Astrophys Space Sci (2011) 331: 657–663
DOI 10.1007/s10509-010-0461-4

ORIGINAL ARTICLE

# Role of deceleration parameter and interacting dark energy in singularity avoidance

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Received: 23 June 2010 / Accepted: 29 July 2010 / Published online: 13 August 2010


**Abstract** A class of non-singular bouncing FRW models are obtained by constraining the deceleration parameter in the presence of an interacting dark energy represented by a time-varying cosmological constant. The models being geometrically closed, initially accelerate for a certain period of time and decelerate thereafter and are also free from the entropy and cosmological constant problems. Taking a constant of integration equal to zero one particular model is discussed in some detail and the variation of different cosmological parameters are shown graphically for specific values of the parameters of the model. For some specific choice of the parameters of the model the ever expanding models of Ozer & Taha and Abdel-Rahman and the decelerating models of Berman and also the Einstein de-Sitter model may be obtained as special cases of this particular model.

**Keywords** Time-dependent cosmological constant · Vacuum energy · Deceleration parameter · Singularity avoidance

## 1 Introduction

One of the most intriguing issues in standard cosmology concerns with the initial singularity or the big bang, where the theory breaks down. If one considers the Robertson-Walker metric

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (1)$$

representing the homogeneous and isotropic universe (at sufficiently large scales) in which the distribution of matter is represented by the energy-momentum tensor of a perfect fluid

$$T_{ij}^{\mathrm{M}} = (\rho + p)U_i U_j + p g_{ij}, \quad (2)$$

where $\rho$ is the energy density of the cosmic matter and $p$ is its pressure, then the Einstein field equations

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} = -8\pi G T_{ij}^{\mathrm{M}}, \quad (3)$$

yield the following two independent equations

$$8\pi G\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \quad (4)$$

$$8\pi G p = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}. \quad (5)$$

These are two equations connecting three unknown functions $R$, $\rho$ and $p$. However the system becomes closed once we assume the usual barotropic equation of state $p(\rho) = w\rho$, $w =$ constant which provides information about the source matter. It is then easy to see that for normal matter with $p > 0$, $\rho > 0$, (4) and (5) require $R$ to vanish at some finite time in the past regardless of the geometry of the universe. At this point, the space-time becomes singular and all the physical variables blow up.

As the quantum gravitational effects are expected to come into play near the singular point, the issue of singularity should be addressed in a theory of quantum gravity. As a

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full theory of quantum gravity is not available at the present stage, we try to solve this problem at the classical level in the framework of general relativity. The solution is provided in terms of bouncing solutions (which are not new to cosmology anyway), the bounce occurring at some finite (classical) value of the scale factor which may escape any quantum contributions. Our aim is to obtain these non-singular solutions by constraining the deceleration parameter which regulates the dynamics of the universe. As we have already mentioned earlier, the Einstein equations taken together with the equation of state of matter, provide a closed system. Thus any extra condition (imposed on the deceleration parameter) makes the system over-deterministic. We plan to compensate this over-determinacy by inserting into the equations another entity, the famous *dark energy*. It is now well established that Einstein's theory requires dark energy to explain the faintness of the supernovae of type Ia (Perlmutter et al. 1999; Reiss et al. 1998; Kowalski et al. 2008; Amanullah et al. 2010). Dark energy is also supported by other observations, for example, the anisotropy measurements of the cosmic microwave background radiation (Spergel et al. 2003, 2007) and the observations of the baryon acoustic oscillations (Wang and Mukherjee 2006; Bond et al. 1997).

Dark energy can be represented by a large-scale scalar field $\phi$ dominated either by potential energy or nearly constant potential energy (hence violating positive energy constraints). Such a *matter* will also have its energy-stress tensor in form $T_{ij}^{\mathrm{DE}} = (\rho_\phi + p_\phi)U_i U_j + p_\phi g_{ij}$ and its equation of state in the form $p_\phi = w_\phi \rho_\phi$, where $w_\phi$ is a function of time in general. Obviously, this provides a number of candidates for dark energy depending upon the dynamics of the field $\phi$ and its potential energy. The simplest and the most favoured one by the cosmological observations is Einstein's cosmological constant $\Lambda$ for which $w_\phi$ reduces to the value-1 (potential energy dominated scalar field). Dark energy can be introduced in Einstein's theory by replacing $T_{ij}^{\mathrm{M}}$ by $T_{ij}^{\mathrm{total}}$ in (3), where $T_{ij}^{\mathrm{total}} = T_{ij}^{\mathrm{M}} + T_{ij}^{\mathrm{DE}} = (\rho_t + p_t)U_i U_j + p_t g_{ij}$ with the understanding that $\rho_t = \rho + \rho_\phi$ and $p_t = p + p_\phi$. In this case, (4) and (5) are modified as

$$8\pi G\rho_t = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \tag{6}$$

$$8\pi G p_t = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}. \tag{7}$$

The Bianchi identities now require that $T_{ij}^{\mathrm{total}}$ has a vanishing divergence. We do not make any additional assumption of minimal coupling (no interaction) between matter and dark energy (which anyway seems ad-hoc and nothing more than a simplifying assumption) and believe that interaction is more natural and is a fundamental principle (Vishwakarma and Narlikar 2007). This leads to

$$\frac{d}{dt}(\rho_t R^3) + p_t \frac{dR^3}{dt} = 0. \tag{8}$$

As we have mentioned earlier, the most favoured candidate of dark energy, from the different cosmological observations, is the cosmological constant $\Lambda$. Hence we limit ourselves in the following to this case only. We know that $\Lambda$ can be represented as the intrinsic energy density of vacuum $\rho_v = \Lambda/8\pi G$ arising from the zero point energy of quantum fluctuations. This however brings about the widely discussed cosmological constant problem that the upper limit of $\rho_v$ from observations is some 120 orders of magnitude below its value predicted by quantum field theory. This problem is alleviated if we have a dynamically decaying $\rho_v$. Due to its coupling with the other matter fields of the universe, a decaying $\rho_v$ (with a large value in the early universe) can relax to its small observed value in course of the expansion of the universe by creating massive or massless particles (Abdussattar and Vishwakarma 1996). We will see in the following that we do have a dynamically decaying $\rho_v$. For the case $\rho_\phi \to \rho_v$, the conservation equation (8) reduces to

$$\frac{d}{dt}(\rho R^3) + p\frac{dR^3}{dt} + R^3\frac{d\rho_v}{dt} = 0, \tag{9}$$

giving the change in the entropy of matter content in the universe

$$T dS \equiv d(\rho R^3) + p dR^3 = -R^3 d\rho_v, \tag{10}$$

which is always increasing for a decaying vacuum energy. Thus, for $\dot{\rho}_v < 0$, there would be a continuous creation of matter out of the vacuum energy.

## 2 Dynamics of the Universe from the deceleration parameter

The two observable parameters $H$ (Hubble parameter) and $q$ (deceleration parameter) defined respectively as $H = \dot{R}/R$ and $q = -R\ddot{R}/\dot{R}^2$ are related by the relation

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{11}$$

This equation can be integrated to give the scale factor $R(t)$ as

$$R(t) = e^\delta \exp\left\{\int \frac{dt}{\int(1+q)dt + \gamma}\right\}, \tag{12}$$

where $\gamma$ and $\delta$ are arbitrary constants of integration. For a possible integration of (12), we observe that (i) $q =$ constant is an easy choice which provides $R(t)$ as an explicit function of $t$, as has also been discussed by Berman (1991). (ii) When $q$ is taken to vary with time, an explicit determination of $R(t)$ leads to a possible choice of $q$ as

$$q=-\frac{\alpha}{t^2}+(\beta-1), \tag{13}$$

which is the main ansatz of the paper. Here $\alpha>0$ is a parameter having the dimension of square of time and $\beta>1$ is a dimensionless constant. Obviously, the different values of $\alpha$ and $\beta$ will give rise to different models. With $q$ given by (13), (12) can be integrated to give the time variation of the scale factor as

$$R(t)=e^{\delta}\exp\left\{\frac{1}{\beta}\int\frac{t}{t^2+\frac{\gamma}{\beta}t+\frac{\alpha}{\beta}}dt\right\}. \tag{14}$$

The integral appearing in (14) can be integrated in the following three different cases: $\gamma\neq 2\sqrt{\alpha\beta}$, $\gamma=2\sqrt{\alpha\beta}$ and $\gamma=0$ giving the following three different forms of the scale factor as

$$R(t)=e^{\delta}\left(t^2+\frac{\gamma}{\beta}t+\frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}} \times\left(\frac{t+\frac{\gamma-\sqrt{\gamma^2-4\alpha\beta}}{2\beta}}{t+\frac{\gamma+\sqrt{\gamma^2-4\alpha\beta}}{2\beta}}\right)^{-\frac{\gamma}{2\beta\sqrt{\gamma^2-4\alpha\beta}}}, \quad \gamma\neq 2\sqrt{\alpha\beta}, \tag{15}$$

$$R(t)=e^{\delta}\left(t+\sqrt{\frac{\alpha}{\beta}}\right)^{\frac{1}{\beta}}\exp\left\{\frac{\sqrt{\alpha}}{\beta^{3/2}}\left(\frac{1}{t+\sqrt{\frac{\alpha}{\beta}}}\right)\right\}, \quad \gamma=2\sqrt{\alpha\beta}, \tag{16}$$

$$R(t)=e^{\delta}\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}}, \quad \gamma=0. \tag{17}$$

We set the origin of the time coordinate (for the purpose of reference) at the bounce of these bouncing models and concentrate only on the expanding part of the solutions which represent the observable universe. It is then easy to see from (15), (16) and (17) that at $t=0$, $R=R_0$ (say) $\neq 0$ but $\dot{R}=0$ and $\ddot{R}=R_0/\alpha=$ constant. Here and henceforth the suffix 'zero' indicates the value of the parameter at $t=0$. This shows that in all the three cases the models are free from initial singularity and start with a finite acceleration. One can choose the parameters suitably to give a sufficiently high initial value $\ddot{R}_0$ so that the majority of the thermal history of the model is akin to the successful thermal history of the standard cosmology (except in the close vicinity of the singularity). Equation (13) suggests that the deceleration parameter $q\to-\infty$ at $t=0$ and reduces to zero at time $t=\sqrt{\alpha/(\beta-1)}$. Obviously, the period of accelerated expansion also depends on the values of $\alpha$ and $\beta$. Afterwards, the model decelerates with deceleration parameter $q$ approaching to $\beta-1$ for sufficiently large values of $t$. This puts a restriction on $\beta$ as $1\leq\beta\leq 2$. If we take $\beta=1$ the universe has an accelerated expansion throughout the evolution as is clear from (13). The time evolution of Hubble parameter for these models is obtained from (14) as

$$H(t)=\frac{1}{\beta}\left\{\frac{t}{t^2+\frac{\gamma}{\beta}t+\frac{\alpha}{\beta}}\right\}. \tag{18}$$

We find that at $t=0$, $H=0$ and consequently from (6)

$$\rho_{t0}=\frac{3k}{8\pi G}\frac{1}{R_0^2}. \tag{19}$$

Equation (19) suggests the value of the curvature parameter

$$k=1, \tag{20}$$

and shows that initially the whole vacuum and the radiation energies are locked up in potential form in the curvature of the space-time.

It is clear from (15), (16) and (17) that although $k=1$ the models are continuously expanding as one can see that $R\to\infty$ as $t\to\infty$. However their subsequent evolution heavily depends on the choice of the parameters $\alpha$, $\beta$, $\gamma$ and $\delta$.

In the next section we consider the case $\gamma=0$.

## 3 Models with $\gamma=0$

From (17) we get $e^{\delta}=R_0(\frac{\beta}{\alpha})^{\frac{1}{2\beta}}$ and consequently

$$R(t)=R_0\left(\frac{\beta}{\alpha}\right)^{\frac{1}{2\beta}}\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}}. \tag{21}$$

If we take $\gamma=0$, $\beta=1$, $e^{2\delta}=2m-1$ and $\alpha=\frac{R_0^2}{2m-1}$, $\frac{1}{2}<m\leq 1$ we get

$$R^2=(2m-1)t^2+R_0^2, \tag{22}$$

which is the nonsingular model obtained by Abdel-Rahman (1992) with a negative deceleration parameter $q=-\frac{R_0^2}{2m-1}\times t^{-2}$ throughout the evolution. The model of Ozer and Taha (1987) is a special case of this model for $m=1$. If the parameters are taken as $\gamma=0$, $\alpha=0$ and $e^{\delta}=D^{\frac{1}{\beta}}$, then we get

$$R=(Dt)^{\frac{1}{\beta}}, \tag{23}$$

which is the singular model obtained by Berman (1991) with a constant deceleration parameter $q=\beta-1$ throughout the evolution.

Equations (6), (7), (20) and (21) may be used to obtain

$$8\pi G\rho_t = -\frac{3}{\beta^2}\frac{\alpha}{\beta}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} + \frac{3}{\beta^2}\frac{1}{(t^2+\frac{\alpha}{\beta})} + \frac{3}{R_0^2}\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}}, \tag{24}$$

$$8\pi G p_t = \left(\frac{3}{\beta^2}-\frac{4}{\beta}\right)\frac{\alpha}{\beta}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} + \left(-\frac{3}{\beta^2}+\frac{2}{\beta}\right)\frac{1}{(t^2+\frac{\alpha}{\beta})} - \frac{1}{R_0^2}\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}}, \tag{25}$$

yielding

$$\rho_t + p_t = \frac{2\beta}{3}\left(\frac{3H^2}{8\pi G}\right) - \frac{1}{8\pi G}\frac{2\alpha}{\beta^2}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} + \frac{1}{8\pi G}\frac{2}{R_0^2}\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}}. \tag{26}$$

This shows that for $\beta = \frac{3}{2}$, the model would indicate $\rho = \rho_c$ at sufficiently large times ($p = 0$).

The total active gravitational mass

$$(\rho_t + 3p_t)R^3 = \frac{6}{8\pi G}\frac{1}{\beta^2}R_0^3\left(\frac{\beta}{\alpha}\right)^{\frac{3}{2\beta}}\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{3-4\beta}{2\beta}} \times\left[(\beta-1)\,t^2-\alpha\right], \tag{27}$$

which is $-ve$, $zero$ or $+ve$ according as $t \lesseqgtr \sqrt{\frac{\alpha}{(\beta-1)}}$. At $t = \sqrt{\frac{\alpha}{(\beta-1)}}$ we have $\rho_r = \rho_v$. There is an obvious phase transition at $t = \sqrt{\frac{\alpha}{(\beta-1)}}$ from vacuum dominated era to matter dominated era.

In the early pure radiation era the equation of state is assumed to be $p = p_r = \frac{1}{3}\rho_r$ , then (24) and (25) yield

$$\rho_r = \frac{3}{16\pi G}\left[\begin{array}{l} -\frac{2\alpha}{\beta^2}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} + \frac{1}{\beta}\frac{1}{(t^2+\frac{\alpha}{\beta})} \\ +\frac{1}{R_0^2}(\frac{\alpha}{\beta})^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}} \end{array}\right], \tag{28}$$

$$\rho_v = \frac{3}{16\pi G}\left[\begin{array}{l} \frac{(\beta-1)}{\beta^2}\frac{2\alpha}{\beta}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} - (\frac{\beta-2}{\beta^2})\frac{1}{(t^2+\frac{\alpha}{\beta})} \\ +\frac{1}{R_0^2}(\frac{\alpha}{\beta})^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}} \end{array}\right]. \tag{29}$$

If we take $\alpha = 0$ and $\beta = 2$, we get back the early radiation dominated era of the standard models with $\rho_v = 0$, $\rho_r = \frac{1}{2t^2}$ and $R \propto \sqrt{t}$.

From (28) and (29) it is easy to see that at $t = 0$

$$\rho_{r0} = \frac{3}{16\pi G}\left[\frac{1}{R_0^2} - \frac{1}{\alpha}\right], \tag{30}$$

$$\rho_{v0} = \frac{3}{16\pi G}\left[\frac{1}{R_0^2} + \frac{1}{\alpha}\right]. \tag{31}$$

Equation (30) suggests that $\alpha \geq R_0^2$. For the parameter, $\alpha = R_0^2$ we get $\rho_{r0} = 0$ and $\rho_{r0} > 0$ for $\alpha > R_0^2$.

The differentiation of (28) and (29) with respect to the cosmic time '$t$' yield

$$\dot{\rho}_r = \frac{3}{16\pi G}\left[\begin{array}{l} \frac{8\alpha}{\beta^2}\frac{t}{(t^2+\frac{\alpha}{\beta})^3} - \frac{2}{\beta}\frac{t}{(t^2+\frac{\alpha}{\beta})^2} \\ -\frac{2}{\beta}\frac{1}{R_0^2}(\frac{\alpha}{\beta})^{\frac{1}{\beta}}\frac{t}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}+1}} \end{array}\right], \tag{32}$$

$$\dot{\rho}_v = \frac{3}{16\pi G}\left[\begin{array}{l} -\frac{(\beta-1)}{\beta^2}\frac{8\alpha}{\beta}\frac{t}{(t^2+\frac{\alpha}{\beta})^3} + 2(\frac{\beta-2}{\beta^2})\frac{t}{(t^2+\frac{\alpha}{\beta})^2} \\ -\frac{2}{\beta}\frac{1}{R_0^2}(\frac{\alpha}{\beta})^{\frac{1}{\beta}}\frac{t}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}+1}} \end{array}\right]. \tag{33}$$

From (32) and (33), it follows that $\dot{\rho}_r = \dot{\rho}_v = 0$ at $t = 0$. The second derivatives of $\rho_r$ and $\rho_v$ at $t = 0$ are obtained as

$$\ddot{\rho}_{r0} = \frac{3}{8\pi G}\frac{1}{\alpha R_0^2}\left[\frac{3\beta R_0^2}{\alpha} - 1\right], \tag{34}$$

$$\ddot{\rho}_{v0} = \frac{3}{8\pi G}\frac{1}{\alpha R_0^2}\left[\frac{(2-3\beta)\,R_0^2}{\alpha} - 1\right]. \tag{35}$$

For $\alpha \geq R_0^2$ the second derivative of $\rho_v$ is always negative implying that $\rho_v$ is maximum at $t = 0$. Moreover, as $\beta < 2$, $\dot{\rho}_v$ is always negative implying that vacuum is continuously decaying and creating radiation throughout the evolution. This further requires $\ddot{\rho}_{r0}$ to be positive which yields $\alpha < 3\beta R_0^2$. Thus, the parameter $\alpha$ is confined to $R_0^2 \leq \alpha < 3\beta R_0^2$.

The radiation temperature $(T)$ is assumed to be related to radiation energy density by the relation

$$\rho_r = \frac{\pi^2}{30}N(T)T^4, \tag{36}$$

in units with $\hbar = c = k_B = 1$. The effective no of spin degrees of freedom $N(T)$ at temperature $T$ is given by $N(T) = N_b(T) + \frac{7}{8}N_f(T)$, where $N_b(T)$ and $N_f(T)$ correspond to bosons and fermions respectively. We assume $N(T)$ to be constant throughout this era.

From (28) and (36) one obtains

$$T = \left(\frac{45}{8\pi^3 GN}\right)^{\frac{1}{4}} \left[\begin{array}{c} -\frac{2\alpha}{\beta^2}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} + \frac{1}{\beta}\frac{1}{(t^2+\frac{\alpha}{\beta})} \\ + \frac{1}{R_0^2}(\frac{\alpha}{\beta})^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}} \end{array}\right]^{\frac{1}{4}}. \quad (37)$$

Equation (37) gives at $t = 0$

$$T_0 = \left(\frac{45}{8\pi^3 GN}\right)^{\frac{1}{4}} \left[\frac{1}{R_0^2} - \frac{1}{\alpha}\right]^{\frac{1}{4}}. \quad (38)$$

Like the radiation energy density the radiation temperature is also zero at $t = 0$ for $\alpha = R_0^2$ and has a finite value for $\alpha > R_0^2$.

As the universe is geometrically closed in our model, it is possible to determine the time $t = t_{cau}$ when the whole universe becomes causally connected. This is given by

$$\int_0^{t_{cau}} \frac{dt}{R(t)} = \int_0^1 \frac{dr}{\sqrt{1-r^2}} = \frac{\pi}{2}. \quad (39)$$

This, by use of (21) yields

$$\int_0^{t_{cau}} \frac{dt}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{2\beta}}} = \frac{\pi}{2} R_0 \left(\frac{\beta}{\alpha}\right)^{\frac{1}{2\beta}}. \quad (40)$$

We find that the global causality is established at $t = t_{cau}$, where $t_{cau}$ can be determined from (40) by giving the particular values of $\alpha$ and $\beta$. For the parameter, $\alpha = R_0^2$ and $\beta = 1$ we get $t_{cau} = 2.3R_0$ as in the case of Ozer and Taha model and if we take $\alpha > R_0^2$ and $\beta = 1$ then we get $t_{cau} > \frac{\pi}{2}R_0$ as in the case of Abdel-Rahman model.

In the present matter dominated era, the matter pressure is negligible i.e. $p = p_m \approx 0$ and $\rho = \rho_m$.

Equations (24) and (25) give

$$\rho_m = \frac{1}{8\pi G}\left[\begin{array}{c} -\frac{4\alpha}{\beta^2}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} + \frac{2}{\beta}\frac{1}{(t^2+\frac{\alpha}{\beta})} \\ + \frac{2}{R_0^2}(\frac{\alpha}{\beta})^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}} \end{array}\right], \quad (41)$$

$$\rho_v = \frac{1}{8\pi G}\left[\begin{array}{c} (-\frac{3}{\beta^2}+\frac{4}{\beta})\frac{\alpha}{\beta}\frac{1}{(t^2+\frac{\alpha}{\beta})^2} + (\frac{3}{\beta^2}-\frac{2}{\beta})\frac{1}{(t^2+\frac{\alpha}{\beta})} \\ + \frac{1}{R_0^2}(\frac{\alpha}{\beta})^{\frac{1}{\beta}}\frac{1}{(t^2+\frac{\alpha}{\beta})^{\frac{1}{\beta}}} \end{array}\right]. \quad (42)$$

If we take $\alpha = 0$ and $\beta = \frac{3}{2}$, we get the subsequent evolution as in the case of matter dominated era of Einstein desitter model with $\rho_v = 0$, $\rho_m = \frac{4}{3t^2}$ and $R \propto t^{2/3}$.

Although, the recent observations favour accelerating models at present but the possibility of decelerating models with small deceleration parameter are not completely ruled out. It has also been shown that the high redshift supernovae Ia can also be explained successfully in the decelerating models if one takes into account the absorption of light by the intergalactic metallic dust ejected from the supernovae explosions (Vishwakarma 2002, 2005). In view of this, in the next section we choose the value of $\beta$ slightly greater than 1, so that the deceleration parameter will be very small close to zero as $t \to \infty$.

## 4 Exemplification: the model $\gamma = 0$, $\alpha = 1.3R_0^2$, $\beta = 1.01$

From (13), (21), (18), (28), (37) and (29) we get in this case

$$q = -\frac{1.3R_0^2}{t^2} + 0.01, \quad (43)$$

$$R(t) \approx 0.883R_0^{0.010}\left(t^2 + 1.287R_0^2\right)^{0.495}, \quad (44)$$

$$H(t) \approx \frac{0.990t}{t^2 + 1.287R_0^2}, \quad (45)$$

$$\rho_r \approx \frac{3}{16\pi G}\left[\begin{array}{c} -\frac{2.549R_0^2}{(t^2+1.287R_0^2)^2} + \frac{0.990}{(t^2+1.287R_0^2)} \\ + \frac{1}{R_0^2}\frac{1.283R_0^{1.980}}{(t^2+1.287R_0^2)^{0.990}} \end{array}\right], \quad (46)$$

$$T \approx \left(\frac{45}{8\pi^3 GN}\right)^{\frac{1}{4}}\left[\begin{array}{c} -\frac{2.549R_0^2}{(t^2+1.287R_0^2)^2} + \frac{0.990}{(t^2+1.287R_0^2)} \\ + \frac{1}{R_0^2}\frac{1.283R_0^{1.980}}{(t^2+1.287R_0^2)^{0.990}} \end{array}\right], \quad (47)$$

$$\rho_v \approx \frac{3}{16\pi G}\left[\begin{array}{c} \frac{0.025R_0^2}{(t^2+1.287R_0^2)^2} + \frac{0.971}{(t^2+1.287R_0^2)} \\ + \frac{1}{R_0^2}\frac{1.283R_0^{1.980}}{(t^2+1.287R_0^2)^{0.990}} \end{array}\right]. \quad (48)$$

The time variation of the different cosmological parameters are depicted in Fig. 1 by taking $R_0 \sim 1$.

In this model the universe starts with a finite radius $R = R_0$ (at $t = 0$). The scale factor R is slowly increasing in the beginning and thereafter follows an almost linear variation with time. The acceleration of the universe is $\approx \frac{0.769}{R_0}$ initially. It gradually decreases and reduces to zero at $t = 11.401R_0$. Thereafter the model decelerates with an increasing deceleration parameter ultimately approaching to 0.01. The radiation energy density increases rapidly within a short period of time to its maximum value and thereafter decreases with time. Like the radiation energy the radiation temperature also increases to its maximum value and then decreases with time. The vacuum energy is maximum at $t = 0$ and decreasing continuously relaxes to its present small value.

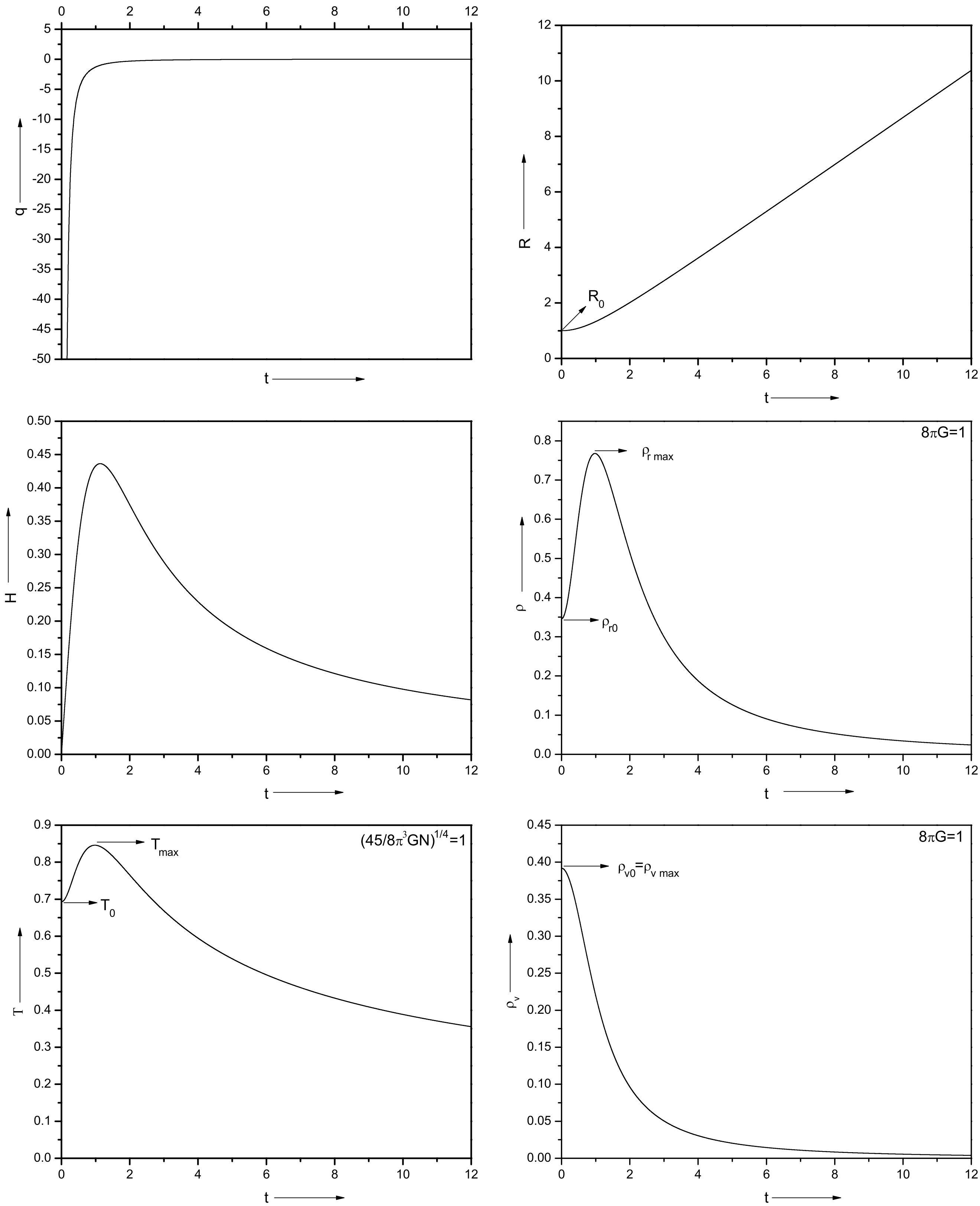


**Fig. 1** Variation of different cosmological parameters with respect to time '$t$'

With the experimental value of the Hubble parameter $H_p = 75$ km/s/Mpc the age of the universe in this model is obtained by taking $R_0 \sim 1$ as

$$t_p \approx 4.074 \times 10^{17}\ \mathrm{S}$$

With this value of $t_p$, we obtain the present values of
Radius of the universe $= R_p \approx 5.590 \times 10^{25}$ m
Critical density $\approx 3.744 \times 10^{-26}\ \frac{\mathrm{kg}}{\mathrm{m}^3}$
Vacuum energy density $\approx 1.861 \times 10^{-26}\ \frac{\mathrm{kg}}{\mathrm{m}^3}$
Cosmological constant $= \Lambda \approx 3.121 \times 10^{-35}\ \mathrm{S}^{-2}$
which are in good agreement with the present observational values.

## 5 Conclusion

In this paper we have obtained a class of non-singular and bouncing FRW cosmological models with a perfect fluid as the source of matter and an interacting dark energy represented by the time-varying cosmological constant by constraining the form of deceleration parameter. The resulting models are geometrically closed but ever expanding and are free from entropy and cosmological constant problems. Out of the three models obtained here we have discussed the evolution of only one of them. The important aspect of the presented model is that it starts from rest with a finite radius and finite acceleration which gradually decreases and reduce to zero after some time; thereafter the model decelerates with gradually increasing deceleration, approaching to a constant value for large values of $t$. There is an obvious provision for the smoothness in the universe during the accelerated phase of the expansion and for the structure formation in the universe during the decelerated phase of the expansion.

**Acknowledgements** One of the author SRP thanks to Prof. R.G. Vishwakarma, *Unidad Académica de Matemáticas, Universidad Autónoma* de Zacatecas, *Zacatecas, ZAC, Mexico* for his useful comments and discussions which helped in improving the paper.

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