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ORIGINAL ARTICLE

Agegraphic reconstruction of modified $F(R)$ and $F(\mathcal{G})$ gravities

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Abstract The cosmological reconstruction of modified $F(R)$ and $F(\mathcal{G})$ gravities with agegraphic dark energy (ADE) model in a spatially flat universe without matter field is investigated by using e-folding "N" as a forward way. After calculating a consistent $F(R)$ in ADE's framework, we obtain conditions for effective equation of state parameter w_{eff}, and see that reconstruction is possible for both phantom and non-phantom era. These calculations also are done for $F(\mathcal{G})$ gravity and the condition for a consistent reconstruction is obtained.

Keywords Modified gravity · Agegraphic dark energy model · Gauss-Bonnet gravity · Reconstruction

1 Introduction

Dark energy problem attracted a great deal of attention at the last decade. Recent astrophysical data suggest that our universe behave under an accelerated expansion with an effective equation of state parameter $-1.48 < w_{\text{eff}} < -0.72$ (Hannestad and Mortsell 2002; Melchiori et al. 2003; Jassal et al. 2005). Up to now, scientists have proposed two prescriptions for this expansion. One group believe that a component of dark energy (DE), which possesses negative pressure, is the source of this expansion. It is understood that about 70 percent of energy content of the current universe is dark energy. Several models of dynamical dark energy, after Λ-CDM model, whose equations of states are no longer a constant but evolve with time, have been proposed by this group. On the other hand, a curvature driven acceleration model which is called, modified gravity, has been proposed by Starobinsky (1980) and Kerner (1982), Duruisseau and Kerner (1986) et al., for the first time, in 1980. Modified gravity approach suggests the gravitational alternative for unified description of inflation, dark energy and dark matter with no need of the hand insertion of extra dark components. It has been shown that these two approaches may related to each other. Many authors have extended a reconstruction technique in order to made a correspondence between an acceptable cosmological model, and a modified gravity (Nojiri and Odintsov 2006a, 2007a, 2007b, 2007c; Capozziello et al. 2006; Elizalde and Saez-Gomez 2009; De la Cruz-Dombriz and Dobado 2006; Cortes and Indurain 2009; Brevik 2006; Granda 2008; Wu and Zhu 2008; Bamba et al. 2008, 2009; Bamba and Geng 2009). Also a reconstruction scheme has been developed in terms of e-folding (or redshift z), and some generalization of such technique for viable $F(R)$ gravity has been done, so that local tests were usually satisfied (Nojiri et al. 2009). By using this technique, some of works have been presented where $F(R)$ and $F(\mathcal{G})$ gravities are reconstructed so that they give the well-known cosmological evolution. The ΛCDM epoch, deceleration/acceleration epoch which is equivalent to presence of phantom and non-phantom matter, late-time acceleration with the crossing of phantom-divide line (Nojiri et al. 2009; Elizalde et al. 2010) and the holographic dark energy model (Setare 2008) are some of examples that have been presented in the recent years. The agegraphic dark energy (ADE) model is one of the interested model, which has been welcomed by many authors. The cosmological behavior, statefinder analysis (Khodam-Mohammadi and Malekjani 2010; Malekjani and Khodam-Mohammadi 2010a) and other cosmological aspects of ADE model

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have been calculated in an interacting/non-interacting spatially flat/non-flat, ordinary/entropy-corrected versions of Friedman-Robertson-Walker (FRW) universe (Wei and Cai 2007, 2008; Kim et al. 2008; Sheykhi 2009a, 2009b, 2010; Karami et al. 2010; Karami and Abdolmaleki 2010; Karami and Sorouri 2010; Malekjani and Khodam-Mohammadi 2010b). ADE model is arisen from combining quantum mechanics with general relativity, directly. This model, proposed by Cai (2007), is based on the line of quantum fluctuations of spacetime, the so-called Károlyházy relation $\delta t=\lambda t_p^{2/3}t^{1/3}$, and the energy-time Heisenberg uncertainty relation $E_{\delta t^3}\sim t^{-1}$. Throughout this paper, we use the Planck unit ($\hbar=c=k_B=1$), where $t_p=l_p=1/m_p$ are Plank's time, length and mass, respectively. These relations enable one to obtain an energy density of the metric quantum fluctuations of Minkowski spacetime as follows (Maziashvili 2007a, 2007b)

$$\rho_q\sim\frac{E_{\delta t^3}}{\delta t^3}\sim\frac{1}{t_p^2t^2}\sim\frac{m_p^2}{t^2}. \tag{1}$$

In ADE, this energy density is considered as density of dark energy component, ρ_d, of spacetime. By considering a FRW universe, due to effect of curvature, one should introduce a numerical factor $3n^2$ in (1) (Cai 2007; Wei and Cai 2005).

In this paper we want to reconstruct a consistent modified gravity so that it gives the cosmological evolution of ADE model. Specially we consider $F(R)$ and modified Gauss-Bonnet (GB) $F(\mathcal{G})$ gravities.

2 The formalism of modified gravity

The action of general modified gravity is

$$S=\int d^4x\sqrt{-g}\left(\frac{R+F(R,\mathcal{G},\Box R,\Box^{-1}R,\ldots)}{2\kappa^2}+\mathcal{L}_{\rm m}\right), \tag{2}$$

where $\kappa^2=8\pi G$, $\mathcal{L}_{\rm m}$ is the matter Lagrangian density and the function $F(R,\mathcal{G},\ldots)$ may contain scalar curvature R, GB term $\mathcal{G}=R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta}-4R_{\mu\nu}R^{\mu\nu}+R^2$ and any contributions of $\Box R$. At follows, we focus our attention only on $F(R)$ and $F(\mathcal{G})$. By varying the action over $g_{\mu\nu}$, the field equations can be obtained (Nojiri and Odintsov 2007a). The field equations corresponding to FRW equations in a spatially flat universe with $R=6\dot{H}+12H^2$, in $F(R)$ gravity is (Nojiri and Odintsov 2009):

$$\begin{aligned}\rho_{\rm eff}=\frac{1}{\kappa^2}\Big[&-\frac{F(R)}{2}+3(H^2+\dot{H})F'(R)\\&-18(4H^2\dot{H}+H\ddot{H})F''(R)\Big]+\rho_{\rm matter},\end{aligned}$$

$$\begin{aligned}p_{\rm eff}=\frac{1}{\kappa^2}\Big[&\frac{F(R)}{2}-(3H^2+\dot{H})F'(R)\\&+6(8H^2\dot{H}+6H\ddot{H}+4\dot{H}^2+\dddot{H})F''(R)\\&+36(4H\dot{H}+\ddot{H})^2F'''(R)\Big]+p_{\rm matter},\end{aligned} \tag{3}$$

and in GB modified gravity, $R+F(\mathcal{G})$, with $\mathcal{G}=24H^2\times(H^2+\dot{H})$, is

$$\begin{aligned}\rho_{\rm eff}=\frac{1}{2\kappa^2}[&-F(\mathcal{G})+\mathcal{G}F'(\mathcal{G})-(24)^2H^4\\&\times(4H^2\dot{H}+H\ddot{H}+2\dot{H}^2)F''(\mathcal{G})]+\rho_{\rm matter},\\
p_{\rm eff}=\frac{1}{2\kappa^2}[&F(\mathcal{G})+(24)^2H^2(3H^4+20H^2\dot{H}^2+6\dot{H}^3\\&+4H^3\ddot{H}+H^2\dddot{H})F''(\mathcal{G})-(24)^3H^5\\&\times(2\dot{H}^2+H\ddot{H}+4H^2\dot{H})^2F'''(\mathcal{G})]+p_{\rm matter}.\end{aligned} \tag{4}$$

Hear $\rho_{\rm eff}$ and $p_{\rm eff}$ are effective energy density and pressure caused by extra gravitational terms due to the modification of the GR Lagrangian. It has been showed that by getting the effective gravitational pressure and energy density, the equation of motion for arbitrary modified gravity can be rewritten in the standard form of FRW in GR as (Nojiri and Odintsov 2009)

$$H^2=\frac{\kappa^2}{3}\rho_{\rm eff},\qquad p_{\rm eff}=-\frac{1}{\kappa^2}(2\dot{H}+3H^2). \tag{5}$$

3 A brief review on ADE model

The metric of a general spatially flat FRW universe is given by

$$ds^2=-dt^2+a^2(t)[dr^2+r^2(d\theta^2+\sin^2\theta d\varphi^2)], \tag{6}$$

where $a(t)$ is the dimensionless scale factor. The energy density of ADE is given by Cai (2007)

$$\rho_D=\frac{3n^2}{\kappa^2T^2}, \tag{7}$$

where n is ADE constant parameter and T is the age of the universe which is given by

$$T=\int_0^t dt=\int_0^a\frac{da}{Ha}, \tag{8}$$

where $\dot{T}=1$. The dimensionless dark energy density is defined as

$$\Omega_D=\frac{\rho_D}{\rho_{cr}}=\frac{n^2}{T^2H^2}, \tag{9}$$

where $\rho_{cr} = 3H^2/\kappa^2$ is critical energy density. Let us consider the dark energy dominated universe. In this case the dark energy evolves according to its conservation law

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = 0, \tag{10}$$

where $w_D = p_D/\rho_D$, is equation of state of ADE, which is (Cai 2007)

$$w_D = -1 + \frac{2\sqrt{\Omega_D}}{3n}; \qquad \Omega_D = 1. \tag{11}$$

4 Reconstruction of $F(R)$ gravity with ADE

Hear we use ADE model to reconstruct a consistent $F(R)$. The first FRW equation of a spatially flat universe containing an agegraphic dark energy without any matter component can be obtained as (Cai 2007)

$$H^2 = \frac{\kappa^2}{3}\rho_D = \frac{n^2}{T^2}. \tag{12}$$

By using a new variable $N = \ln\frac{a}{a_0}$, which is often called e-folding, instead of the cosmological time t, we have $\frac{d}{dt} = H\frac{d}{dN}$ and therefore $\frac{d^2}{dt^2} = H^2\frac{d^2}{dN^2} + H\frac{dH}{dN}\frac{d}{dN}$. In a dark energy dominated universe, without any matter component, (3) can be written as

$$\kappa^2\rho_{\text{eff}} = -\frac{F(R)}{2} + 3(H^2 + HH')F'(R) - 18(4H^3H' + H^2H'^2 + H^3H'')F''(R). \tag{13}$$

Here $H' \equiv dH/dN$ and $H'' \equiv d^2H/dN^2$. Using $G(N) = H^2$, the (13) may be written as:

$$\kappa^2\rho_{\text{eff}} = -\frac{F(R)}{2} + 3\left[G(N) + \frac{1}{2}G'(N)\right]F' - 9G(N)[4G'(N) + G''(N)]F''(R), \tag{14}$$

and the scalar of curvature become

$$R = 3G'(N) + 12G(N). \tag{15}$$

Note that from (15), N generally is a function of R. In order to reconstruct $F(R)$ gravity with ADE, by comparing (12) and (5), it is required to get $\rho_D = \rho_{\text{eff}}$. Also the age of the universe T is a function of N. Using

$$G(N) = \frac{n^2}{T^2(N)},$$
$$G'(N) = \frac{-2\sqrt{G(N)}}{T(N)} = \frac{-2n}{T^2(N)},$$
$$G''(N) = \frac{4}{T^2(N)}, \tag{16}$$

and (15), $T(N(R))$ can be calculated as a function of R as

$$T(N(R)) = \sqrt{\frac{6n(2n-1)}{R}}. \tag{17}$$

Now from (17) and (16), all functions G, G' and G'' may be rewritten as a function of R. Inserting those in (14), one can obtain

$$2R^2F''(R) + (n-1)RF'(R) - (2n-1)F(R) - nR = 0. \tag{18}$$

This differential equation should be solved to find a consistent modified gravity with ADE in flat space. Its solution is

$$F(R) = C_+R^{m_+} + C_-R^{m_-} - R, \tag{19}$$

where $m_\pm$ are

$$m_\pm = \frac{3 - n \pm \sqrt{n^2 + 10n + 1}}{4}, \tag{20}$$

and $C_\pm$ are any arbitrary constant which are given by initial conditions. In order to generating an accelerating expansion at the present universe, let us consider that $F(R)$ could be a small constant at present universe, which is,

$$F(R_0) = -3R_0, \qquad \lim_{R\to 0} F(R) = 0, \qquad F'(R_0) \sim 0. \tag{21}$$

Hear R_0 is current curvature $R_0 \sim (10^{-33}\ \text{eV})^2$ (Nojiri and Odintsov 2007d). Therefore constants $C_\pm$ are

$$C_\pm = \mp\frac{R_0(n - 5 \pm \sqrt{n^2 + 10n + 1})}{R_0^{m_\pm}\sqrt{n^2 + 10n + 1}}. \tag{22}$$

As we see from (20) and (22), by choosing $n^2 + 10n + 1 \geq 0$, a consistent $F(R)$ can be found. Therefore we can obtain two following conditions

$$n \geq -0.1 \quad \text{or} \quad n \leq -9.9. \tag{23}$$

By getting $w_{\text{eff}} = w_D$, and from (11) and (5), the effective EOS may be obtained as: $w_{\text{eff}} = -1 + 2/3n$. Hence, the constant ADE parameter as a function of w_{eff} can be written as: $n = 2/3(1 + w_{\text{eff}})$. Therefore the conditions (23) may be rewritten as

$$9w_{\text{eff}}^2 + 78w_{\text{eff}} + 73 \geq 0;$$
$$w_{\text{eff}} \geq -1.067 \quad \text{or} \quad w_{\text{eff}} \leq -7.6. \tag{24}$$

In this case a transition between deceleration ($w_{\text{eff}} > -1/3$)-acceleration ($w_{\text{eff}} < -1/3$) phase of the universe has been permitted and it is possible that the dark energy dominated universe may live at effective phantom era ($w_{\rm eff} < -1$; $n < 0$). As we see from (24), a quintessence era, where ($w_{\rm eff} > -1$; $n > 0$) can also be permitted in forward reconstruction method.

It is worthwhile to mention that the differential equation (18) can also be obtained from another way, followed by Setare (2008, 2010) and Karami and Khaledian (2010). By given a quintessence scale factor form as: $a = a_0 t^h$ with $h > 0$, or by properly shifting of time, Phantom scale factor form: $a = a_0(t_s - t)^h$ with $h < 0$, which tell us that there will be a Big Rip singularity at $t = t_s$ (Nojiri and Odintsov 2006b). Using latter form of scale factor, we can easily find

$$T(t) = (t_s - t), \qquad H(t) = \frac{-h}{(t_s - t)};$$
$$R(t) = \frac{12h^2 - 6h}{(t_s - t)^2}, \tag{25}$$

where h is an arbitrary negative constant. Using (12), we see that $h = n$. From (25), we have $(t_s - t)^2 = 6n(2n - 1)/R$ and $\rho_D = \rho_{\rm eff} = nR/(2\kappa^2(2n - 1))$ and finally (3) for $\rho_{\rm m} = 0$, is exactly similar to (18) which is obtained in the forward way. We must mention that by this method, the reconstruction is permitted only in phantom era ($h < 0$), or in quintessence era ($h > 0$), according to choose each form of mentioned scale factor, separately.

5 Reconstruction of $F(\mathcal{G})$ gravity with ADE

In $F(\mathcal{G})$ gravity, like $F(R)$ gravity, by using the variable N instead of the cosmological time t, (4), without any matter component, may be rewritten as

$$\kappa^2\rho_{\rm eff} = -\frac{F(\mathcal{G})}{2} + 12H^2(H^2 + HH')F'(\mathcal{G}) - (12)^2H^6(6H'^2 + 8HH' + 2HH'')F''(\mathcal{G}). \tag{26}$$

Using $G(N) = H^2$, the GB term is

$$\mathcal{G} = 12G(N)[2G(N) + G'(N)], \tag{27}$$

and the (26) may be written as

$$\kappa^2\rho_{\rm eff} = -\frac{F(\mathcal{G})}{2} + 6[2G^2(N) + G(N)G'(N)]F'(\mathcal{G}) - (12)^2G(N)^2[G'^2(N) + 4G(N)G'(N) + G(N)G''(N)]F''(\mathcal{G}). \tag{28}$$

Note that from (27), N generally is a function of $\mathcal{G}$. In order to reconstruct $F(\mathcal{G})$ gravity with ADE, it is required to get $\rho_D = \rho_{\rm eff}$. Also from (27) and (16), $T(N(\mathcal{G}))$ can be calculated as a function of $\mathcal{G}$ as

$$T(N(\mathcal{G})) = \left(\frac{24n^3(n-1)}{\mathcal{G}}\right)^{\frac{1}{4}}. \tag{29}$$

Now from (29) and (16), all functions G, G' and G'' can be calculated as a function of $\mathcal{G}$. Inserting those into (28), we obtain

$$8\mathcal{G}^2F''(\mathcal{G}) + 2(n-1)\mathcal{G}F'(\mathcal{G}) - 2(n-1)F(\mathcal{G}) - \sqrt{6n(n-1)\mathcal{G}} = 0. \tag{30}$$

Its solution can be obtained as

$$F(\mathcal{G}) = C_1\mathcal{G} + C_2\mathcal{G}^{(-\frac{n-1}{4})} - \frac{\sqrt{6n(n-1)}}{n+1}\sqrt{\mathcal{G}}, \tag{31}$$

and as a function of $w_{\rm eff}$, it is rewritten as

$$F(\mathcal{G}) = C_1\mathcal{G} + C_2\mathcal{G}^{(\frac{1}{12}\frac{1+3w_{\rm eff}}{1+w_{\rm eff}})} - \frac{\sqrt{-12(1+3w_{\rm eff})}}{5+3w_{\rm eff}}\sqrt{\mathcal{G}}. \tag{32}$$

We see that a consistent $R + F(\mathcal{G})$ gravity may be existed, provided that $w_{\rm eff} \le -1/3$.

6 Conclusion

In this paper we show that a consistent modified $F(R)$ and $F(\mathcal{G})$ gravities may be reconstructed forwardly so that it gives the cosmological evolution of ADE model in a no matter spatially flat universe with no need of the hand insertion of extra dark components. After calculating a consistent $F(R)$ with ADE, we obtain conditions for $w_{\rm eff}$ and see that reconstruction is possible for both phantom and non-phantom era. These calculations have also been done for $F(\mathcal{G})$ gravity and the condition for a consistent $F(\mathcal{G})$ is obtained. Although it is possible that dark energy dominated universe live (or enter) at effective phantom era like non-phantom era, deceleration phase of the universe ($w_{\rm eff} > -1/3$) is not achieved in this case.

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