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ORIGINAL ARTICLE

Well behaved class of charge analogue of Heintzmann's relativistic exact solution

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Abstract We present a well behaved class of Charge Analogue of Heintzmann (Z. Phys. 228:489, 1969) solution. This solution describes charge fluid balls with positively finite central pressure and positively finite central density ; their ratio is less than one and causality condition is obeyed at the centre. The outmarch of pressure, density, pressure-density ratio and the adiabatic speed of sound is monotonically decreasing, however, the electric intensity is monotonically increasing in nature. The solution gives us wide range of constant K $(1.25 \leq K \leq 15)$ for which the solution is well behaved and therefore, suitable for modeling of super dense star. For this solution the mass of a star is maximized with all degrees of suitability and by assuming the surface density $\rho_b = 2 \times 10^{14}$ g/cm^3. Corresponding to $K = 1.25$ and $X = 0.42$, the maximum mass of the star comes out to be $3.64 M_\Theta$ with linear dimension 24.31 km and central red-shift 1.5316.

The charge analogue of Heint-solution has simple algebraic expressions. In order to study the behavior of physical parameters from centre to boundary we use the analytic method with the help of the developed theorem. However, the charge analogue of exact solutions, so far obtained, the numerical methods have been used to study the behavior of physical parameters from centre to boundary.

Keywords Charge fluid · Reissner–Nordstrom · General relativity · Exact solution

1 Introduction

It is well known that the Reissner-Nordstrom solution for the external field of a ball of charged mass has two distinct singularities at finite radial positions other than at the centre. Thus the solution describes a bridge (worm hole) between two asymptotically flat spaces and an electric flux flowing across the bridge. Graves and Brill (1960) pointed out that the region of minimum radius or the throat of worm hole pulsates periodically between these two surfaces due to Maxwell pressure of the electric field. Consequently, unlike Schwarzschild's exterior solution of chargeless matter, in *Reissner–Nordstrom* solution has no surface which can catastrophically hit the geometric singularity at $r = 0$. All these aspects show that the presence of some charge in a spherical material distribution provides an additional resistance against the gravitational contraction by means of electric repulsion.

The above result has been supported by a physically reasonable charge spherical model of Bonnor (1965), that a dust distribution of arbitrarily large mass and small radius can remain in equilibrium against the pull of gravity by a repulsive force produced by a small amount of charge. Thus it is desirable to study the implications of Einstein–Maxwell field equations with reference to the general relativistic prediction of gravitational collapse. For this purpose charged fluid ball models are required. Eventually the external field of such ball is to be matched with *Reissner–Nordstrom* solution.

For obtaining significant charged fluid ball models of Einstein–Maxwell field equations, the Astrophysicists have been using exact solutions with finite central parameters of

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Einstein field equations, as seed solutions. There are two types of exact solutions of this category.

Type 1 If the solutions are well behaved (Delgaty and Lake 1998; Pant et al. 2010), these solutions their self completely describe interior of the *Neutron star or* analogous super dense astrophysical objects *with chargeless matter*. Delgaty and Lake (1998) studied most of the exact solutions so far obtained and pointed out that only *nine* solutions are regular and well behaved; out of which only six are well behaved in curvature coordinates and the rest three solutions are in isotropic coordinates. In previous papers (Pant et al. 2010; Pant 2010), we obtained a new well behaved solution in isotropic coordinates and two new well behaved solutions in curvature coordinates respectively.

Type 2 If the solutions are not well behaved, but with finite central parameters; such solutions are taken as seed solutions of super dense star with charge matter, since at centre the charge distribution is zero.

(1) Schwarzschild's interior solution. The solution is insignificant as it gives us infinite speed of sound throughout within the ball. However, charge Analogues of the solution is well behaved for wide rang of constant (Gupta and Kumar 2005; Gupta and Gupta 1986; Florides 1983, etc.).
(2) Adler (1974) solution and Durgapal and Fuloria solution (1985). The solutions are not well behaved as the speed of sound is monotonically increasing from centre to boundary (well behaved conditions; Delgaty and Lake 1998; Pant et al. 2010). However, charge Analogues of the solution is well behaved for wide rang of constant (Singh and Yadav 1978).

Many of the authors electrified the well known exact solutions which are not well behaved, as seed solutions e.g. Kuchowicz solutions (1968) by Nduka (1977), Tolman solution (1939) by Cataldo and Mitskievic (1992) and Durgapal and Fuloria solution (1985) by Gupta and Maurya (2010) etc. These coupled solutions are well behaved and completely describe interior of the Neutron star or pulsar with the charge matter. The mass of the such modeled super dense object can be maximized by assuming surface density is equal to typical nuclear density i.e. $\rho_b = 2 \times 10^{14}$ g/cm^3.

In the present paper we have charged the Heintzmann's (1969) solution, later rediscovered by Durgapal (1982) and Pant (2010), which is not well behaved with chargeless matter as the speed of sound is monotonically increasing from centre to boundary. The charge analogue of Heint solution is well behaved in all respects.

For well behaved nature of the solution in curvature coordinates, the following conditions should be satisfied (augmentation of (Delgaty and Lake 1998 and Pant et al. 2010) conditions).

(i) The solution should be free from physical and geometrical singularities i.e. finite and positive values of central pressure, central density and non zero positive values of e^λ and e^ν i.e. $p_0 > 0$ an $\rho_0 > 0$. For such solutions the tangent 3-space at the centre is flat and it is an essential condition. For curvature coordinates, mathematically it is expressed as $(e^{-\lambda})_{r=0} = 1$ and $(e^\nu)_{r=0} =$ positive constant (Leibovitz 1969; Pant 2010).

(ii) The solution should have positive and monotonically decreasing expressions for fluid parameters (p and ρ) with the increase of r i.e.
   (a) $p' = 0 \Rightarrow r = 0$ and $(p'')_{r=0} < 0$ and p' *is negative valued function for* $r > 0$.
   (b) $\rho' = 0 \Rightarrow r = 0$ and $(\rho'')_{r=0} < 0$ and ρ' *is negative valued function for* $r > 0$.

(iii) The solution should have positive and monotonically decreasing expression for fluid parameter $\frac{p}{\rho c^2}$ with the increase of r i.e.

$$\left(\frac{p}{\rho c^2}\right)' = 0 \quad \Rightarrow \quad r = 0 \quad \text{and} \quad \left(\frac{p}{\rho c^2}\right)''_{r=0} < 0$$

$$\text{and} \quad \left(\frac{p}{\rho c^2}\right)' \text{ is negative valued function for } r > 0.$$

(iv) The solution should have positive and monotonically decreasing expressions for fluid parameter $(\frac{dp}{d\rho})$ with the increase of r. Thus in view of (ii) we infer that $(\frac{dp}{d\rho})$ should be monotonically increasing with the increase of density ρ i.e.

$$\frac{d^2 p}{d\rho^2} > 0.$$

(v) The solution should have causality condition with in the ball i.e. $0 < \frac{1}{c^2}(\frac{dp}{d\rho}) \le 1$.

(vi) The solution should have positive value of ratio of pressure-density and less than 1 with in the ball i.e. $0 < \frac{p}{\rho c^2} \le 1$.

(vii) The central red shift Z_0 and surface red shift Z_b should be positive and finite i.e.

$$Z_0 = (e^{-\nu/2} - 1)_{r=0} > 0 \quad \text{and}$$

$$Z_b = [e^{\lambda_b/2} - 1] > 0 \quad \text{and both should be bounded.}$$

(viii) Electric intensity is positive and monotonically increasing from centre to boundary and at the centre the Electric intensity is zero. Also the pressure at the boundary is zero.

Aforesaid conditions can be simplified by virtue of the following theorem.

Theorem *If $f(r)=g(x)$; $(\frac{dg}{dx})_{x=0}$ and $(\frac{d^2g}{dx^2})_{x=0}$ are non zero finite; where $x=r^2$, then*

(i) *maxima of $f(r)$ will exist at $r=0$ if $(\frac{dg}{dx})_{x=0}$ is finitely negative.*
(ii) *Minima of $f(r)$ will exist at $r=0$ if $(\frac{dg}{dx})_{x=0}$ is finitely nonzero positive.*

Proof For maxima and minima we have

$$\frac{d}{dr}f(r)=\frac{dg}{dx}\frac{dx}{dr}=2r\frac{dg}{dx}=0 \quad\Rightarrow\quad r=0,$$

$$\text{as}\left(\frac{dg}{dx}\right)_{x=0}\neq 0$$

$$\left(\frac{d^2}{dr^2}f(r)\right)_{r=0}=\left(\frac{d}{dr}\left\{2r\frac{dg}{dx}\right\}\right)_{r=0}$$
$$=\left(2\frac{dg}{dx}+4r^2\frac{d^2g}{dx^2}\right)_{r=0}=\left(2\frac{dg}{dx}\right)_{x=0},$$

provided $(\frac{d^2g}{dx^2})_{x=0}$ is finite.

For maxima at the centre

$$(r=0)\quad\Rightarrow\quad\left(\frac{d^2}{dr^2}f(r)\right)_{r=0}=\left(2\frac{dg}{dx}\right)_{x=0}<0.$$

For minima at the centre

$$(r=0)\quad\Rightarrow\quad\left(\frac{d^2}{dr^2}f(r)\right)_{r=0}=\left(2\frac{dg}{dx}\right)_{x=0}>0.\qquad\square$$

In the forth coming section we shall use this theorem for showing the monotonically decreasing or increasing nature of various physical parameters for well behaved nature of the solution.

2 Einstein-Maxwell equation for charged fluid distribution

Let us consider a spherical symmetric metric in curvature coordinates

$$ds^2=-e^{\lambda}dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2)+e^{\nu}dt^2 \tag{1}$$

where the functions $\lambda(r)$ and $\nu(r)$ satisfy the Einstein-Maxwell equations

$$-\frac{8\pi G}{c^4}T^i_j=R^i_j-\frac{1}{2}R\delta^i_j$$
$$=-\frac{8\pi G}{c^4}\Big[(c^2\rho+p)v^iv_j-p\delta^i_j$$
$$+\frac{1}{4\pi}\left(-F^{im}F_{jm}+\frac{1}{4}\delta^i_jF_{mn}F^{mn}\right)\Big] \tag{2}$$

where ρ, p, v^i, F_{ij} denote energy density, fluid pressure, velocity vector and skew-symmetric electromagnetic field tensor respectively.

In view of the metric (1), the field equation (2) gives (Dionysiou 1982)

$$\frac{\nu'}{r}e^{-\lambda}-\frac{(1-e^{-\lambda})}{r^2}=\frac{8\pi G}{c^4}p-\frac{q^2}{r^4} \tag{3}$$

$$\left(\frac{\nu''}{2}-\frac{\lambda'\nu'}{4}+\frac{\nu'^2}{4}+\frac{\nu'-\lambda'}{2r}\right)e^{-\lambda}=\frac{8\pi G}{c^4}p+\frac{q^2}{r^4} \tag{4}$$

$$\frac{\lambda'}{r}e^{-\lambda}+\frac{(1-e^{-\lambda})}{r^2}=\frac{8\pi G}{c^2}\rho+\frac{q^2}{r^4} \tag{5}$$

where, prime ($'$) denotes the differentiation with respect to r and $q(r)$ represents the total charge contained within the sphere of radius r.

Now let us set

$$e^{\nu}=B(1+c_1r^2)^3 \tag{6}$$

which is the same as that of the metric obtained by Heintzmann (1969).

Putting (6) into (3)–(5), we have

$$\frac{6Y}{1+x}-\frac{(1-Y)}{x}+\frac{c_1q^2}{x^2}=\frac{1}{c_1}\frac{8\pi G}{c^4}p \tag{7}$$

$$\frac{(1-Y)}{x}-2\frac{dY}{dx}-\frac{c_1q^2}{x^2}=\frac{1}{c_1}\frac{8\pi G}{c^2}\rho \tag{8}$$

and Y satisfying the equation

$$\frac{dY}{dx}+\frac{2x^2-2x-1}{x(1+x)(1+4x)}Y=\frac{(1+x)}{x(1+4x)}\left(\frac{2c_1q^2}{x}-1\right) \tag{9}$$

where $x=c_1r^2$, $e^{-\lambda}=Y$.

Our task is to explore the solutions of (9) and obtain the fluid parameters p and ρ from (7) and (8).

3 New class of solutions

In order to solve the differential equation (9), we consider the electric intensity E of the following form

$$\frac{E^2}{c_1}=\frac{c_1q^2}{x^2}=\frac{K}{2}x\sqrt{1+4x} \tag{10}$$

where K is a positive constant. The electric intensity is so assumed that the model is physically significant and well behaved i.e. E remains regular and positive throughout the sphere. In addition, E vanishes at the centre of the star. Thus we have,

$$K\geq 0,\qquad c_1>0 \tag{10a}$$

In view of (10) differential equation (9) yields the following solution

$$Y=e^{-\lambda}=\frac{K}{3}.\frac{x(1+x)^2}{\sqrt{1+4x}}+\frac{(2-x)}{2(1+x)}+\frac{Ax}{(1+x)\sqrt{1+4x}} \tag{11a}$$

where A is an arbitrary constant of integration.

$$e^{\nu}=B(1+x)^3 \tag{11b}$$

Using (11a), (11b) into (7) and (8), we get the following expressions for pressure and energy density

$$\frac{1}{c_1}\frac{8\pi G}{c^4}p=\frac{K}{6}\cdot\frac{(26x^2+19x+2)}{\sqrt{1+4x}}+\frac{A(1+7x)}{(1+x)^2\sqrt{1+4x}}+\frac{9}{2}\cdot\frac{(1-x)}{(1+x)^2} \tag{12}$$

$$\frac{1}{c_1}\frac{8\pi G}{c^2}\rho=-\frac{K}{2}\cdot\frac{(32x^3+34x^2+13x+2)}{(1+4x)^{\frac{3}{2}}}-\frac{3A(1+3x)}{(1+x)^2(1+4x)^{\frac{3}{2}}}+\frac{3(x+3)}{2(1+x)^2} \tag{13}$$

4 Properties of the new class of solutions

The central values of pressure and density are given by

$$\frac{1}{c_1}\frac{8\pi G}{c^4}p_0=\frac{K}{3}+A+\frac{9}{2} \tag{14}$$

$$\frac{1}{c_1}\frac{8\pi G}{c^2}\rho_0=-K-3A+\frac{9}{2} \tag{15}$$

For p_0 and ρ_0 must be positive and $\frac{p_0}{\rho_0}\le 1$, we have

$$-\left(\frac{2K+27}{6}\right)<A\le-\frac{K}{3},\quad \text{and}\quad K\ge 0, A<0 \tag{16}$$

Differentiating (12) and (13) w.r.t. x, we get:

$$\frac{1}{c_1}\cdot\frac{8\pi G}{c^4}\frac{dp}{dx}=\frac{K}{2}\frac{(52x^2+30x+5)}{(1+4x)^{\frac{3}{2}}}-3A\frac{(14x^2+x-1)}{(1+x)^3(1+4x)^{\frac{3}{2}}}+\frac{9}{2}\frac{(x-3)}{(1+x)^3} \tag{17}$$

$$\frac{1}{c_1}\cdot\frac{8\pi G}{c^2}\frac{d\rho}{dx}=-\frac{K}{2}\cdot\frac{(464x^3-108x^2+42x+1)}{(1+4x)^{\frac{5}{2}}}+3A\frac{(30x^2+23x+5)}{(1+x)^3(1+4x)^{\frac{5}{2}}}-\frac{3}{2}\cdot\frac{(x+5)}{(1+x)^3} \tag{18}$$

$$\left(\frac{1}{c_1}\cdot\frac{8\pi G}{c^4}\frac{dp}{dx}\right)_{x=0}=\frac{5K}{2}+3A-\frac{27}{2}$$
$$\left(\frac{1}{c_1}\cdot\frac{8\pi G}{c^4}\frac{dp}{dx}\right)_{x=0}<0\quad(-ve) \tag{19}$$

The expression of right hand side of (19) is negative, thus the pressure p is maximum at the centre and monotonically decreasing.

$$\left(\frac{1}{c_1}\frac{8\pi G}{c^2}.\frac{d\rho}{dx}\right)_{x=0}=-\frac{K}{2}+15A-\frac{15}{2} \tag{20}$$

$$\left(\frac{1}{c_1}\cdot\frac{8\pi G}{c^2}\frac{d\rho}{dx}\right)_{x=0}<0 \tag{21}$$

The expression of right hand side of (20) is negative, thus the density ρ is maximum at the centre and monotonically decreasing.

And hence the velocity of sound v is given by the following expression

$$v^2=\frac{dp}{d\rho}$$
$$\frac{1}{c^2}\frac{dp}{d\rho}=\frac{(1+4x)[\frac{K}{2}(52x^2+30x+5)(1+x)^3-3A(14x^2+x-1)+\frac{9}{2}(x-3)(1+4x)^{\frac{3}{2}}]}{-\frac{K}{2}(464x^3-108x^2+42x+1)(1+x)^3+3A(30x^2+23x+5)-\frac{3}{2}(x+5)(1+4x)^{\frac{5}{2}}} \tag{22}$$

$(\frac{1}{c^2}\frac{dp}{d\rho})_{r=0}\le 1$, for all values of K and A satisfied by (16).

Using (12) and (13)

$$\frac{1}{c^2}\frac{p}{\rho}=\frac{\alpha}{\beta} \tag{23}$$

where

$$\alpha=\frac{K}{6}\cdot\frac{(26x^2+19x+2)}{\sqrt{1+4x}}+\frac{A(1+7x)}{(1+x)^2\sqrt{1+4x}}+\frac{9}{2}\cdot\frac{(1-x)}{(1+x)^2}$$

$$\beta = -\frac{K}{2}\cdot\frac{(32x^3+34x^2+13x+2)}{(1+4x)^{\frac{3}{2}}} - \frac{3A(1+3x)}{(1+x)^2(1+4x)^{\frac{3}{2}}} + \frac{3(x+3)}{2(1+x)^2}$$

Differentiating (22) w.r.t. x

$$\frac{1}{c^2}\frac{d}{dx}\left(\frac{p}{\rho}\right) = \frac{\beta\frac{d\alpha}{dx} - \alpha\frac{d\beta}{dx}}{\beta^2} \tag{24}$$

$$\frac{1}{c^2}\frac{d}{dx}\left(\frac{p}{\rho}\right)_{x=0} = \frac{(-\frac{7}{3}K^2 - 15AK + 20K - 6A - 24A^2 - 27)}{(-K - 3A + \frac{9}{2})^2} \tag{25}$$

The expression of right hand side of (25) is negative, thus the pressure-density ratio $\frac{p}{\rho c^2}$ is maximum at the centre and monotonically decreasing for all values of K and A satisfied by (16).

Differentiating (22) w.r.t. x, we get,

$$\frac{1}{c^2}\left(\frac{d}{dx}\left(\frac{dp}{d\rho}\right)\right) = \frac{d}{dx}\left(\frac{\psi}{\xi}\right) = \frac{\xi\frac{d\psi}{dx} - \psi\frac{d\xi}{dx}}{\xi^2} \tag{26}$$

$$\psi = (1+4x)\left[\frac{K}{2}(52x^2+30x+5)(1+x)^3 - 3A(14x^2+x-1) + \frac{9}{2}(x-3)(1+4x)^{\frac{3}{2}}\right] \tag{26a}$$

$$\xi = -\frac{K}{2}(464x^3 - 108x^2 + 42x + 1)(1+x)^3 + 3A(30x^2 + 23x + 5) - \frac{3}{2}(x+5)(1+4x)^{\frac{5}{2}} \tag{26b}$$

$$\frac{1}{c^2}\left(\frac{d}{dx}\left(\frac{dp}{d\rho}\right)\right)_{x=0} = \frac{40K^2 + 378AK - 291K - 864A - 72A^2 - 54}{(-\frac{K}{2} + 15A - \frac{15}{2})^2} \tag{27}$$

The expression of right hand side of (27) is to be negative, thus the square of adiabatic speed of sound $\frac{dp}{d\rho}$ is maximum at the centre and monotonically decreasing.

The expression for gravitational red-shift (z) is given by

$$z = \frac{(1+x)^{-3/2}}{\sqrt{B}} - 1 \tag{28}$$

The central value of gravitational red shift to be non zero positive finite, we have

$$1 > \sqrt{B} > 0 \tag{28a}$$

Differentiating (28) w.r.t. x, we get,

$$\left[\frac{dz}{dx}\right]_{x=0} = \frac{-3}{2\sqrt{B}} < 0 \tag{28b}$$

The expression of right hand side of (28b) is negative, thus the gravitational red-shift is maximum at the centre and monotonically decreasing.

Differentiating (10) w.r.t. x, we get,

$$\frac{d}{dx}\left(\frac{E^2}{c_1}\right) = \frac{K}{2}\left[\frac{(1+6x)}{\sqrt{1+4x}}\right] \tag{29}$$

$$\left[\frac{d}{dx}\left(\frac{E^2}{c_1}\right)\right]_{x=0} = \frac{K}{2} \quad (+ve) \tag{29a}$$

The expression of right hand side of (29a) is positive, thus the electric intensity is minimum at the centre and monotonically increasing for all values of $K > 0$. Also at the centre it is zero.

5 Boundary conditions

The solutions so obtained are to be matched over the boundary with Reissner-Nordstrom metric:

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{2GM}{r} + \frac{e^2}{r^2}\right)dt^2 \tag{30}$$

which requires the continuity of e^λ, e^ν and q across the boundary $r = r_b$

$$e^{-\lambda(r_b)} = 1 - \frac{2GM}{c^2 r_b} + \frac{e^2}{r_b^2} \tag{31}$$

$$e^{\nu(r_b)} = 1 - \frac{2GM}{c^2 r_b} + \frac{e^2}{r_b^2} \tag{32}$$

$$q(r_b) = e \tag{33}$$

$$p(r_b) = 0 \tag{34}$$

The condition (34) can be utilized to compute the values of arbitrary constants A as follows:

On setting $x_{r=r_b} = X = c_1 r_b^2$ (r_b being the radius of the charged sphere)

Pressure at $p_{(r=r_b)} = 0$ gives

$$A = -\frac{K}{6}\frac{(1+X)^2(126X^2+19X+2)}{(1+7X)} - \frac{9}{2}\cdot\frac{(1-X)\sqrt{1+4X}}{(1+7X)} \tag{35}$$

The expression for mass can be written as

$$\frac{GM}{c^2} = \frac{r_b}{2}\left[1 + KX^2\frac{(1+4X)^{\frac{3}{2}}}{(1+7X)} - \frac{(1+X)}{(1+7X)}\right] \tag{36}$$

Table 1 The variation of various physical parameters at the centre, surface density, electric field intensity on the boundary, mass and linear dimension of stars with different values of K and X

K	$X=c_1r_b^2$	$\frac{1}{c_1}\frac{8\pi G}{c^4}p_0$	$\frac{1}{c_1}\frac{8\pi G}{c^2}\rho_0$	$\frac{1}{c^2}\frac{p_0}{\rho_0}$	$\frac{1}{c^2}(\frac{dp}{d\rho})_{x=0}$	z_0	$(\frac{E^2}{c_1})_{r_b}$	$\frac{8\pi G}{c^2}\rho_b r_b^2$	$\frac{M}{M_\Theta}$	$\approx 2r_b$ in km
1.25	0.395	0.6747	18.655	0.0329	0.3204	1.4324	0.396	0.6402	3.52	24.62
1.25	**0.42**	**0.3983**	**19.304**	**0.02065**	**0.3152**	**1.5316**	**0.429**	**0.5503**	**3.64**	**24.31**
1.25	0.46	0.01037	20.468	0.005563	0.3070	1.6965	0.484	0.3848	3.28	20.33
4	0.1	0.566	24.30	0.0232	0.2125	0.3824	0.236	0.53046	1.58	23.87
4	0.15	0.168	25.49	0.00659	0.2169	0.5748	0.374	0.4832	2.03	22.78
4	0.164	0.0031	25.99	0.00012	0.2165	0.6344	0.422	0.4497	2.10	21.98
15	0.005	0.012	47.96	0.000252	0.02837	0.0199	0.037	0.0199	0.02	4.62
15	**0.01**	**0.0043**	**47.98**	**0.000089**	**0.02850**	**0.0399**	**0.076**	**0.0399**	**0.06**	**6.54**

Table 2 The Derivatives of various physical parameters at the centre with different values of K and X

K	$X=c_1r_b^2$	$(\frac{1}{c_1}\cdot\frac{8\pi G}{c^4}\frac{dp}{dx})_{x=0}$	$(\frac{1}{c_1}\frac{8\pi G}{c^2}\cdot\frac{d\rho}{dx})_{x=0}$	$\frac{1}{c^2}\frac{d}{dx}(\frac{p}{\rho})_{x=0}$	$\frac{1}{c^2}(\frac{d}{dx}(\frac{dp}{d\rho}))_{x=0}$	$[\frac{dz}{dx}]_{x=0}$
1.25	0.395	−23.28	−72.654	−1.3154	−0.00067	−3.64858
1.25	0.42	−23.93	−75.899	−1.3589	−0.00976	−3.79754
1.25	**0.46**	**−25.0939**	**−81.719**	**−1.43**	**−0.0251**	**−4.04476**
4	0.1	−19.3	−88.50	−1.13	−0.7644	−2.07
4	0.15	−20.49	−94.47	−1.24	−0.7349	−2.36
4	**0.164**	**−20.99**	**−96.95**	**−1.28**	**−0.7237**	**−2.44**
15	0.005	−4.4638	−151.31	−0.6842	−1.918	−1.52
15	**0.010**	**−4.4871**	**−157.43**	**−0.6872**	**−1.919**	**−1.55**

In view of (31) and (32) we get,

$$B=-\frac{K}{2}\frac{X^2\sqrt{1+4X}}{(1+X)^2(1+7X)}+\frac{1}{(1+X)^2(1+7X)} \tag{37}$$

The surface density is given by

$$\frac{8\pi G}{c^2}\rho_b r_b^2=\frac{X}{2(1+X)^2(1+4X)^{\frac{3}{2}}}\times\Big[-K\cdot(32X^3+34X^2+13X+2)(1+X)^2 - 6A(1+3X)+3(X+3)(1+4X)^{\frac{3}{2}}\Big] \tag{38}$$

Centre red shift is given by

$$z_0=B^{-1/2}-1 \tag{39}$$

6 Discussion

In view of and Table 1, it has been observed that all the physical parameters (p, ρ, $\frac{p}{\rho c^2}$, $\frac{dp}{d\rho}$ and z) are positive at the centre and within the limit of realistic equation of state. From Table 2, the first derivative of all the parameters is negative at the centre. Thus by virtue of the theorem all the parameters are monotonically decreasing in nature for ($1.25 \le K \le 15$). In view of (29a), the electric intensity is minimum at the centre and monotonically increasing. for all values of $K>0$. Therefore, the solution is well behaved for ($1.25 \le K \le 15$), however, corresponding to any value of K satisfying the inequalities $0 \le K < 1.25$, the nature of adiabatic sound speed is non decreasing in nature.

Further, corresponding to any value of $K>15$, there exists no value of X for which centre pressure is positive.

For this solution the mass of a star is maximized with all degree of suitability and by assuming the surface density $\rho_b = 2\times 10^{14}$ g/cm^3. Corresponding to $K=1.25$ and $X=0.42$ the maximum mass of the star comes out to be $3.64M_\Theta$ with linear dimension 24.31 km and central red shift 1.5316. It has been thus observed that for higher values of K approaching to 15, though the solution is well behaved but the mass of the star is very less than the Chandrasekhar limit.

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