-6}
.2	7.9749×10^{-5}	8.5604	1.3979	0.1373	0.0036	9.3160×10^{-6}
.4	6.7304×10^{-5}	7.2898	1.8491	1.0101	0.0041	9.2326×10^{-6}
.6	4.7951×10^{-5}	5.6050	2.8915	3.0048	0.0050	8.5549×10^{-6}
.8	2.4290×10^{-5}	3.8138	5.7319	6.1086	0.0060	6.3690×10^{-6}
1	0.0000	2.0553	∞	10.0850	0.0071	0.0000

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