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ORIGINAL ARTICLE

Ion acoustic shock waves in weakly relativistic and dissipative plasmas with nonthermal electrons and thermal positrons

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Abstract Ion acoustic shock waves in dissipative plasmas are studied. The target medium contains Relativistic ions, nonthermal electrons and positrons with Maxwell–Boltzmann distribution. Kinematic viscosity among the plasma constituents has been considered. The effects of kinematic viscosity on the general behavior of plasma and specifically on nonthermal distribution of electrons are studied. Results would be helpful for understanding the localized structures that may occur in space plasmas.

Keywords Shock waves · Ion acoustic · Relativistic · Soliton · Nonthermal

1 Introduction

The dynamics of ion-acoustic waves, which is one of the basic wave processes in plasmas, has been studied for several decades both theoretically and experimentally. Nonlinear theory for these waves was first considered in Sagdeev (1966) where the basic features have been studied using a mechanical analogy. The first experimental observation of ion-acoustic solitons was made by Ikezi et al. (1970). Observations made by the Viking spacecraft (Boström 1992) and Freja satellite (Dovner et al. 1994) have proved the existence of electrostatic solitary structures in the magnetosphere with density depressions. Experiments on ion-acoustic waves (IAWs) were widely studied in double plasma devices and Q-machines (Tran 1979; Nakamura 1982; Lonngren 1983). Recently, a great deal of attention has been devoted to the study of different types of collective processes in electron–positron–ion plasmas. In contrast to the usual plasmas consisting of electrons and positive ions, it has been known that the nonlinear waves in the plasmas containing positrons, behave differently (Rizzato 1988). In fact, electron–positron–ion plasmas have been appeared in the early universe (Rees 1983) active galactic nuclei (Miller and Witta 1987) pulsar magnetospheres (Michel 1982) and the solar atmosphere (Tandberg-Hansen and Emslie 1988). However, most of these investigations on linear and nonlinear phenomena are confined to nonrelativistic plasmas. But we know that when the electron or ion velocity approaches the velocity of light, relativistic effects can modify the wave behavior (Singh and Honzawa 1993; Malik et al. 1994; Gill et al. 2004). Relativistic plasmas occur in a variety of situations, e.g. in laser–plasma interaction (Arons 1979), space-plasma phenomena (Grabbe 1989), plasma sheet boundary layer of earth's magnetosphere (Vette 1970), in the Van Allen radiation belts (Ikezi 1973). Gill et al. have explored the dynamics of ion acoustic solitons in weakly relativistic e-p-i plasmas (Gill et al. 2007). Using the Korteweg–de Vries (KdV) and Kadomstev–Petviashvili (KP) equations, the behavior of solitonic solutions in one-dimensional and two-dimensional propagation has been described. They arise due to delicate balance of nonlinearity and dispersion. However, when a medium is both dispersive and dissipative, the propagation characteristics of small-amplitude perturbations

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can then be adequately described by Korteweg–de Vries–Burger (KdVB) in one dimension and by the Kadomtsev–Petviashvilli–Burgers (KPB) equation in a two-dimensional geometry. The dissipative Burger term in both nonlinear KdVB and KPB equations arises by considering the kinematic viscosity among the plasma constituents (Mamun and Shukla 2002; Xue 2003; Sahu and Roychoudhury 2007). Ion-acoustic shock waves were first observed in a novel plasma device called Double Plasma device (Taylor et al. 1970). Asif Shah et al. (2009) have derived the Korteweg–de Vries-Burger (KdVB) equation ion acoustic shock waves in weakly relativistic electron–positron–ion plasma. Electrons, positrons were considered isothermal and ions are relativistic. More recently, it had been found that the electrons and ions distributions play a crucial role in characterizing the physics of the nonlinear waves (Shukla and Stenflo 1993; Popel et al. 1995; Ghosh and Bharuthram 2008; Pakzad 2009a; Pakzad 2009b). Numerous observations of space plasmas (Mamun 2000; Carins et al. 1995; Pakzad 2009a) indicate clearly the presence of nonthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. The latter may arise due to the effect of external forces acting on the natural space environment plasmas or to the wave-particle interaction which ultimately leads to the same distributions. As a consequence, a high-energy tail appears in the distribution function of the particles. Only few investigations have been reported on the study of ion acoustic shock waves in relativistic plasmas (Shah and Saeed 2009; Masood et al. 2008; Masood et al. 2009). The aim of the present paper is study of ion-acoustic shock waves in plasmas consisting of relativistic ions, Maxwell–Boltzmann distributed positrons and nonthermal electrons.

2 Derivation of KdV Burger equation

We consider one-dimensional, collisionless, unmagnetized plasma comprising of relativistic ions, thermal positrons and nonthermal electrons. We assume that the phase velocity of ion-acoustic wave is much smaller than the electron and positron thermal velocities and larger than the ion thermal velocity, so we can ignore the electron and positron inertia and write down the equation of motion for the ions. The nonlinear dynamics of the low frequency ion-acoustic solitons in the three component plasma is governed by the following set of equations.

$$\begin{aligned}
&\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0 \\
&\frac{\partial (\gamma u)}{\partial t} + u\frac{\partial (\gamma u)}{\partial x} + \frac{\partial \phi}{\partial x} - \eta \frac{\partial^2 u}{\partial x^2} = 0 \qquad (1) \\
&\frac{\partial^2 \phi}{\partial x^2} = n_e - n - n_p
\end{aligned}$$

where n is the ion number density, u is the ion fluid velocity, η is the kinematic viscosity and ϕ is electrostatic potential. The above equations are not Lorentz invariant. But they can be derived from the covariant fluid equations in the wave frame (Lee and Choi 2007). γ is relativistic factor and for a weakly relativistic plasma it is approximated by its expansion up to second term ($u \ll c$)

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} \cong 1 + \frac{u^2}{2c^2} \qquad (2)$$

Relativistic effects may be induced by different reasons which may affect on some parts of plasmas. It is possible that the fluid velocity of the relativistic particles is close to the light velocity while other parts move in lower velocities. For example, the relativistic effects can be induced by thermal effects of the particles under concern (T_i in our case). One can consider low velocity pair (electrons and positrons) production by relativistic ions. This feature has been studied by Becker et al. (1986).

In order to adding the effects of nonthermal electrons, one can write (Carins et al. 1995)

$$n_e = (1 - \beta\phi + \beta\phi^2)e^{\phi} \qquad (3)$$

where $\beta = \frac{4\alpha}{1+3\alpha}$ in which α is a parameter that determines the population of nonthermal (fast) electrons (Taylor et al. 1970). Positrons are assumed to be in thermal equilibrium, with the density of

$$n_p = pe^{-\sigma\phi} \qquad (4)$$

In (1), the densities of the plasma species are normalized by the unperturbed electron density n_{e0}, the ion velocity is normalized by the ion acoustic speed $c_i = \sqrt{T_e/m}$, space variables are normalized by the electron Debye length $\lambda_D = \sqrt{T_e/4\pi n_0 e^2}$, time variable is normalized by the electron plasma period $T = \sqrt{m_e/4\pi n_{e0} e^2}$ and electrostatic potential is normalized by the quantity (T_e/e). The coefficient of kinematic viscosity η is incorporated in the parameter, $\eta = \frac{\eta}{\lambda_{Dcs}}$. Also, $p = n_{p0}/n_{e0}$ represents the positron concentration in e-p-i plasma and$\sigma = T_e/T_p$, is the temperature ratio of electron to positron.

In order to finding the equation of motion for nonlinear ion acoustic wave, we use the reductive perturbation method and define the following stretched coordinates

$$\xi = \varepsilon(x - \lambda t), \qquad \tau = \varepsilon^3 t, \qquad \eta = \varepsilon^{\frac{1}{2}}\eta_0 \qquad (5)$$

where ε is a small parameter which characterizes the strength of the nonlinearity, and λ is the phase velocity of the wave. Dependent variables are expanded as follows

$$\begin{aligned}
&n = (1 - p) + \varepsilon^2 n_1 + \varepsilon^4 n_2 \\
&u = u_0 + \varepsilon^2 u_1 + \varepsilon^4 u_2 \qquad (6) \\
&\phi = \varepsilon^2 \phi_1 + \varepsilon^4 \phi_2
\end{aligned}$$

On substituting (6) into (1), using (5) and collecting the terms in the different powers of ε, we obtain the following equations at the lowest order of ε

$$n_1 = \frac{(1-p)u_1}{\lambda - u_0}, \qquad u_1 = \frac{\phi_1}{(\lambda - u_0)\gamma_1}$$
$$n_1 = (1-\beta+p\sigma)\phi_1 \tag{7}$$
$$(\lambda-u_0)^2 = \frac{1-p}{\gamma_1[1-\beta+p\sigma]}$$

and for the higher orders of ε, we have

$$-(\lambda-u_0)\frac{\partial n_2}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + \frac{\partial(n_1u_1)}{\partial \xi} + (1-p)\frac{\partial u_2}{\partial \xi} = 0$$
$$-(\lambda-u_0)\gamma_1\frac{\partial u_2}{\partial \xi} + \gamma_1\frac{\partial u_1}{\partial \tau} + \frac{\partial \phi_2}{\partial \xi}$$
$$+[\gamma_1 - 2\gamma_2(\lambda-u_0)]u_1\frac{\partial u_1}{\partial \xi} - \eta_0\frac{\partial^2 u_1}{\partial \xi^2} = 0 \tag{8}$$
$$\frac{\partial^2\phi_1}{\partial\xi^2} = (1-\beta+p\sigma)\phi_2 + (1-p\sigma^2)\frac{1}{2}\phi_1^2 - n_2$$

where $\gamma_1 = 1 + \frac{3u_0^2}{2c^2}$ and$\gamma_2 = \frac{3u_0}{2c^2}$.

Finally the KdV-Berger equation is derived from (7) and (8)

$$\frac{\partial\phi_1}{\partial\tau} + A\phi_1\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} - C\frac{\partial^2\phi_1}{\partial\xi^2} = 0 \tag{9}$$

where

$$A = \left(\frac{3}{2(\lambda-u_0)} - \frac{\gamma_2}{\gamma_1}\right) - \frac{\gamma_1(\lambda-u_0)^3}{2(1-p)}[1-\beta-p\sigma^2]$$
$$B = \frac{(\lambda-u_0)^3\gamma_1}{2(1-p)}, \qquad C = \frac{\eta_0}{2\gamma_1} \tag{10}$$

For $\eta_0 = 0$, (9) is reduced to KdV equation and in this case it has solitonic solutions. By introducing the new variable $\chi = \xi - U\tau$, where u is a constant velocity, the solution of (9) in the stationary frame is

$$\phi_1 = \phi_0 \operatorname{sech}^2\frac{(\chi)}{w} \tag{11}$$

The soliton amplitude, ϕ_0 and its width, w are given as following

$$\phi_0 = \frac{3U}{A}, \qquad w = 2\sqrt{\frac{B}{U}} \tag{12}$$

Equation (9) is the well known KdV-Burger equation describing the nonlinear propagation of the ion acoustic shock waves in relativistic plasmas with nonthermal electrons. Apparently, coefficients of the nonlinear and dispersion terms depend upon the different parameters like the relative temperature, relative density, relativistic factor and nonthermal parameter. It is also seen that the Burger term (C) arises due to the effect of ion kinematic viscosity. The mentioned results are comparable with the results of Asif Shah et al. (2009) for relativistic plasmas with the Boltzmann distribution of electrons. The obtained results are in agreement with the results in Nejoh (1987b) and Gill et al. (2007) for e-i and e-p-i relativistic plasmas with the Boltzmann distribution of electrons, respectively. Our results are as the same as reported results in Washimi and Taniuti (1996) and Popel et al. (1995) for e-i and e-p-i plasmas with Boltzmann distributed electrons but without dissipative effects. Some researchers have studied one-dimensional (Gill et al. 2007; Nejoh 1987b; Washimi and Taniuti 1996; Das and Paul 1985; Nejoh and Sanuki 1994; Kuehl and Zhang 1991) and two-dimensional (Singh and Honzawa 1993; EL-Labany et al. 1996; Nejoh and Sanuki 1994) ion acoustic solitary waves in weakly relativistic plasmas containing ions and electrons and our results are comparable with the results of their studies. The obtained results are in agreement with the results of Nejoh (1987a, 1987b) for electron-ion plasmas ($p = 0$) containing of Boltzmann distributed electrons ($\beta = 0$). For the cold plasmas with Boltzmann electron distribution our results reduce to results of Nejoh and Sanuki (1994). Our results are also changed to Washimi and Taniuti (1996) (Das and Paul 1985) for nonrelativistic plasmas with Boltzmann distributed electrons, without positron and cold (warm) ions.

We have used the numerical computation for doing quantitative analysis on the results. Relativistic factor has been represented with $\mu = \frac{u_0}{c}$. In this case we have $\gamma_1 = 1 + \frac{3}{2}\mu^2$ and

$$\gamma_2 = \frac{3\mu}{2c}$$

3 Discussion

The KdV-Burger equation is widely used in the plasma physics and theoretical physics. There has been presented different solutions for this equation using several methods. The tangent hyperbolic method is a powerful method for the computation of exact traveling wave solutions. More recently, Asif Shah et al. (2009) have derived the monotonic shock waves solution theoretically by employing the tangent hyperbolic method (Wazwaz 2008). They used the transformation $\zeta = \kappa(\xi - v\tau)$ (where κ and v are wave number and wave velocity, respectively) and presented the solution in terms of independent variable χ as

$$\phi_1 = \frac{12B}{A}[1-\tanh^2(\chi)] - \frac{36C}{15A}\tanh(\chi) \tag{13}$$

Now we can use (13) for doing a quantitative analysis on the results. It is clear that the profile of the wave (13) is depend

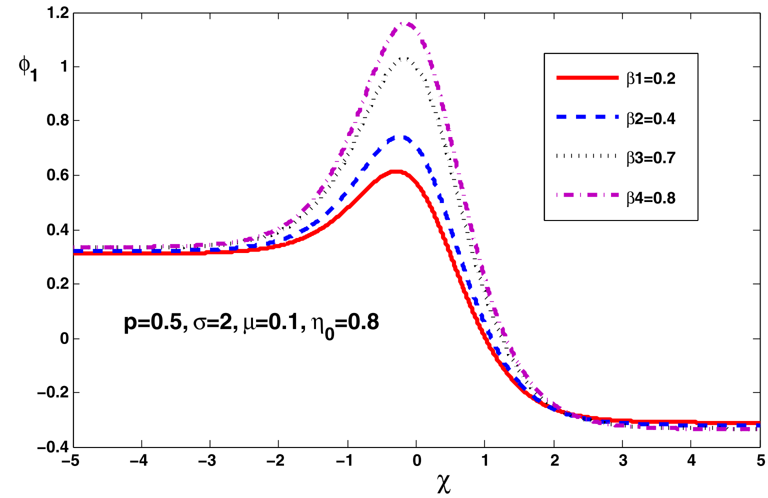

Fig. 1 Variation of ϕ_1 with respect to χ for $\beta = 0.4$, $p = 0.7$, $\mu = 0.1$, $\sigma = 10$ and different values of η_0

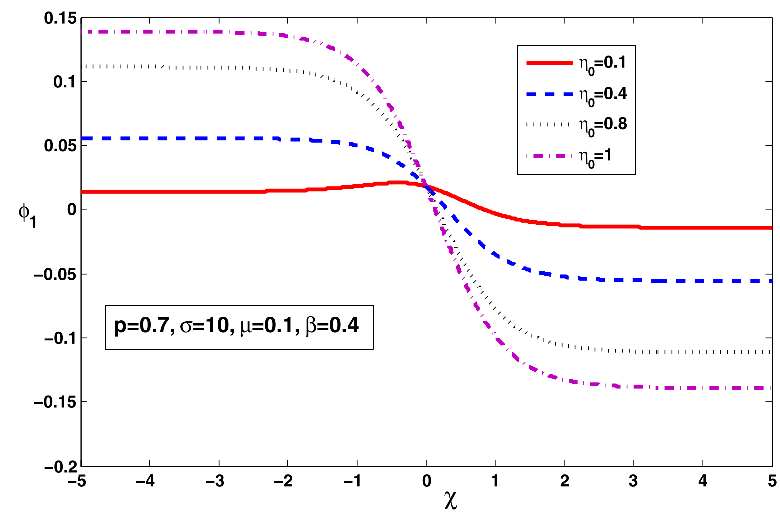

Fig. 2 Variation of ϕ_1 with respect to χ for $p = 0.5$, $\sigma = 2$, $\mu = 0.1$, $\eta_0 = 0.8$ and different values of β

to the plasma parameters $(p, \beta, \sigma, \mu, \eta_0, \chi)$. In this analysis, we study the effects of the plasma parameters on the shock waves behavior. We have considered $\mu \leq 0.4$ for weakly relativistic plasmas in our calculations. Figures 1–4 show the ϕ_1 profiles with different values of plasma parameters. Figure 1 shows ϕ_1 with respect to χ for $\beta = 0.4$, $\sigma = 10$, $p = 0.7$, $\mu = 0.1$ and different values of η_0. It is shown that the solitary wave is changed to monotonic shock wave, when η_0 is increased from $\eta_0 = 0.1$ until $\eta_0 = 1$. For $\eta_0 = 0.1$ the wave is more similar to solitonic profile than a shock wave and for $\eta_0 = 1$, a shock wave structure is appeared. In fact, if the dissipation term is negligible compared to the dispersion term, then solitonic structure will be established by balancing the effects of dispersive and nonlinear terms. On the other hand, if the coupling becomes very strong the shock waves will appear by balancing the effects of dissipative and dispersive terms. So the presence of a Burger term leads to the formation of a shock wave from initial solitonic profile. Also, it is obvious that with a stronger dissipation, the shock wave structure becomes steeper. The effects of nonthermal electrons on the solitonic and shock wave solutions of (13) have been presented in Fig. 2. It is clear that the amplitude of the solitons and shock waves increases with an increasing β. In fact, it is obvious the waves are similar to solitons when nonthermal electrons are exist.

The effects of relativistic ions on the behavior of shock waves are investigated in Fig. 3, where we have plotted ϕ_1 with respect to χ for $\beta = 0.05$, $\sigma = 0.8$, $p = 0.9$, $\eta_0 = 0.9$ and different values of μ. It is clear that the shock amplitude decreases with an increasing relativistic factor. This is congruent with the observation of Gill et al. (2007) made in relativistic e-p-i plasma. Figure 3 presents that the wave profiles are very close to each other and the height of the wave slightly increases when μ increases. Figure 4 shows transition from a solitary wave to a shock wave situation when σ_1 increases. It is observed that the amplitude of shock profile decreases when σ increases. This means that the amplitude of shock waves in the medium increases when the electrons (positrons) temperature increases (decreases). It is also seen that the profile of the wave is close to soliton pro-

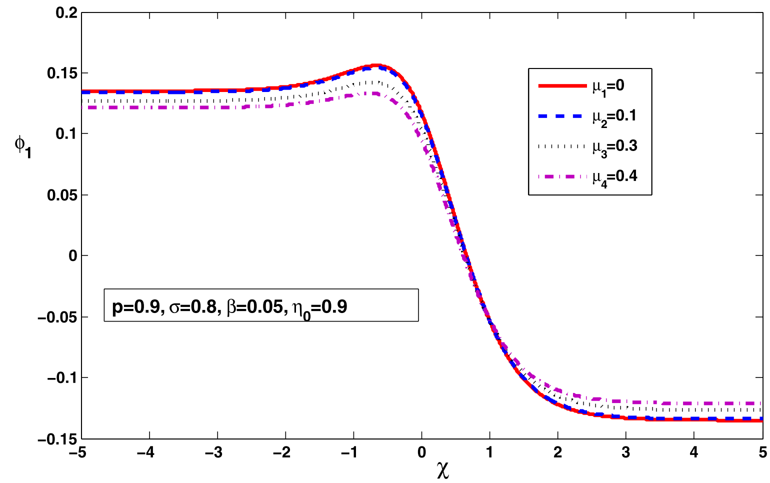

Fig. 3 Variation of ϕ_1 with respect to χ for $p = 0.9$, $\sigma = 0.8$, $\beta = 0.05$, $\eta_0 = 0.9$ and different values of μ

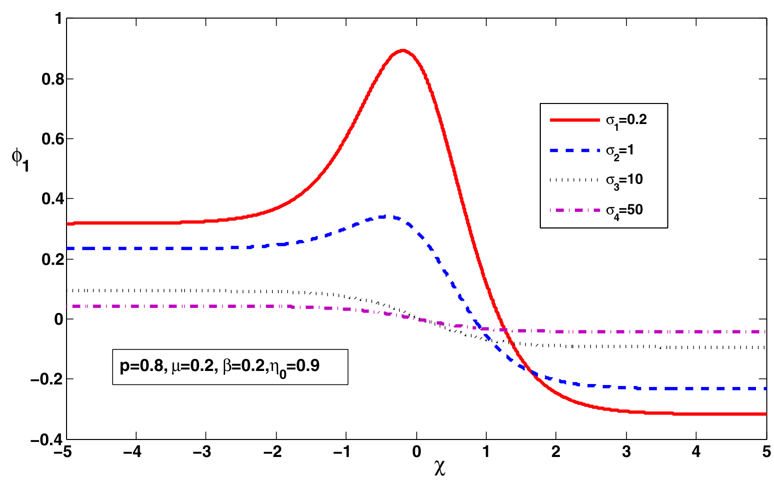

Fig. 4 Variation of ϕ_1 with respect to χ for $p = 0.8$, $\beta = 0.2$, $\mu = 0.2$, $\eta_0 = 0.9$ and different values of σ

file for low (small) values of σ. At low temperatures, the dispersion is dominant and the nonlinear profile is in the form of solitonic and at higher temperatures the dissipation effects are dominant and thus a shock profile appears. It is obvious that the shock wave has strong dependence on the relative temperature (σ). More investigations show that the amplitude of the waves decreases when positron concentration increases. It is clear that the ion concentration decreases with an increasing positron concentration in the plasma, so we expect to observe propagation of ion acoustic soliton and shock wave with smaller amplitudes (Shah and Saeed 2009; Popel et al. 1995; Pakzad 2009a).

4 Conclusions

Effects of nonthermal electrons and viscosity on the nonlinear propagation of ion acoustic waves in e-p-i plasmas have been investigated. In order to studying the behavior of nonlinear ion acoustic wave the reductive perturbation method has been used and the KdV-Burger equation has been derived. The dissipative Burger term in the nonlinear KdV-Burger equation arises by considering the kinematic viscosity among the plasma constituents. Numerical analysis shows that both soliton and shock waves structures may appear. It was found that the establishing of positive and negative values for the wave height depends on the presence and amount of nonthermal electrons. It was shown that in plasmas with strong kinematic viscosity the monotonic structures can be appeared. Also, we observed that the amplitude of the waves increases when the nonthermal parameter increases. The ranges of different plasma parameters used in this investigations are very wide ($0 < \mu < 0.4$, $0 < \eta_0 < 15$, $0 < \beta < 1$, $p = 0.1$, $\sigma = 1$) and are relevant to (Gill et al. 2007; Shah and Saeed 2009; Popel et al. 1995; Ghosh and Bharuthram 2008; Pakzad 2009a). Streaming ions with energies between 0.1 to 100 MeV are frequently observed in solar atmosphere and interstellar space. Only few investigations have been reported on IASs, and other nonlinear wave structures in relativistic plasmas (Das and Paul 1985). As electron-positron plasmas with ion possessing relativistic velocities are frequently observed in astrophysical and space environments, thus studying of ion-acoustic solitons in such plasmas is an interesting case. The results of the present investigation would help us to explain the basic features of localized ion acoustic solitary waves in polar coups region of pulsars and around active galactic nuclei where e-p-i plasmas with adiabatic ions can exist. These results can also help us to understand some features of nonlinear structures in plasmas containing nonthermal electrons, which may exist in certain heliospheric environments.

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