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ORIGINAL ARTICLE

Yukawa-type effects in satellite dynamics

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Abstract Many of today's gravity theories predict the existence of a non-Newtonian Yukawa-type correction to the gravitational potential. New experimental techniques, such as Sagnac interferometry, may help in exploring the range $\lambda \geq 10^{14}$ m where such forces are possibly measurable. It is expected that future space missions will operate in this range, which has not been examined for a very long time. Restricting ourselves to an Earth orbiting satellite we follow a perturbing-potential approach applied on the Lagrange planetary equations, in order to study the effect of such a non-Newtonian potential in the range $\lambda \cong 1.073 R_{\mathrm{E}}$. This is achieved by calculating the time rates of change of the orbital elements for the earth orbiting satellite GRACE-A. All these time rates have been calculated on the Keplerian and the precessing Keplerian ellipse of the body under study. Of all the orbital elements, the argument of the perigee is most affected by this potential.

Keywords Yukawa potential · Lagrange planetary equations · Sagnac interferometry · GRACE mission

1 Introduction

It is common knowledge that the theory of general relativity is one of the foundations of modern physics, and most of its predictions have been confirmed by a wide variety of experiments (Ciufolini and Wheeler 1995). Because many of the present theories of gravitation and elementary particles predict forces coupled to gravitation, the investigation for the deviations from Newtonian gravity is of interest. For example, Mücket and Treder (1977) considered a logarithmic correction to the post-Newtonian gravitational potential acting on a satellite which is moving on the vicinity of a primary body. This correction predicts a perihelion shift that depends neither on the primary mass M_{p} nor on the semimajor axis a. Similarly, another potential, which is widely used in the study of various celestial mechanics scenarios, is the Maneff potential (Maneff 1924). Haranas and Mioc (2009) have studied the motion of a satellite in such a potential with results similar to those predicted by general relativity. This potential provided unexpected results which, statistically as well as observationally, match better the astronomical reality when compared to those of the classical Newtonian model (Mioc 2002; Stoica and Mioc 1997).

To the physics community a force generated by such a potential is known as the fifth force. This force can no longer be described with the $1/r$ Newtonian potential, but, instead, an additional term must be incorporated into the original $1/r$ potential term. For the last decade or so, the fifth force has witnessed luck of evidence supporting its existence (Fischbach and Talmadge 1999). On top of that, forces of this kind generally violate the weak equivalence principle (Fischbach et al. 1986). Therefore, if more work is going to be done in this domain, somebody will have to go into considerable details and analysis in justifying such a research. Gibbons and Whiting (1981) put forward the requirement that the agreement of planetary motions with Newtonian gravity does not actually exclude the existence of non-Newtonian forces over large distances. At this point, we should mention that most of the research done on the subject today, as

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presented by Fischbach and Talmadge (1999), is for a force range related to the interval 10 m $\leq \lambda \leq$ 1000 m, where λ is the distance range beyond which the effects of this force becomes unimportant. This is known as the geophysical window. The domain $\lambda \geq 10^{14}$ m remains unexplored (Camacho 2004). Experimental techniques, such as space interferometers, might be a new method for exploring this domain of orbital range.

In this paper, we deal with the Yukawa potential effect on satellites. By evaluating the corresponding Yukawa potential on the Keplerian and on the precessing Keplerian orbital ellipses respectively, we derive the Lagrange planetary equations up to a first-order perturbation. Finally, we apply the derived equations to estimate the corresponding time rates of change of the elements for an orbit reported by GRACE mission

2 The theory of non-Newtonian gravity

Consider a two-body problem concerning the motion of a secondary of mass m under the influence of a primary of mass $M_{\rm p}$. Then the non-Newtonian effects on the secondary conventionally described in terms of the modified potential energy (Fischbach et al. 1991)

$$V(r) = -\frac{G_\infty M_{\rm p} m}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right) = V_{\rm N}(r) + V_{\rm Yk}(r) \qquad (1)$$

In this notation, r is the distance between these bodies, G_∞ is the Newton's gravitational constant, $\alpha = \frac{kK}{G_\infty M_{\rm p} m}$, where k and K are the coupling constants of the new force to the bodies relative to gravity (Ciufolini and Wheeler 1995), and λ is the range of this interaction. $V_{\rm Yk}(r)$ is the Yukawa correction to the Newtonian potential energy. The corresponding force can be written as

$$\begin{aligned}\mathbf{F}(r) &= -\frac{G_\infty M_{\rm p} m}{r^2}\left[1 + \alpha\left(1 + \frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}\right]\mathbf{r} \\ &= \left[F_{\rm N}(r) + F_{\rm Yk}(r)\right]\mathbf{r} \qquad (2)\end{aligned}$$

or

$$\mathbf{F}(r) = -\frac{G(r) M_{\rm p} m}{r^2}\mathbf{r} \qquad (3)$$

where

$$G(r) = G_\infty\left[1 + \alpha\left(1 + \frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}\right]. \qquad (4)$$

Therefore, the presence of the non-Newtonian term can be seen as converting the Newtonian gravitational constant G_∞ into a function of the distance. Hence, the results of investigating the new force are often expressed by the deviation of $G(r)/G_\infty$ from unity (Fischbach and Talmadge 1999). For experiments in the laboratory $r/\lambda \ll 1$ and (4) becomes

$$G(r) \cong G(0) = G_\infty(1 + \alpha) \qquad (5)$$

so that $G(0)$ is the usual laboratory value. For Earth orbiting satellites like LAGEOS, $\alpha_{\min} = 1.38 \times 10^{-11}$ and $\lambda = 6.081 \times 10^6$ m (Kolosnitsyn and Melnikov 2004). Pitjeva (1999) has estimated by using a radar that random Mercury perihelion motions are of the order of $0.052''$/cy, which implies that a minimum value of the Yukawa coupling constant is $\alpha_{\min} = 3.57 \times 10^{-10}$ for $\lambda = 2.89 \times 10^{10}$ m.

3 Perturbations

In a two-body problem the secondary body moves under the dominant force of the primary one. However, other bodies exert forces, which depend on the relative positions of the objects and disturb the motion of the two-body system. The resulting deviations from the unperturbed orbit are usually very small. Given the elements describing the relative position of the two bodies at any instant, the perturbations can be calculated as the time rates of change of these elements. If $R \neq 0$ is the disturbing function, one may use the Lagrange equations to estimate these time rates of change. The Lagrange equations are written as follows (Kaula 2000; Vallado and McClain 2007):

$$\frac{{\rm d}a}{{\rm d}t} = \frac{2}{na}\frac{\partial R}{\partial M}, \qquad (6)$$

$$\frac{{\rm d}e}{{\rm d}t} = \frac{(1-e^2)}{na^2 e}\frac{\partial R}{\partial M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega}, \qquad (7)$$

$$\frac{{\rm d}\omega}{{\rm d}t} = \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} - \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i}, \qquad (8)$$

$$\frac{{\rm d}i}{{\rm d}t} = \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega}, \qquad (9)$$

$$\frac{{\rm d}\Omega}{{\rm d}t} = \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i}, \qquad (10)$$

$$\frac{{\rm d}M}{{\rm d}t} = n - \frac{(1-e^2)}{na^2 e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial a} \qquad (11)$$

where a is the semimajor axis of the orbit of the secondary body, e is its eccentricity, i is its orbital inclination, Ω is the longitude of the node, ω is the argument of the perigee of the orbiting body, $n = \sqrt{G_\infty M_{\rm p}}a^{-3/2}$is the mean motion and M is the mean anomaly. In order to use (6)–(11) the disturbing function R must be written in terms of the orbital elements.

4 The Yukawa perturbing function and the Lagrange equations

In the case of an Earth orbiting satellite, the non-Newtonian Yukawa perturbing potential per unit of mass can be written as follows (Fischbach and Talmadge 1999):

$$R_{\mathrm{Yk}} = -\frac{\mu_{\mathrm{p}}\alpha}{r} e^{-\frac{r}{\lambda}} \tag{12}$$

where $\mu_{\mathrm{p}} = GM_{\mathrm{p}}$. For such a satellite orbiting the Earth on a Keplerian ellipse with $a \approx \lambda$, we can find that, on the unperturbed Keplerian ellipse, this disturbing function is expressed as follows:

$$R_{\mathrm{Yk}} = -\frac{\mu_{\mathrm{p}}\alpha(1+e\cos(\omega+f))}{a(1-e^2)} e^{-\frac{a(1-e^2)}{\lambda(1+e\cos(\omega+f))}}. \tag{13}$$

Then, we evaluate the partial derivatives of R_{Yk} with respect to a, e, ω, i, Ω and M. In order to calculate $\partial R_{\mathrm{Yk}}/\partial M$ we have to express the true anomaly f as a function of M. This is accomplished by using the following series expansion (Murray and Dermott 1999):

$$f \cong M + 2e\sin(M) + \frac{5}{4}e^2\sin(2M) + \mathrm{O}(e^3). \tag{14}$$

Following these evaluations (we omit the details), we replace the due derivatives $\partial R_{\mathrm{Yk}}/\partial s$, $s \in \{a, e, \omega, i, \Omega, M\}$, for $\partial R/\partial s$ to (6)–(11) after setting $a = a_0$, $e = e_0$, $\omega = \omega_0$, $i = i_0$, $\Omega = \Omega_0$, $M = M_0$ in order to obtain the Lagrange equations for the time rates of the orbital elements up to a first-order perturbation. Finally, we see that these equations are

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2\mu_{\mathrm{p}}\alpha e^{-A_0} e_0 N_0 \sin F_0}{n_0 a_0 \lambda (1+e_0\cos F_0)} \left(1+\frac{1}{A_0}\right), \tag{15}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{\mu_{\mathrm{p}}\alpha e^{-A_0}\sin F_0}{n_0 a_0^2 \lambda(1+e_0\cos F_0)} \left(\sqrt{1-e_0^2} - (1-e_0^2)N_0\right) \times \left(1+\frac{1}{A_0}\right), \tag{16}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{\mu_{\mathrm{p}}\alpha(1+A_0)}{n_0 a_0^2 e_0\sqrt{1-e_0^2}} \times e^{-A_0}\left[\frac{2e_0}{\lambda A_0} + \frac{1}{a_0}(\cos F_0 - e_0 B_0 \sin F_0)\right], \tag{17}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{\mu_{\mathrm{p}}\alpha e_0 \cot i_0 e^{-A_0}\sin F_0}{n_0 a_0^2 \lambda\sqrt{1-e_0^2}(1+e_0\cos F_0)} \left(1+\frac{1}{A_0}\right), \tag{18}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = 0, \tag{19}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n_0 + \frac{\mu_{\mathrm{p}}\alpha(1+A_0)}{n_0 a_0^2 e_0} \times e^{-A_0}\left[\frac{2e_0}{\lambda A_0} + \frac{1}{a_0}(\cos F_0 - e_0 B_0\sin F_0)\right], \tag{20}$$

where

$$F_0 = \omega_0 + n_0 t + 2e_0\sin(n_0 t) + \frac{5e_0^2}{4}\sin(2n_0 t). \tag{21}$$

$$N_0 = 1 + 2e_0\cos(n_0 t) + \frac{5e_0^2}{2}\cos(2n_0 t), \tag{22}$$

$$A_0 = \frac{a_0(1-e_0^2)}{\lambda(1+e_0\cos F_0)}, \tag{23}$$

$$B_0 = 2\sin(n_0 t) + \frac{5e_0}{2}\sin(2n_0 t). \tag{24}$$

Via the procedure described above, the disturbing function is approximated by

$$R_{\mathrm{Yk}} = -\frac{\mu_{\mathrm{p}}\alpha}{\lambda A_0}e^{-A_0}. \tag{25}$$

5 The precessing perturbing function and the Lagrange equations

Next, we derive the Lagrange equations on the precessing orbital ellipse. In this case, the motion of the secondary body can be described by the following Binet-type differential equation (Fischbach and Talmadge 1999):

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\theta^2} + u = \frac{G_\infty M_{\mathrm{p}} m^2}{L^2}\left[1+\alpha\left(1+\frac{1}{\lambda u}\right)e^{-\frac{1}{\lambda u}}\right], \tag{26}$$

where $u = 1/r, \theta = \omega + f$ and L is the angular momentum of the body. The first-order solution of this equation is the following (Fischbach and Talmadge 1999):

$$r = \frac{a_0(1-e^2)}{[1+\alpha e^{-\frac{a_0}{\lambda}}(1+\frac{a_0}{\lambda}-\frac{a_0^2}{\lambda^2})][1+e\cos[q_0(\omega+f)]]}, \tag{27}$$

where a_0 is the unperturbed semimajor axis of the orbit and

$$q_0 = \left[1-\alpha\left(\frac{a_0^2}{\lambda^2}\right)e^{-\frac{a_0}{\lambda}}\right]^{1/2} \cong 1 - \frac{\alpha}{2}\left(\frac{a_0^2}{\lambda^2}\right)e^{-\frac{a_0}{\lambda}}. \tag{28}$$

Equation (27) represents a precessing ellipse radial vector, $q_0 \neq 1$. Then, the Yukawa disturbing potential on this ellipse in this way is

$$R_{PYk} = -\frac{\mu_{\mathrm{p}}\alpha[1+\alpha e^{-\frac{a_0}{\lambda}}(1+\frac{a_0}{\lambda}-\frac{a_0^2}{\lambda^2})][1+e\cos(q(\omega+f))]}{a_0(1-e^2)} \times e^{-\frac{a_0(1-e^2)}{\lambda[1+\alpha e^{-\frac{a_0}{\lambda}}(1+\frac{a_0}{\lambda}-\frac{a_0^2}{\lambda^2})][1+e\cos(q(\omega+f))]}}. \tag{29}$$

By following a procedure similar to the one described in the previous section, we obtain the result that, on the precessing Keplerian ellipse, the time rates of the orbital elements are

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2\mu_{\mathrm{p}}\alpha e_0}{n_0 a_0^2(1-e_0^2)} q_0 \sin(q_0 F_0) N_0 A_1 (1+B_1) e^{-B_1}, \quad (30)$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{\mu_{\mathrm{p}}\alpha}{n_0 a_0^3\sqrt{1-e_0^2}} q_0 \sin(q_0 F_0)\big(1 - N_0\sqrt{1-e_0^2}\big) \times A_1(1+B_1)e^{-B_1}, \quad (31)$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{\mu_{\mathrm{p}}\alpha}{n_0 a_0^2 e_0\sqrt{1-e_0^2}}(1+B_1) \times e^{-B_1}\left[\frac{2e_0}{\lambda B_1} + \frac{1}{a_0}A_1\big(\cos(q_0F_0) - e_0 q_0 B_0 \sin(q_0 F_0)\big)\right], \quad (32)$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{\mu_{\mathrm{p}}\alpha e_0 \cot i_0}{n_0 a_0^3(1-e_0^2)^{3/2}} q_0 \sin(q_0 F_0) A_1(1+B_1)e^{-B_1}, \quad (33)$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = 0, \quad (34)$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n_0 + \frac{\mu_{\mathrm{p}}\alpha}{n_0 a_0^2 e_0\sqrt{1-e_0^2}}(1+B_1) \times e^{-B_1}\left[\frac{2e_0}{\lambda B_1} + \frac{1}{a_0}A_1\big(\cos(q_0F_0) - e_0 q_0 B_0 \sin(q_0 F_0)\big)\right], \quad (35)$$

while the disturbing function is approximated by

$$R_{PYk} = -\frac{\mu_{\mathrm{p}}\alpha}{\lambda B_1} e^{-B_1} \quad (36)$$

where

$$A_1 = 1 + \alpha e^{-\frac{a_0}{\lambda}}\left(1 + \frac{a_0}{\lambda} - \frac{a_0^2}{\lambda^2}\right), \quad (37)$$

$$B_1 = \frac{a_0(1-e_0^2)}{\lambda A_1(1+e_0\cos(q_0F_0))}. \quad (38)$$

6 Numerical results

Next, we apply (15)–(20) and (30)–(35) to calculate the time rates of the orbital elements for the satellite GRACE-A. GRACE-A is an Earth orbiting satellite that is a part of the Gravity Recovery and Climate Experiment. The data that are reported in the website of this experiment (http://www.csr.utexas.edu/grace) are $a_0 = 6876.4816$, $e_0 = 0.00040989$, and therefore $n_0 = 0.001100118$ rad/s $= 15.113$ rev/d, $i_0 = 89.025446°$, $\omega_0 = 302.414244°$, $\Omega_0 = 354.447149°$, $M_0 = 80.713591°$. Unfortunately, no improved estimates for λ and α from the GRACE mission are available yet (Iorio 2002). Following Lucchesi (2003), we have used for λ the maximum distance range, $1.073R_{\mathrm{E}} = 6843.737$ km, while, for the coupling constant α, the maximum value 4.6×10^{-10} and the value 4.0×10^{-12} have been utilized. Our results are given in Tables 1 and 2.

Comparing the corresponding entries in Tables 1 and 2, we can see that there is no significant difference between the orbital time rates of change on the Keplerian ellipse and those on the precessing one. The orbital element most affected is the argument of the perigee. For the same value of α, there is, approximately, a 0.05% difference between the respective rates of change on the Keplerian and the precessing Keplerian ellipses. It is also seen that, in both cases, the higher value of α results in higher rates of change than the lower one does. Again, among all the orbital elements, the argument of the perigee is affected the most.

Table 1 First-order perturbation effects of the Yukawa correction on the time rates of the orbital elements of the satellite GRACE-A calculated on the unperturbed Keplerian ellipse

GRACE-A						
$\lambda \cong 1.073R_{\mathrm{E}}$	$\frac{da}{dt}$ [m/d]	$\frac{de}{dt}$ [d^{-1}]	$\frac{d\omega}{dt}$ [°/d]	$\frac{di}{dt}$ [°/d]	$\frac{d\Omega}{dt}$ [°/d]	$\frac{dM}{dt}$ [rev/d]
$\alpha = 4.6 \times 10^{-10}$	-1.530×10^{-4}	-2.220×10^{-11}	−0.0024128000	-1.152×10^{-11}	0	6.700×10^{-6}
$\alpha = 4.0 \times 10^{-12}$	-1.331×10^{-6}	-1.931×10^{-13}	−0.0000209808	-9.406×10^{-14}	0	5.830×10^{-8}

Table 2 First-order perturbation effects of the Yukawa correction on the time rates of the orbital elements of the satellite GRACE-A calculated on the precessing Keplerian ellipse

GRACE-A						
$\lambda \cong 1.073R_{\mathrm{E}}$	$\frac{da}{dt}$ [m/d]	$\frac{de}{dt}$ [d^{-1}]	$\frac{d\omega}{dt}$ [°/d]	$\frac{di}{dt}$ [°/d]	$\frac{d\Omega}{dt}$ [°/d]	$\frac{dM}{dt}$ [rev/d]
$\alpha = 4.6 \times 10^{-10}$	-1.530×10^{-4}	-2.220×10^{-11}	−0.0024140200	-1.152×10^{-11}	0	6.700×10^{-6}
$\alpha = 4.0 \times 10^{-12}$	-1.330×10^{-6}	-1.931×10^{-13}	−0.0000209915	-9.406×10^{-14}	0	5.830×10^{-8}

Similarly, for the same values of α the mean anomaly time rates of change calculated on the Keplerian ellipse and the precessing one are almost identical.

In a year's time, the total effect of the Yukawa correction on the perigee will be of about $-0.880°$ on the Keplerian ellipse and $-0.881°$ on the precessing one, respectively, when $\alpha = 4.6 \times 10^{-10}$. This effect could possibly be detected by today's technology.

In the nearest future, when new values for α and λ will be obtained from the GRACE mission itself, these effects can be more accurately predicted.

7 Conclusions

For a Yukawa-type correction to the gravitational potential, which is predicted by today's various gravitational theories, the Lagrange planetary equations are used in order to study the motion of an orbiting body in this type of potential. In particular, we have studied the motion of the Earth orbiting satellite GRACE-A, by deriving the rates of change of its orbital elements on the Keplerian ellipse and on the precessing one. We find that, in this case, among all the orbital elements, the argument of the perigee is affected the most. There is no significant difference between these rates on the unperturbed Keplerian ellipse and the precessing one, respectively, except for those related to the argument of the perigee, which differ from each other by about 0.05%. For $\lambda \cong 1.073 R_E$ a larger value for b produces higher rates of change.

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