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ORIGINAL ARTICLE

Dust acoustic solitary and shock waves in coupled dusty plasmas with variable dust charge and vortex-like ion distribution

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Abstract The modified Kodomtsev-Petviashvili-Burger (mKP-Burger) and Kodomtsev-Petviashvili-Burger equations are derived in strongly coupled dusty plasmas containing iso-nonthermal ions; Boltzmann distributed electrons and variable dust charge. We use reductive perturbation method and discuss on solitary waves and shock waves solutions of these equations.

Keywords Shock · Soliton · Coupling

1 Introduction

In recent years, the study of nonlinear waves in dusty plasmas has become one of the most important problems of plasma physics (Morfill and Thomos 1996; Rao et al. 1990). Rao, Shukla and Yu theoretically predicted the existence of dust acoustic waves (DAWs) in unmagnetized dusty plasmas (Rao et al. 1990). Usually the dust grains are of micrometer or sub-micrometer size and their masses are very large. Experimental observations have confirmed the existence of linear and nonlinear feature of both dust acoustic waves (Waleed 2006). Also a number of authors have investigated properties of one-dimensional linear and nonlinear dust acoustic waves in coupled unmagnetized dusty plasmas (Kaw and Sen 1998; Shukla and Stenflo 2003). Shukla and Stenflo derived governing hydrodynamic equations for nonlinearity propagation of DAW in strongly coupled unmagnetized dusty plasmas (Shukla and Stenflo 2003). Shukla and Mamun have derived Kordeweg-de Vries-Burger (KdV-Burger) equation by reductive perturbation method and they have studied the properties of the solitons and shock waves for strongly coupled unmagnetized dusty plasmas (Shukla and Mamun 2001). Also Mamun et al. have studied dusty plasma with a Boltzmann electron distribution, an iso-nonthermal (vortex-like) ion distribution and strongly correlated grains in a liquid-like state and discussed on properties of shock waves structures in it (Mamun et al. 2004). They have derived modified KdV-Burger equation by using a set of generalized hydrodynamic (GH) equations. Cylindrical KP-Burger equation in strongly coupled dusty plasmas with two-temperature nonthermal ions has been studied by Wang and Zhang (2006). Strongly coupled plasma is very interested in science, because of its applications in the interior of heavy planets, plasmas produced by laser compression of matter and non ideal plasmas for industrial applications (Nakamura 2002). In dusty plasmas, if the dissipation is weak at the characteristic dynamical time scales of the system, then the balance between nonlinear and dispersion effects can result in the formation of symmetrical solitary waves. Also shock waves will be propagated in this system if dissipation effect is strong. Thus solitary waves and shock waves (oscillatory and monotone types) can be produced in dusty plasmas. In the most of laboratory plasmas charged dust grains are strongly coupled and may be either in a liquid or solid phase (Barkan et al. 1995). Also, since ions may be trapped by the large amplitude DAW potential, so they can follow non-isothermal distribution. Therefore, in this paper, we study the dust acoustic waves in unmagnetized strongly coupled dusty plasmas with variable dust charge, Boltzmann electrons distribution and iso-nonthermal distributed ions.

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The GH equations are presented in Sect. 2. In Sect. 3 we derive the modified Kadomtsev-Petviashvili Burger equation by using the reductive perturbation method. The solitonic and shock waves solutions for this equation are studied in Sect. 4. The numerical solve for solitary and shock waves were presented in Sect. 5. In Sect. 6 we have derived KP equation for critical value and introduced solitonic and shock wave solutions for this situation in Sect. 7. Finally the main results from this investigation have been given in Sect. 8.

2 Basic equations

We consider an unmagnetized strongly coupled dusty plasma with Boltzmann distributed electrons, iso-nonthermal distributed ions and negatively charged dust grains. We assume that the electrons and ions are weakly coupled compare with the dust grains. The dynamics of the DAW in our coupled dusty plasma are given by GH equations (Kaw and Sen 1998; Shukla and Stenflo 2003; Shukla and Mamun 2002) as follows

$$\begin{cases} \frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) + \frac{\partial}{\partial y}(n_d v_d) = 0 \\ \left[1 + \tau_m \frac{\partial}{\partial t}\right]\left[n_d\left(\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + v_d \frac{\partial u_d}{\partial y} - z_d \frac{\partial \phi}{\partial x}\right)\right] \\ \quad = \eta_1\left(\frac{\partial^2 u_d}{\partial x^2} + \frac{\partial^2 u_d}{\partial y^2}\right) \\ \left[1 + \tau_m \frac{\partial}{\partial t}\right]\left[n_d\left(\frac{\partial v_d}{\partial t} + u_d \frac{\partial v_d}{\partial x} + v_d \frac{\partial v_d}{\partial y} - z_d \frac{\partial \phi}{\partial y}\right)\right] \\ \quad = \eta_1\left(\frac{\partial^2 v_d}{\partial x^2} + \frac{\partial^2 v_d}{\partial y^2}\right) \\ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = n_d Z_d + n_e - n_i \end{cases} \tag{1}$$

Where n_d, n_e and n_i are the number density of dust particle, electrons and ions that normalized with $n_{\circ d}$. Z_d refers to the number of charges residing on the dust grains surface and normalized with $Z_{\circ d}$. Charge neutrality at equilibrium requires that

$$Z_{\circ d} n_{\circ d} + n_{\circ e} = n_{\circ i} \tag{2}$$

u_d and v_d are velocity components of the dust particle in x- and y-direction, respectively. Velocities normalized by the dust acoustic speed $C_d = \sqrt{\frac{Z_d T_i}{m_d}}$, in which T_i is temperature of ions and m_d is dust particle mass. ϕ is electrostatic potential normalized by $\frac{T_i}{e}$ (e is the magnitude of the electron charge). Space and time variables are normalized by Debye length $\lambda_D = \sqrt{\frac{T_i}{4\pi n_{\circ d} Z_{\circ d} e^2}}$ and inverse of dust plasma frequency $\omega_{Pd}^{-1} = \sqrt{\frac{m_d}{4\pi n_{\circ d} Z_d^2 e^2}}$. The normalized densities of the Boltzmann distributed electron and iso-nonthermal distributed ion (Schamel and Bujarbarua 1980) are given by

$$n_e = \frac{\mu}{1-\mu} e^{\sigma_i \phi} \tag{3}$$

$$n_i = \frac{1}{1-\mu}\left[1 - \phi - \frac{4(1-a)}{3\sqrt{\pi}}(-\phi)^{\frac{3}{2}} + \frac{1}{2}\phi^2 + \cdots\right] \tag{4}$$

where $\sigma_i = \frac{T_i}{T_e}$ in which, T_e is temperature of electrons and $\mu = \frac{n_{\circ e}}{n_{\circ i}}$. In this distribute, a is ratio of the free temperature to the trapped ion temperature ($a = \frac{T_h}{T_{ht}}$). If $a < 0$, it represents a vortex-like excavated trapped ion distribution, and if $a = 1$ ($a = 0$) we have Maxwellian (flat-topped) ion distribution. τ_m refers to the viscoelastic relaxation time normalized by the dust plasma period ω_{pd}^{-1} and is given as

$$\tau_m = \eta_1 \frac{T_e}{T_d}\left[1 - \mu_d + \frac{4}{15}u(\Gamma)\right]^{-1} \tag{5}$$

where

$$\mu_d = 1 + \frac{1}{3}u_{(\Gamma)} + \frac{\Gamma}{9}\frac{\partial u_{(\Gamma)}}{\partial \Gamma} \tag{6}$$

is the compressibility (Ichimaru and Tanaka 1986), in which Γ is coulomb parameter and $u(\Gamma)$ is a measure of the excess internal energy of the system. For weakly coupled plasma with $\Gamma \ll 1$, $u(\Gamma)$ can be written as $u(\Gamma) \cong -(\frac{\sqrt{3}}{2})\Gamma^{\frac{3}{2}}$ (Ichimaru et al. 1987). For $1 < \Gamma < 100$, Slattery et al. have analytically derived a relation $u_{(\Gamma)} \approx -0.89\Gamma + 0.95\Gamma^4 + 0.19\Gamma^{-\frac{1}{4}} - 0.81$ (Slattery et al. 1980). η_1 is the normalized viscosity coefficient given as

$$\eta_1 = \frac{1}{m_d n_{\circ d}\omega_{Pd}\lambda_D^2}\left[\eta_b + \frac{4}{3}\zeta_b\right] \tag{7}$$

where η_b and ζ_b are transport coefficients of shear and bulk viscosities, is the normalized longitudinal viscosity coefficient.

The dust charge variable $Q_d = m_d Z_d$ is determined by the charge current equation (Melandso et al. 1993)

$$\left(\frac{\partial}{\partial t} + \vec{\nabla}.\vec{V}\right)Q_d = I_e + I_i \tag{8}$$

where $\vec{V} = (u_d, v_d)$. Notice that the characteristic time for dust motion is $\sim 10^{-3}$ s (Barkan et al. 1995) while the dust charging time is typically about of $\sim 10^{-9}$ s (Winske 1995). There fore, the dust charge reaches their equilibrium position quickly. Thus $\frac{dQ_d}{dt} \ll I_e, I_i$ and current balance equation (8) reads (Melandso 1996)

$$I_e + I_i \approx 0 \tag{9}$$

The electron and ion currents are (Nejoh 1997)

$$\begin{aligned} I_e &= -e\pi r^2\left(\frac{8T_e}{\pi m_e}\right)^{\frac{1}{2}} n_e \exp\left(\frac{e\Phi}{T_e}\right) \\ I_i &= e\pi r^2\left(\frac{8T_i}{\pi m_i}\right)^{\frac{1}{2}} n_i\left(1 - \frac{e\Phi}{T_i}\right) \end{aligned} \tag{10}$$

where Φ denotes the dust grain surface potential relative to the plasmas potential ϕ. If the thermal velocities of electrons and ions are larger than their streaming velocities then from (9) we have

$$\sqrt{\frac{\sigma_i}{\mu_i}}\left[1-\phi-\frac{4(1-a)}{3\sqrt{\pi}}(-\phi)^{\frac{3}{2}}+\frac{1}{2}\phi^2\right](1-\psi) - \mu\exp(\sigma_i\phi)\exp(\sigma_i\psi)=0 \tag{11}$$

where $\psi=e\frac{\Phi}{T_i}$, $\mu_i=\frac{m_i}{m_e}\cong 1840$. The dust charge $Q_d=C\Phi$ is calculated by using (11) in which C is capacitance of dust grains ($C=r$). Z_d is defined as

$$Z_d=\frac{\psi}{\psi_\circ} \tag{12}$$

where $\psi=\psi_\circ$ at $\phi=0$ is the dust surface floating potential with respect to the unperturbed plasma potential at an infinite place. By substituting $\phi=0$ into (11) to obtain $\psi_\circ$

$$\sqrt{\frac{\sigma_i}{\mu_i}}(1-\psi_\circ)-\mu\exp(\sigma_i\psi_\circ)=0 \tag{13}$$

Z_d can be expended respect to ϕ as follows

$$Z_d=1+\gamma_1\phi+\gamma_2\phi^2 \tag{14}$$

where $\gamma_1\equiv\frac{\psi'_\circ}{\psi_\circ}$, $\gamma_2\equiv\frac{\psi''_\circ}{2\psi_\circ}$ come from expanding ψ near $\psi_\circ$ and we can write

$$\begin{aligned}\psi'_\circ&=\frac{(1+\sigma_i)(\psi_\circ-1)}{1+\sigma_i(1-\psi_\circ)}\\ \psi''_0&=\frac{1+\psi'_\circ-\sigma_i^2(1-\psi_\circ)(1+\psi'_\circ)^2}{1+\sigma_i(1-\psi_\circ)}\end{aligned} \tag{15}$$

3 Modified KP-Burger equation

According to the general method of reductive perturbation theory (Washimi and Taniuchi 1996) choose the independent variables as

$$\begin{aligned}&\xi=\varepsilon^{1/4}(x-\lambda t),\qquad \eta=\varepsilon^{1/2}y,\qquad \tau=\varepsilon^{3/4}t\\ &\eta_l=\varepsilon^{1/4}\eta_\circ,\qquad \tau_m=\varepsilon^{1/4}\tau_{m\circ}\end{aligned} \tag{16}$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system and λ is the phase velocity of the wave along the x direction and normalized by dust acoustic velocity. Now we expand dependent variables as follows

$$\left[\begin{aligned}&n_d=1+\varepsilon n_{1d}+\varepsilon^{3/2}n_{2d}+\cdots\\ &u_d=\varepsilon u_{1d}+\varepsilon^{3/2}u_{2d}+\cdots\\ &v_d=\varepsilon^{5/4}v_{1d}+\varepsilon^{7/4}v_{2d}+\cdots\\ &\phi=\varepsilon\phi_1+\varepsilon^{3/2}\phi_2+\cdots\\ &Z_d=1+\varepsilon\gamma_1\phi_1+\varepsilon^{3/2}\gamma_1\phi_2+\cdots\end{aligned}\right. \tag{17}$$

Substituting (17) into (1) and collecting the terms in different powers of ε the following equations can be obtained at the lower order of ε

$$\begin{aligned}&n_{1d}=-\frac{\phi_1}{\lambda^2},\qquad u_{1d}=-\frac{\phi_1}{\lambda}\\ &\frac{1}{\lambda^2}=\gamma_1+\frac{\mu\sigma_i+1}{1-\mu},\qquad -\lambda\frac{\partial v_{1d}}{\partial\xi}=\frac{\partial\phi_1}{\partial\eta}\end{aligned} \tag{18}$$

At the higher order of ε, we have

$$\left|\begin{aligned}&\tfrac{\partial n_{1d}}{\partial\tau}-\lambda\tfrac{\partial n_{2d}}{\partial\xi}+\tfrac{\partial u_{2d}}{\partial\xi}+\tfrac{\partial v_{1d}}{\partial\eta}=0\\ &\lambda^2\tau_{m\circ}\tfrac{\partial^2u_{1d}}{\partial\xi^2}+\lambda\tau_{m\circ}\tfrac{\partial^2\phi_1}{\partial\xi^2}-\lambda\tfrac{\partial u_{2d}}{\partial\xi}+\tfrac{\partial u_{1d}}{\partial\tau}-\tfrac{\partial\phi_2}{\partial\xi}\\ &\quad=\eta_\circ\tfrac{\partial^2u_{1d}}{\partial\xi^2}\\ &\tfrac{\partial^2\phi_1}{\partial\xi^2}=n_{2d}+\gamma_1\phi_2+\tfrac{\mu}{1-\mu}(\sigma_i\phi_2)\\ &\qquad-\tfrac{1}{1-\mu}\left[-\phi_2-\tfrac{4(1-a)}{3\sqrt{\pi}}(-\phi_1)^{\frac{3}{2}}\right]\end{aligned}\right. \tag{19}$$

Finally from (18) and (19) yields modified KP-Burger equation

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}-A\frac{\partial}{\partial\xi}(-\phi_1)^{\frac{3}{2}}+B\frac{\partial^3\phi_1}{\partial\xi^3}-C\frac{\partial^2\phi_1}{\partial\xi^2}\right]+D\frac{\partial^2\phi_1}{\partial\eta^2}=0 \tag{20}$$

where the coefficients are

$$\begin{aligned}&A=\frac{2(1-a)}{3\sqrt{\pi}(1-\mu)}\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{-\frac{3}{2}}\\ &B=\frac{1}{2}\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{-\frac{3}{2}}\\ &C=\frac{\eta_\circ}{2},\qquad D=\frac{1}{2}\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{-\frac{1}{2}}\end{aligned} \tag{21}$$

Equation (20) shows the well known Burger equation which describes the nonlinear propagation of the DIA shock waves in a coupling dusty plasma with nonisothermal ions. The above mentioned equations for $\gamma_1=0$ is reduced to Mamun et al. (2004) in propagation in one dimension. This equation has not known exact solution, but there are some solutions in special cases. If the dissipation term is negligible compare with the nonlinearity and dispersion terms, then solitonic structure will be established by balancing the effects of dispersive and nonlinear terms. On the other hand if the coupling becomes very strong the shock waves will appear. The dissipative term changes only with changing the parameter η_0. Indeed, the dissipative term increases with increasing η_0. But the nonlinear and dispersive terms are functions of μ, σ_i and α. Equation (21) shows clearly that the nonlinear and dispersive coefficients, "A", "B" and "D", decrease when μ, σ_i and γ_1 increase.

4 Solitonic and shock waves solutions of mKP-Burger

If the coupling force is absent; i.e. ($\eta_\circ = 0$, $C = 0$) the dissipation effect is negligible in comparison with that of nonlinearity and dispersion we will have the modified KP equation

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau} - A\frac{\partial}{\partial \xi}(-\phi_1)^{\frac{3}{2}} + B\frac{\partial^3 \phi_1}{\partial \xi^3}\right] + D\frac{\partial^2 \phi_1}{\partial \eta^2} = 0 \qquad (22)$$

We introduce the variable (Moslem 2006)

$$\chi = l\xi + m\eta - u\tau \qquad (23)$$

where χ is the transformed coordinate relative to a frame which moves with the velocity u. "l" and "m" are the directional cosines of the wave vector "k" along the ξ and η respectively, in the way that $l^2 + m^2 = 1$. By integrating (22) respect to the variable χ and using the vanishing boundary condition for ϕ_1 and its derivatives up to the second-order for $|\chi| \to \infty$, we have

$$\left(\frac{d\phi_1}{d\chi}\right)^2 = \frac{1}{2}\left[\frac{lu - m^2 D}{Bl^4}\right]\phi_1^2 - \frac{2A}{5Bl^2}(-\phi_1)^{\frac{5}{2}} \qquad (24)$$

Equation (24) has solitonic solutions and one-soliton solution for this equation is given by Wang and Zhang (2006), Gill et al. (2006)

$$\phi_1 = -\phi_\circ \sec h^4\left[\frac{\chi}{W}\right] \qquad (25)$$

Where $\phi_\circ = [\frac{15(u-D)}{8Al}]^2$ is the amplitude while $w = 4\sqrt{\frac{Bl^3}{u-D}}$ is the width of the soliton.

For investigating the stability conditions of this solution, we use a method based on the energy considerations (Krall and Trivelpiece 1973). Thus we are going to find the Sagdeev potential for this situation. In order to obtain the Sagdeev potential, from (24) yield the nonlinear equation of motion as

$$\frac{1}{2}\left[\frac{d\phi_1}{d\chi}\right]^2 + V(\phi_1) = 0 \qquad (26)$$

where

$$V_{(\phi_1)} = -\frac{1}{4}\left(\frac{lu - m^2 D}{Bl^4}\right)\phi_1^2 + \frac{A}{5Bl^2}(-\phi_1)^{5/2} \qquad (27)$$

It is clear that $V(\phi_1) = 0$ and $\frac{dV(\phi_1)}{d\phi_1} = 0$ at $\phi_1 = 0$. A stable solitonic solution must satisfy the following conditions (Moslem 2006; Annou 1998)

(I) $[\frac{d^2 V}{d\phi_1^2}]_{\phi_1 = 0} < 0$.

(II) There must exists a nonzero crossing point $\phi_1 = \phi_\circ$ that $V(\phi_1 = \phi_0) = 0$.

(III) There must exists a ϕ_1 between $\phi_1 = 0$ and $\phi_1 = \phi_\circ$ to make $V(\phi_1) < 0$.

Thus, from (26) and (27) we have

$$\left.\frac{d^2 V(\phi_1)}{d\phi_1}\right|_{\phi_1 = 0} = -\frac{h}{l^4 b} < 0 \qquad (28)$$

The parameters, l and b are positive, Therefore $h > 0$ or

$$ul - m^2 D > 0 \qquad (29)$$

It is clear that the width (W) of a stable solitary wave is real.

We found that $h > 0$ and also the parameter "A" is always positive. By these conditions the term $\phi_\circ = \frac{3h}{l^2 A}$ is positive. Therefore only rarefactive solitons can be propagated.

Now let us find the stability conditions for the above solution. From the (29) we have

$$\begin{aligned} &u > \frac{m^2}{l} D \quad \text{or} \\ &u > \left(\frac{1 - l^2}{l}\right) D \end{aligned} \qquad (30)$$

If $\frac{1-l^2}{l} > 1$ then $u > D$ and when $\frac{1-l^2}{l} < 1$ we have $u < D$. Thus the soliton is stable if

$$\begin{cases} u \geq D & \text{when } 0 < l \leq 0.62 \\ 0 < u < D & \text{when } 0.62 < l < 1 \end{cases} \qquad (31)$$

We can see that the amplitude of the soliton ($\phi_\circ$) increases when "u" is increased, while its width decreases with an increasing velocity "u". On the other hand, from the definition of the soliton amplitude and its width, one can find that the amplitude (width) decreases (increases) with an increasing value for the parameter "l". This means that the parameters "u" and "l" have important roles in the stability of soliton. Thus a soliton is stable when the effects of these two phenomena cancel out each other. Finally for the case $\frac{m^2}{l} = 1$ and $u > D$, we have

$$\begin{aligned} &w = 4\sqrt{\frac{Bl^3}{u - D}}, \qquad \phi_\circ = \left[\frac{15(u - D)}{8Al}\right]^2 \\ &\phi_1 = -\phi_\circ \sec h^2\left(\frac{\chi}{w}\right) \end{aligned} \qquad (32)$$

And the potential is

$$V_{(\phi_1)} = -\frac{1}{4}\left(\frac{u - D}{Bl^3}\right)\phi_1^2 + \frac{A}{5Bl^2}(-\phi_1)^{\frac{5}{2}} \qquad (33)$$

Now we discuss on the shock wave solutions of (20). Generally shock waves appear when the coefficient "C" in (20) is not zero ($\eta_\circ \neq 0$). Indeed the coupling force is responsible for existing of shock wave solutions. By using the co-moving coordinate system that is defined with

$\chi = \xi + \eta - u\tau$ in which u is the velocity of wave and integrating with respect to the variable χ, we have

$$\frac{d^2\phi_1}{d\chi^2} + \left(\frac{D-u}{B}\right)\phi_1 - \frac{A}{B}(-\phi_1)^{\frac{3}{2}} - \frac{C}{B}\frac{d\phi_1}{d\chi} = 0 \tag{34}$$

The constant of integration has been taken equal to zero in (34). Boundary condition is set as

$$\chi \to -\infty: \quad \phi_1 = \frac{d\phi_1}{d\chi} = \frac{d^2\phi_1}{d\chi^2} = 0$$

$$\chi \to -\infty \to \phi_1 = \phi_c, \qquad \frac{d\phi_1}{d\chi} = \frac{d^2\phi_1}{d\chi^2} = 0 \quad \Rightarrow \tag{35}$$

$$\left(\frac{D-u}{B}\right)\phi_c - \frac{A}{B}(-\phi_c)^{\frac{3}{2}} = 0 \quad \Rightarrow \quad \phi_c = -\left(\frac{D-u}{A}\right)^2$$

By using $\phi_1 = \phi_c + \phi'$, for $|\phi_c| \gg |\phi'|$ (34) can be linearized as Xue and Zhang (2007)

$$\frac{d^2\phi'}{d\chi^2} - \left(\frac{D-u}{B}\right)\phi' - \frac{C}{B}\frac{d\phi'}{d\chi} = 0 \tag{36}$$

It can be seen that the solutions of (36) is proportional to $\exp(H\chi)$ where H is given by

$$H = \frac{C}{2B}\left[1 \mp \left(1 - \frac{4B(u-D)}{C^2}\right)^{\frac{1}{2}}\right] \tag{37}$$

Equation (37) indicates that for $C^2 < 4B(u-D)$ (H is imaginary) the shock wave has an oscillatory profile and for $C^2 > 4B(u-D)$ (H is real) we have a monotonic type shock wave (Xue 2003). Thus both two types of solutions can be created as

(I) For $C^2 \ll 4B(u-D)$ oscillatory shock wave solution which is given by Xue and Zhang (2007)

$$\phi_1 = -\left(\frac{u-D}{A}\right)^2 + \phi_\circ \exp\left(-\frac{C}{2B}\chi\right) \times \cos\left(\sqrt{\frac{u-D}{B}}\chi\right) \tag{38}$$

where ϕ_0 is a constant. Generally ϕ_0 has been chosen equal to $(-\phi_c)$.

(II) For $C^2 > 4B(u-D)$ monotonic shock wave solution which is given by Moslem (2006)

$$\phi_1 = \left(\frac{u-D}{A}\right)\left[1 - \tan h\left(\left(\frac{u-D}{2C}\right)\chi\right)\right] \tag{39}$$

Comparing (25) with (38) and (39) one can find that the amplitude of both solutions are the same functions of medium parameters. Equations (38) and (39) correspond to the dust acoustic shock waves in two cases (oscillatory shock wave and monotonic shock wave). The nature of these shock structures depends on the relative values between the dispersive and dissipative coefficients B and C, respectively. In this case, when C is very small, the first few oscillations at the wave front will be close to solitary waves, thus the oscillating shock waves can be appeared. But if the value of C is large, the motion of the wave will give monotonic shock waves. It is obvious that amplitude of both oscillatory and monotonic shock waves are depend on coefficient of nonlinear term, "A". The amplitude of shock waves increases when σ_i increases. This means that the amplitude of shock waves in the medium increases when the temperature of ions increases or the temperature of electrons decreases. Also the amplitude of this type of solution increases with an increasing μ. Therefore the amplitude increases when the density of ions increases (or the density of electrons decreases). The amplitude of shock waves increases when the temperature of free ions (which is modeled by parameter a) increases. We see that A increases with an increasing γ_1. Thus we can conclude that amplitude of waves increases when dust charge increases. Also, it is found that the strength of shock waves is depending on η_0, such that oscillatory shock wave can be reduced to monotonic shock wave with increasing η_0.

5 Numerical solve

It is well known that shock waves can be described by means of the Burgers term. In the preceding section, we have seen that the parameter $C = \frac{\eta_\circ}{2}$, which is due to the coupled dust particles, does not play any role in our analysis of DIA solitary waves. In order to study the transition from solitary to shock waves, we can start out from the KdV-Burger equation and use the initial solitonic solution in our model. Thus we choose $\phi_1 = -\phi_\circ \sec h^4[\frac{\zeta}{W}]$ where $\zeta = \xi + \eta$ with $\phi_\circ = 0.24$ and $W = 4.6$ and $u = 2$. Now we can study the effect of time on the solitons with different values of C. The following figures explore the differences in solitary and shock wave structure, due to variation of viscosity coefficient. Figure 1 show the initial soliton or pulse after some time breaks up into a group of solitary waves ($C = 0$). It is obvious only solitary waves may be appearing in this plasma. In this case, the amplitude of solitary waves increases with increasing time. It is also observed that the solitary waves.

Figures 2 and 3 show how solitary waves are converted into shock-like structures when one accounts for the effect of correlation among the dust particles. In Fig. 2 we have study a weakly coupled dusty plasma ($C = 0.2$). These figures show that the system develops shock-like structures with oscillations close to the shock front. Figure 3 shows the effect of viscosity on the structure of wave for $C = 2$. These figures show how solitary waves are reduced into monotonic shock structures in the dusty plasma with strong correlation

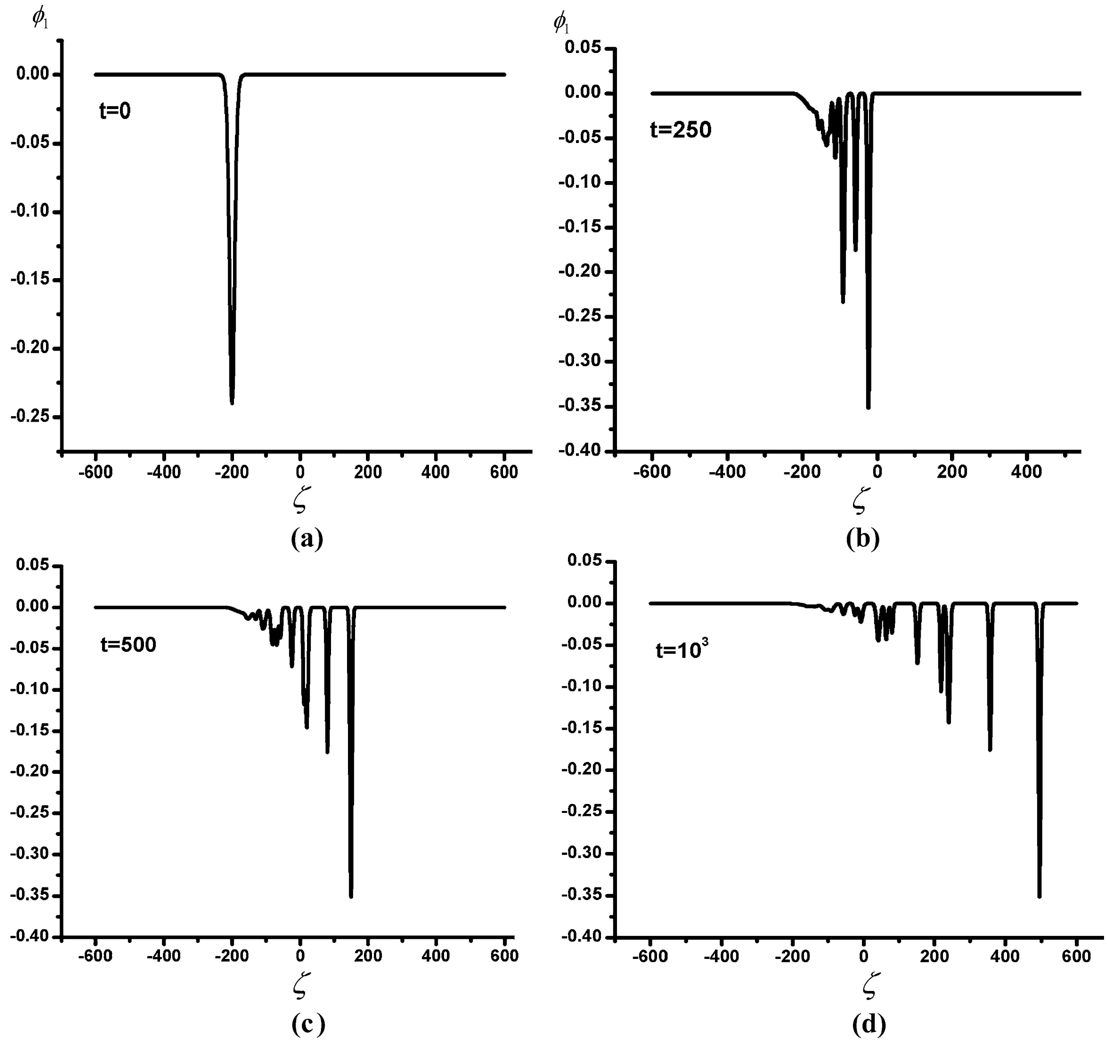

Fig. 1 The solitary waves for $C = 0$

among the dust particles that are in a liquid phase. Both strength and steepness of the shock increase by increasing the viscosity. The transition from a solitary wave to oscillatory or monotonic shock wave depends on the magnitude of the dissipation coefficient C. With the stronger dissipation, the shock wave structure becomes steeper (monotonic-type) and for weaker dissipation the shock wave has an oscillatory behavior.

6 Reduce to KP-Burger equation

The parameter "**A**" approaches zero when $a \to 1$. Thus the amplitude of solitary and shock waves increases to infinity in this case. In this case we use (6) but with a new set of stretching coordinate as follows

$$\begin{aligned} &\xi = \varepsilon^{1/2}(x - \lambda t), \qquad \tau = \varepsilon^{3/2} t, \qquad \eta = \varepsilon y \\ &\eta_1 = \varepsilon^{1/2}\eta_\circ, \qquad \tau_m = \varepsilon^{1/2}\tau_{m\circ} \end{aligned} \tag{40}$$

$$\begin{cases} n_d = 1 + \varepsilon n_{1d} + \varepsilon^{3/2} n_{2d} + \varepsilon^2 n_{3d} + \cdots \\ u_d = \varepsilon u_{1d} + \varepsilon^{3/2} u_{2d} + \varepsilon^2 u_{3d} + \cdots \\ v_d = \varepsilon^{5/4} v_{1d} + \varepsilon^{7/4} v_{2d} + \varepsilon^{9/4} v_{3d} + \cdots \\ \phi = \varepsilon\phi_1 + \varepsilon^{3/2}\phi_2 + \varepsilon^2\phi_3 + \cdots \\ Z_d = 1 + \varepsilon\gamma_1\phi_1 + \varepsilon^{3/2}\gamma_1\phi_2 + \varepsilon^2(\gamma_1\phi_3 + \gamma_2\phi_1) \end{cases} \tag{41}$$

Substituting the above expansions into (1) and collecting different order of ε, after some calculate we derive the following equation

$$\begin{aligned} &\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau} + E\phi_1\frac{\partial\phi_1}{\partial\xi} - F\frac{\partial}{\partial\xi}(-\phi_1\phi_2) \right. \\ &\quad \left. + G\frac{\partial^3\phi_1}{\partial\xi^3} - H\frac{\partial^2\phi_1}{\partial\xi^2}\right] + P\frac{\partial^2\phi_1}{\partial\eta^2} = 0 \end{aligned} \tag{42}$$

where

$$E = \frac{1}{2}\left(\gamma_1 + \frac{\mu\sigma_i + 1}{1-\mu}\right)^{-\frac{3}{2}}\left[\frac{(1-\mu)(1-\mu\sigma_i^2)}{(1+\mu\sigma_i)^2} - 2\gamma_2\right]$$

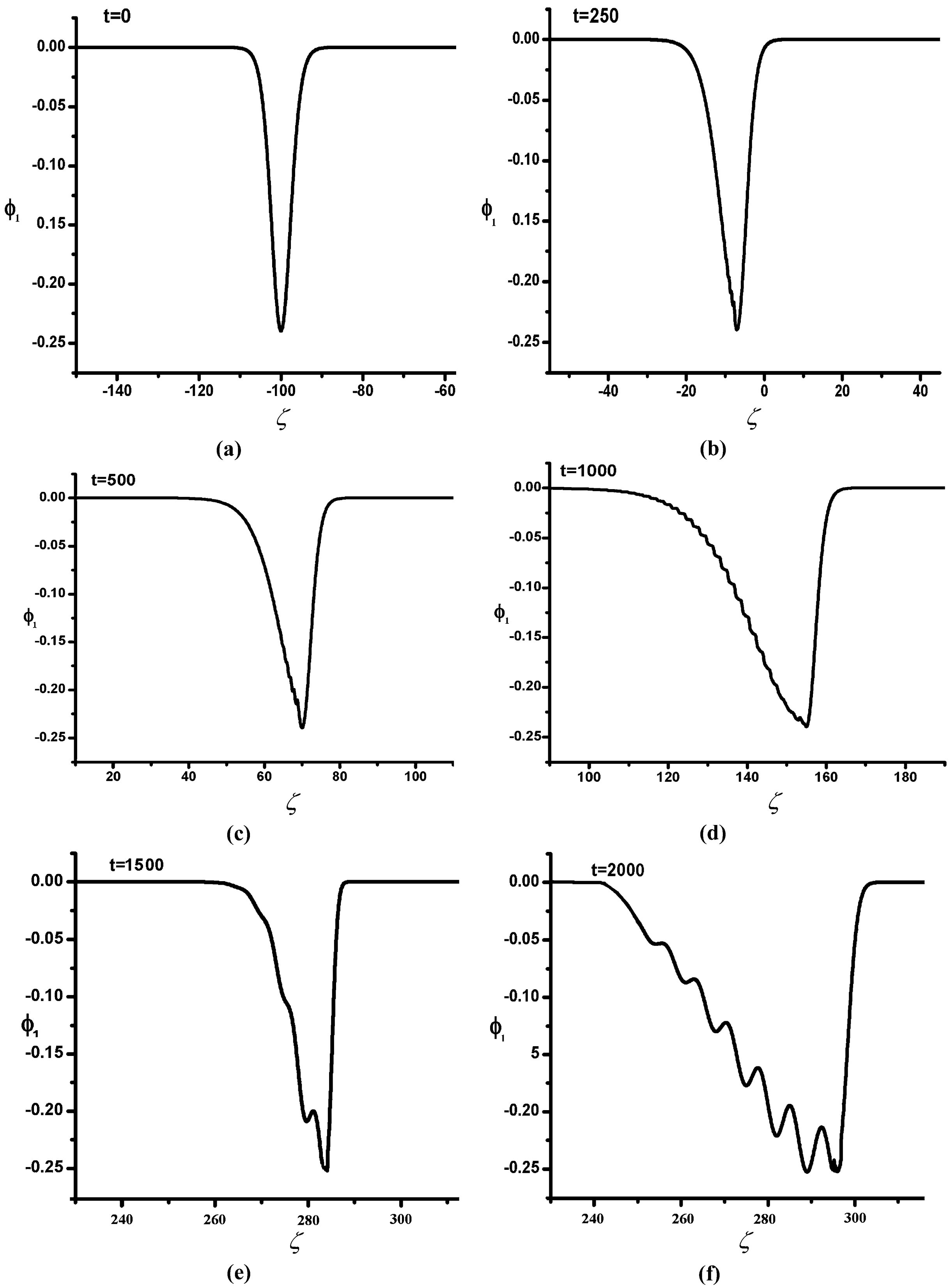

Fig. 2 The oscillatory shock waves for $C = 0.2$

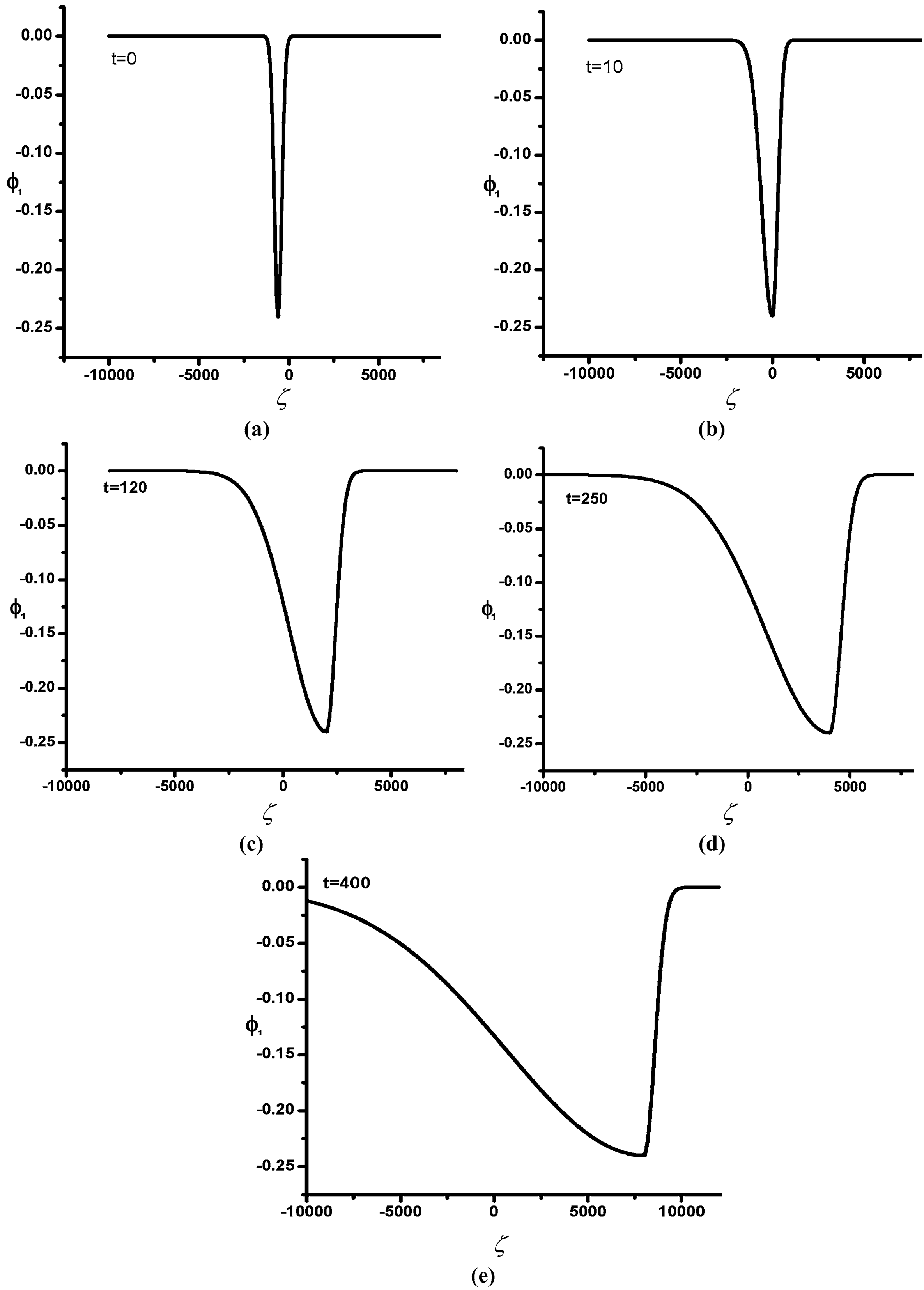

Fig. 3 The monotonic shock waves for $C = 2$

$$+\frac{3}{2}\gamma_1\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{-\frac{1}{2}}-\frac{1}{2}\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{\frac{1}{2}}$$

$$F=\frac{2(1-a)}{3\sqrt{\pi}(1-\mu)}\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{-\frac{3}{2}} \tag{43}$$

$$G=\frac{1}{2}\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{-\frac{3}{2}}, \qquad H=\frac{\eta_\circ}{2}$$

$$P=\frac{1}{2}\left(\gamma_1+\frac{\mu\sigma_i+1}{1-\mu}\right)^{-\frac{1}{2}}$$

For $a\to 1$ we have $F=0$ and (20) reduce to a KP-Burger equation as follows

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+E\phi_1\frac{\partial\phi_1}{\partial\xi}+G\frac{\partial^3\phi_1}{\partial\xi^3}-H\frac{\partial^2\phi_1}{\partial\xi^2}\right]+P\frac{\partial^2\phi_1}{\partial\eta^2}=0 \tag{44}$$

7 Solitonic and shock waves solutions of KP-Burger

For weakly coupled dusty plasma, that is the dissipation effect is negligible in comparison with that of nonlinearity and dispersion we will have the KP equation

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+E\phi_1\frac{\partial\phi_1}{\partial\xi}+G\frac{\partial^3\phi_1}{\partial\xi^3}\right]+P\frac{\partial^2\phi_1}{\partial\eta^2}=0 \tag{45}$$

The solitary solution for (45) can be written as Gill et al. (2006)

$$\phi=\phi_\circ\sec h^2\left(\frac{\xi+\eta-u\tau}{w}\right) \tag{46}$$

In which $\phi_\circ=\frac{3(u-P)}{E}$ and $w=2\sqrt{\frac{G}{(u-P)}}$ are the soliton amplitude and its width, respectively. Note that for $\mu=\mu_c$ both compressive and rarefactive of solitary waves can be exist.

Equation (44) also has shock wave solutions as Xue and Zhang (2007), Moslem (2006).

(i) Monotonic shock wave:

$$\phi_1=\frac{2(u-P)}{E}\left[1-\tan h\left(-\frac{(u-P)}{G}\chi\right)\right] \quad \text{for } G^2>4E(u-P) \tag{47}$$

(ii) Oscillatory shock wave:

$$\phi_1=\frac{2(u-P)}{E}+\phi_0\exp\left(\frac{H}{2G}\chi\right)\times\cos\left(\sqrt{\frac{2(u-P)}{G}}\chi\right) \quad \text{for } G^2\ll 4E(u-P) \tag{48}$$

where ϕ_0 is a constant.

8 Conclusion

In this paper, we studied propagating of nonlinear waves in unmagnetized and strongly coupled dusty plasma containing nonthermal ions, Boltzmann distributed electrons and variable dust charge. We derived mKP-Burger and discussed on solitonic solutions and also shock waves. As long as the dispersive term and the dissipative term as well as the nonlinear term are balanced, the shock wave (both monotonic and oscillatory types) structure forms; otherwise, the soliton forms due to the balance between the dispersive term and the nonlinear term. The dissipation caused by the dust-dust correlation also has an effect on the amplitude of the shock wave. With the strong dissipation, the shock wave structure becomes steeper (monotonic) and for weak dissipation the shock wave structure has oscillatory behavior. Propagation of monotonic and oscillatory shock waves in the dusty plasma depend on the plasma parameters and it can shown by numerical computation. We have also numerically examined the salient features of dust-acoustic solitary and shock waves that may exist in the plasma. We used the initial solitonic solution and show the shock wave structure (oscillatory and monotonic) can be appear for weak and strong coupling between the dust particles. For modified KP-Burger, neither solitonic solution nor shock waves cannot be found when "A" becomes zero. In this case amplitude of waves becomes infinity and we derived KP-Burger equation. This equation has stable solitary and shock waves solutions.

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