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ORIGINAL ARTICLE

Arbitrary amplitude electron-acoustic solitary waves in the presence of excess superthermal electrons

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Abstract The problem of arbitrary amplitude electron-acoustic solitary (EAS) waves in a plasma having cold fluid electrons, hot superthermal electrons and stationary ions is addressed. The domain of their allowable Mach numbers enlarges as the spectral index κ increases revealing therefore that the "maxwellisation" process of the hot component favors the propagation of the EAS waves. As the superthermal character of the plasma is increased, the potential pulse amplitude increases while its width is narrowed, i.e, the superthermal effects makes the electron-acoustic solitary structure more spiky. As the spectral index κ decreases, the hot electrons are locally expelled and pushed out of the region of the soliton's localization. A decrease of the fractional number density of the hot electrons relative to that of the cold ones number density would lead to an increase of the depth as well as the width of the localized EAS wave. Our results should help to understand the salient features of large amplitude localized structures that may occur in the plasma sheet boundary layer and may provide an explanation for the strong spiky waveforms that have been observed in auroral electric fields.

Keywords Electron-acoustic waves · Solitary waves · Pseudo-potential analysis · Superthermal electrons · Plasma sheet boundary layer

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1 Introduction

The electron-acoustic wave, which is one of the basic wave processes in plasmas, is a high-frequency (in comparison with the ion plasma frequency) wave that occurs in a plasma having, in addition to positively charged ions, two electron components with widely disparate temperatures (Watanabe and Taniuti 1977; Tokar and Gary 1984; Gary and Tokar 1985). The relatively cold inertial electrons oscillate against a thermalized background of inertialess hot electrons providing the necessary restoring force. Electron acoustic waves may also exist in an electron-ion plasmas with ions hotter than electrons (Fried and Gould 1961). During the last decade, there has been a vast body of theoretical literature on electron-acoustic waves to explain the space observations of solitary waves with either negative (Singh and Lakhina 2004; Tagare et al. 2004) or positive potentials (Berthomier et al. 2000; Mace and Hellberg 2001; Berthomier et al. 2003). Attempts have been made to explain the broadband electrostatic noise (a common wave activity in the plasma sheet boundary layer of the Earth's magnetotail region) as being solitary electron-acoustic structures with negative potential in two-temperature electron plasma (Mace et al. 1991). More recently, Verheest et al. (2005) demonstrated that the inclusion of the hot electron inertia can lead to compressive electron-acoustic solitons and may render the analysis much more intricate (Verheest et al. 2007). There are several other papers dealing with electron-acoustic waves (Bharuthram and Shukla 1988; Bharuthram 1993; Singh and Lakhina 2001; Cattaert et al. 2005; Gill et al. 2007; Lakhina et al. 2008a, 2008b; Pottelette and Berthomier 2009; Pakzad and Tribeche 2010). However, numerous observations of space plasmas (Scudder et al. 1981; Marsch et al. 1982) indicate clearly the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. The latter may arise due to the effect of external forces acting on the natural space environment plasmas or to the wave–particle interaction which ultimately leads to κ-like distributions. As a consequence, a high-energy tail appears in the distribution function of the particles. The aim of the present paper is therefore to show the existence of arbitrary amplitude electron-acoustic solitary potentials in a plasma having stationary ions, cold inertial electrons and hot superthermal electrons. It is worth to note that some theoretical work focused on the effects of superthermal particles on different types of linear and nonlinear collective processes in plasmas (Mann et al. 1998; Barghouthi et al. 2001; Leubner 2002; Kumar and Sikka 2007; Shizgal 2007; Aoutou et al. 2008; Younsi and Tribeche 2008; Baluku and Hellberg 2008; Tribeche et al. 2009; Saini et al. 2009; Mace and Hellberg 2009; Aoutou et al. 2009; Tribeche and Boubakour 2009; Hellberg et al. 2009; Boubakour et al. 2009).

2 Theoretical model and basic equations

Let us consider a collisionless unmagnetized plasma with cold fluid electrons, hot superthermal kappa-distributed electrons and stationary ions of density n_c, n_h, and n_i, respectively. Thus, at equilibrium, we have $n_{c0} + n_{h0} = n_{i0}$ or $\alpha = n_{h0}/n_{c0} = n_{i0}/n_{c0} - 1$, where the subscript "0" stands for unperturbed quantities. The dynamics of one-dimensional electron-acoustic oscillations is governed by the following adimensional equations

$$\frac{\partial N_c}{\partial T} + \frac{\partial (N_c U_c)}{\partial X} = 0 \tag{1}$$

$$\frac{\partial U_c}{\partial X} + U_c \frac{\partial U_c}{\partial X} = \alpha \frac{\partial \Psi}{\partial X} \tag{2}$$

$$\frac{\partial^2 \Psi}{\partial X^2} = \frac{N_c}{\alpha} + N_h - \left(1 + \frac{1}{\alpha}\right) \tag{3}$$

Here and in the following, $j = c, h, i$ stands for cold electrons, hot electrons, and ions, respectively, e is the elementary charge, m_j are the masses, and T_j the temperatures. The electrostatic potential Ψ, the cold electron fluid velocity U_c, and the particle densities N_j are normalized by T_h/e, $C_e = (T_h/\alpha m_e)^{1/2}$, and n_{j0}, respectively. The time and space variables are in units of the cold electron plasma period $\omega_{pc}^{-1} = (m_e/4\pi n_{c0} e^2)^{1/2}$ and the hot electron Debye length $\lambda_{Dh} = (T_h/4\pi n_{h0} e^2)^{1/2}$, respectively. To model the effects of the hot superthermal electrons, we refer to an appropriate Bernstein-Greene-Kruskal solution that solves the collisionless Vlasov equation. Thus, we choose (Thorne and Summers 1991)

$$f_h(v_x) = C_h \left(1 - \frac{e\phi}{\kappa m_e \theta_{\text{the}}^2} + \frac{v_x^2}{2\kappa \theta_{\text{the}}^2}\right)^{-\kappa - 1} \tag{4}$$

where the normalization is provided for any value of the spectral index $\kappa > 1/2$ by

$$C_h = \frac{n_{e0}}{(2\pi \kappa \theta_{\text{the}}^2)^{1/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa + 1/2)} \tag{5}$$

Here, the parameter κ shapes predominantly the superthermal tail of the distribution, the quantity Γ stands for the standard gamma function, and

$$\theta_{\text{the}} = \left(\frac{\kappa - 1/2}{\kappa} \frac{T_h}{m_e}\right)^{1/2} \tag{6}$$

In the limit $\kappa \to \infty$, distribution (4) reduces to the well known Maxwell-Boltzmann velocity distribution. Integrating $f_h(v_x)$ over all velocity space, we get

$$N_h = \frac{n_h}{n_{h0}} = \left(1 - \frac{\Psi}{\kappa - 1/2}\right)^{-\kappa - 1/2} \tag{7}$$

To study the time-independent arbitrary amplitude electron-acoustic solitary waves, we assume that all the dependent variables in (1)–(3) depend only on a single variable $\xi = X - MT$ (where again ξ is normalized by λ_{Dh} and $M =$ solitary wave speed$/C_e$). Now, under the appropriate boundary conditions, viz., $\Psi \longrightarrow 0$, $U_c \longrightarrow 0$, and $N_c \longrightarrow 1$ at $\xi \longrightarrow \pm\infty$, Equations (1) and (2) can be integrated to give

$$N_c = \frac{1}{\sqrt{1 + 2\alpha\Psi/M^2}} \tag{8}$$

Substituting for N_c from (8) into Poisson's equation (3), and multiplying both sides of the resulting equation by $d\Psi/d\xi$, integrating once, and imposing the appropriate boundary conditions for localized solutions, namely, $\Psi \longrightarrow 0$ and $d\Psi/d\xi \longrightarrow 0$ at $\xi \longrightarrow \pm\infty$, we obtain the quadrature

$$\frac{1}{2}\left(\frac{d\Psi}{d\xi}\right)^2 + V(\Psi) = 0 \tag{9}$$

where the Sagdeev potential (Sagdeev 1966) for our purposes reads as

$$V(\Psi) = -\frac{M^2}{\alpha^2}\left\{\left(1 + \frac{2\alpha\Psi}{M^2}\right)^{1/2} - 1\right\} - \left(1 - \frac{\Psi}{\kappa - \frac{1}{2}}\right)^{-\kappa + 1/2} + \left(1 + \frac{1}{\alpha}\right)\Psi + 1 \tag{10}$$

Equation (9) can be regarded as an "energy integral" of an oscillating particle of unit mass, with a velocity $d\Psi/d\xi$

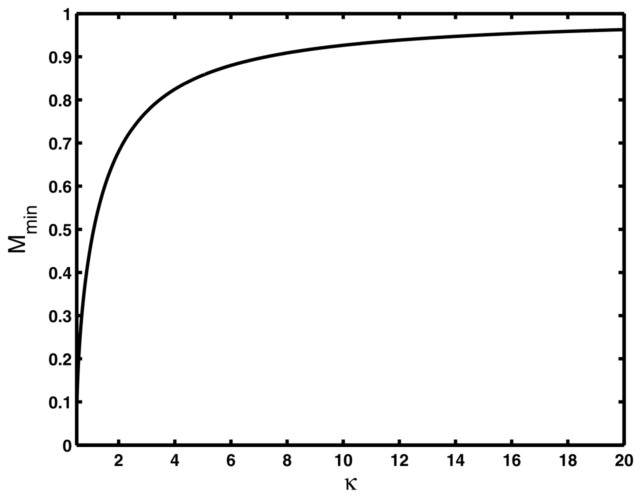

Fig. 1 Plot of the lower limit $M_{\min}$ of the allowable Mach numbers versus the spectral index κ [see (12)]

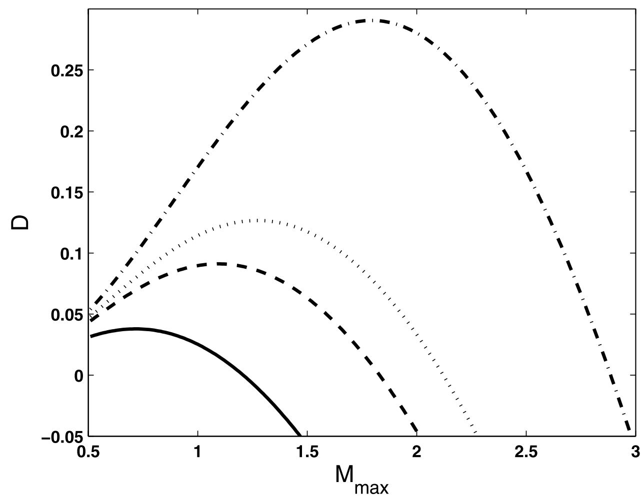

Fig. 2 Plot of D versus $M_{\max}$ for different values of the spectral index $\kappa = 0.6$ (*solid line*), 0.8 (*dashed line*), and 1 (*dotted line*) [see (13)], with $\alpha = 1.5$. The Maxwell-Boltzmann case ($\kappa \to \infty$) is represented by a *dash-dotted line*

and position Ψ in a potential $V(\Psi)$. Prior to the numerical integration, it is instructive to discuss the conditions under which (9) leads to soliton solutions by analyzing the Sagdeev potential (10). It is clear from (10) that $V(\Psi) = 0$ and $dV(\Psi)/d\Psi = 0$ at $\Psi = 0$. Solitary wave solutions of (9) exist if (i) $(d^2V/d\Psi^2)_{\Psi=0} < 0$, so that the fixed point at the origin is unstable; (ii) there exists a nonzero Ψ_m at which $V(\Psi_m) = 0$; and (iii) $V(\Psi) < 0$ when Ψ lies between 0 and Ψ_m. The second condition simply means that a quasiparticle of zero total energy will be reflected at the position $\Psi = \Psi_m$. The third condition means that V has to be a potential trough in which the quasiparticle can be trapped and experience oscillations. Condition (i) for the existence of localized structures requires the Mach number to satisfy (recall that $\kappa > 1/2$)

$$M^2 > \frac{\kappa - 1/2}{\kappa + 1/2} \tag{11}$$

It is clear that the lower limit of M

$$M_{\min} = \left(\frac{\kappa - 1/2}{\kappa + 1/2}\right)^{1/2} \tag{12}$$

is smaller than the lower limit in a plasma without superthermal electrons ($\kappa \to \infty$), $M_{\min} = 1$ (see Fig. 1). The upper limit of M can be found by the condition $V(\Psi_c) \geq 0$, where $\Psi_c = -M^2/2\alpha$ is the minimum value of Ψ for which the cold electron density N_c is real. Thus, we have

$$D(M_{\max}, \alpha, \kappa) = 1 + M_{\max}^2 \left(\frac{1-\alpha}{2\alpha^2}\right) - \left(1 + \frac{M_{\max}^2/2\alpha}{\kappa - 1/2}\right)^{-\kappa+1/2} \geq 0 \tag{13}$$

For Boltzmann distributed hot electrons ($\kappa \to \infty$), (13) reduces to the following transcendental relation

$$1 + M_{\max}^2 \left(\frac{1-\alpha}{2\alpha^2}\right) - \exp\left(-\frac{M_{\max}^2}{2\alpha}\right) \geq 0 \tag{14}$$

Keeping α at a constant value, the effect of the hot electrons superthermality on the allowable Mach numbers is investigated. Interestingly, one finds that the effect of increasing the spectral index κ is to shift the upper limit of M toward higher values, enlarging therefore the domain of allowable Mach numbers (ΔM in what follows) as can be seen on Fig. 2. This domain passes from $0.301 < M < 1.207$ ($\Delta M = 0.906$) for $\kappa = 0.6$, to $0.480 < M < 1.803$ ($\Delta M = 1.323$) for $\kappa = 0.8$, to $0.577 < M < 2.121$ ($\Delta M = 1.544$) for $\kappa = 1$, to $1 < M < 2.909$ ($\Delta M = 1.909$) for Maxwell-Boltzmann distributed hot electrons. Therefore, one can conclude that the "maxwellisation" process of the hot superthermal component (Brodsky et al. 1988) favors the propagation of electron-acoustic solitary waves.

We now proceed with the presentation of our numerical results. Equation (9) is integrated numerically assuming the initial value $\Psi_0 = \Psi(\xi = 0) = 0$ and a small edge electric field $E_0 = -(\frac{d\Psi}{d\xi})(\xi = 0) = -10^{-12}$. For the sake of comparison, we have plotted the spatial variation of Ψ for various values of the spectral index $\kappa = 1$, 1.5, and 2 in Fig. 3. The electrostatic potential exhibits spatially localized electron-acoustic structures as is evident from the well structure of the Sagdeev potential in Fig. 4. The following parameters $\alpha = 0.5$ and $M = 1.2$ have been chosen. It can be seen that as κ decreases, i.e, the superthermal character of the plasma is increased, the potential pulse amplitude increases while its width is narrowed, i.e, the superthermal

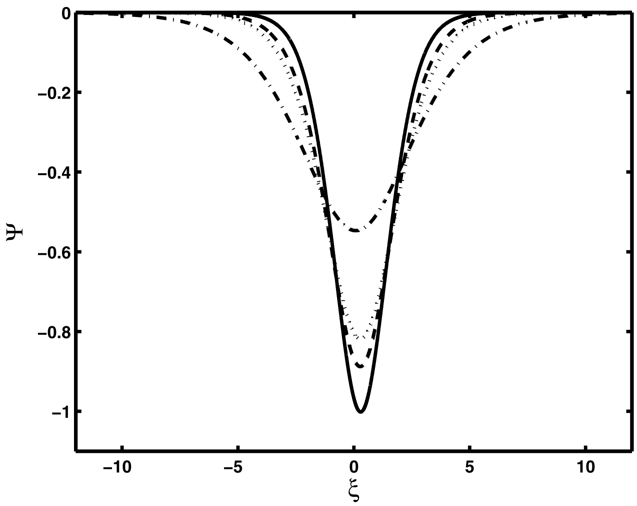

Fig. 3 Soliton-like solution for the electrostatic potential Ψ for different values of the spectral index $\kappa = 1$ (*solid line*), 1.5 (*dashed line*), and 2 (*dotted line*), with $\alpha = 0.5$ and $M = 1.2$. The Maxwell-Boltzmann case ($\kappa \to \infty$) is represented by a *dash-dotted line*

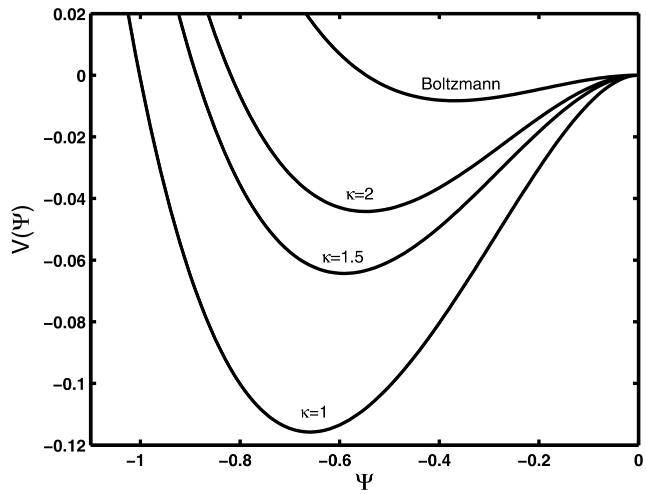

Fig. 4 Plot of the Sagdeev potential associated with the nonlinear localized structure of Fig. 3

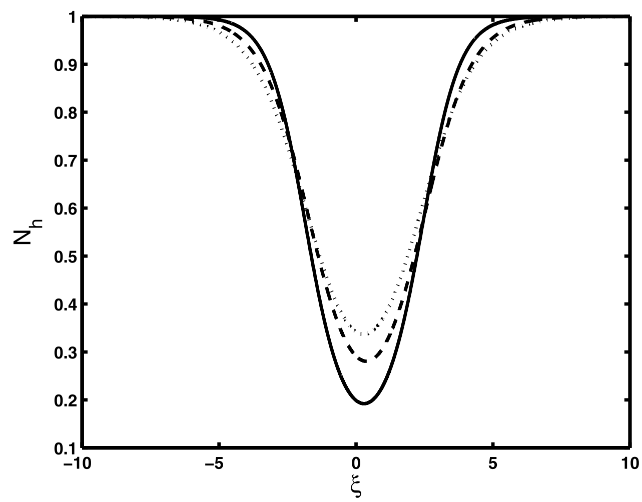

Fig. 5 Spatial profile of the hot electron density for different values of the spectral index $\kappa = 1$ (*solid line*), 1.5 (*dashed line*), and 2 (*dotted line*), with $\alpha = 0.5$ and $M = 1.2$

effects makes the electron-acoustic solitary structure more spiky. Consequently, cusped electron-acoustic solitons may arise as the electrons evolve far away from their thermodynamic equilibrium. This may be attributed to the fact that as the spectral index κ decreases, the hot electrons are locally expelled (Fig. 5) and pushed out of the region of the soliton's localization, a phenomenon that can generate an intense and spiky electric field in the medium. Keeping M and κ at constant values, Fig. 6 indicate that a decrease of the fractional number density of the hot electrons relative to that of the cold ones number density ($\alpha = n_{h0}/n_{c0} \to 0$) would lead to an increase of the depth as well as the width of the localized electron-acoustic solitary wave. Our findings are in good agreement with recently published results (Pakzad 2009a, 2009b, 2010; Alinejad 2010a, 2010b), in the sense that a departure from thermodynamic equilibrium not only affects the salient features of the localized solitary structures but also modifies the domain of their admissible Mach numbers.

3 Weak amplitude analysis

The nature of small-amplitude, $|\Psi| \ll 1$, electron-acoustic localized structures may be obtained explicitly. Expanding in Ψ and retaining leading-order terms, we find that (10) reduces to

$$V(\Psi) = A_1\Psi + A_2\Psi^2 + A_3\Psi^3 + O(\Psi^4) \tag{15}$$

where

$$A_1 = 0$$

$$A2 = \frac{2\kappa+1}{2(2\kappa-1)M^2} - \frac{2\kappa+1}{2(2\kappa-1)} \tag{16}$$

$$A_3 = \frac{12\alpha\kappa - 12\alpha\kappa^2 - 3\alpha}{6(2\kappa-1)^2 M^4} - \frac{4\kappa^2+8\kappa+3}{6(2\kappa-1)^2}$$

Performing the last step in deriving soliton solutions, namely, solving the energy equation (15), one gets

$$\Psi(\xi) = \Psi_m \mathrm{Sech}^2\left(\frac{\xi - \xi_0}{\Delta}\right) \tag{17}$$

where $\Psi_m = -A_2/A_3$, and $\Delta = (-2/A_2)^{1/2}$ represent, respectively, the amplitude and the width of the localized soliton. Solution (17) represents a small-amplitude stationary electron-acoustic solitary wave provided $A_2 < 0$ or $M > 1$.

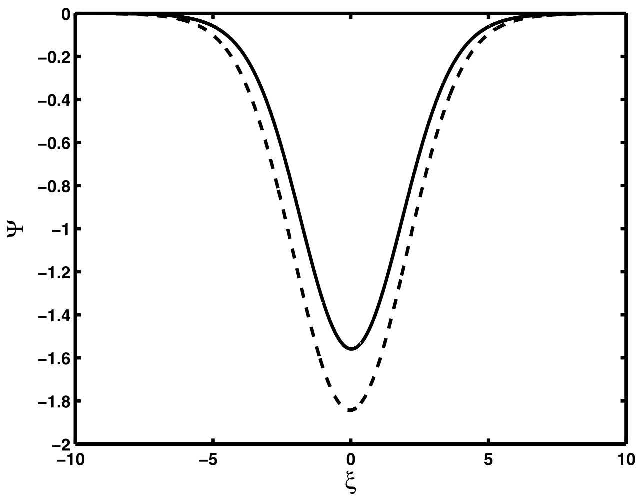

Fig. 6 Soliton-like solution for the electrostatic potential Ψ for two different values of the density ratio $\alpha = 0.01$ (*dashed line*) and 0.1 (*solid line*), with $\kappa = 1.5$ and $M = 1.2$

Now retaining terms up to Ψ^4, (10) may be expanded as

$$V(\Psi) = A_1\Psi + A_2\Psi^2 + A_3\Psi^3 + A_4\Psi^4 + O(\Psi^5) \tag{18}$$

where

$$A_4 = \frac{120\alpha^2\kappa^3 - 180\alpha^2\kappa^2 + 90\alpha^2\kappa - 15\alpha^2}{24M^6(2\kappa-1)^3} - \frac{8\kappa^3 + 36\kappa^2 + 46\kappa + 15}{24(2\kappa-1)^3} \tag{19}$$

Double layers solutions of (18) exist if (i) $V(0) = V(\Psi_m) = 0$; (ii) $(dV/d\Psi)_{\Psi=0} = (dV/d\Psi)_{\Psi=\Psi_m} = 0$; and $(d^2V/d\Psi^2)_{\Psi=0,\Psi_m} < 0$, where 0 and Ψ_m are two extreme points of the Sagdeev potential $V(\Psi)$. Applying the first two boundary conditions, we obtain $2\Psi_m = -A_3/A_4$ and $V(\Psi)$ can be rewritten as $V(\Psi) = A_4\Psi^2(\Psi_m - \Psi)^2$. Performing the last step in deriving double layer solution, one gets

$$\Psi(X) = \frac{\Psi_m}{2}\left[1 - \tanh\left(\frac{2\xi}{\Delta}\right)\right] \tag{20}$$

where $\Delta = \sqrt{-8/A_4}/|\Psi_m|$ represents the width of the double layer provided $A_4 < 0$, i.e,

$$\frac{15(8\alpha^2\kappa^3 - 12\alpha^2\kappa^2 + 6\alpha^2\kappa - \alpha^2)}{8\kappa^3 + 36\kappa^2 + 46\kappa + 15} < M^6 \tag{21}$$

4 Conclusion

To conclude, we have addressed the problem of arbitrary amplitude electron-acoustic solitary (EAS) waves in a plasma having cold fluid electrons, hot superthermal electrons and stationary ions. Our results show that in a such plasma, localized EAS structures can exist. The domain of their allowable Mach numbers enlarges as the spectral index κ increases revealing therefore that the "maxwellisation" process of the hot component favors the propagation of the EAS waves. As the superthermal character of the plasma is increased, the potential pulse amplitude increases while its width is narrowed, i.e, the superthermal effects makes the electron-acoustic solitary structure more spiky. Consequently, cusped electron-acoustic solitons may arise as the electrons evolve far away from their thermodynamic equilibrium. This may be attributed to the fact that as the spectral index κ decreases, the hot electrons are locally expelled and pushed out of the region of the soliton's localization, a phenomenon that can generate intense and spiky electric fields in the medium. A decrease of the fractional number density of the hot electrons relative to that of the cold ones number density would lead to an increase of the depth as well as the width of the localized electron-acoustic solitary wave. Considering the wide relevance of nonlinear oscillations, our results should help to understand the salient features of large amplitude localized structures that may occur in the plasma sheet boundary layer (Sarafopoulos et al. 1997) and may provide an explanation for the strong spiky waveforms that have been observed in auroral electric fields (Ergun et al. 1998).

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References

Alinejad, H.: Astrophys. Space Sci. **325**, 209 (2010a)
Alinejad, H.: Astrophys. Space Sci. **327**, 131 (2010b)
Aoutou, K., Tribeche, M., Zerguini, T.H.: Phys. Plasmas **15**, 013702 (2008)
Aoutou, K., Tribeche, M., Zerguini, T.H.: Phys. Plasmas **16**, 083701 (2009)
Baluku, T.K., Hellberg, M.A.: Phys. Plasmas **15**, 123705 (2008)
Barghouthi, I.A., Pierrard, V., Barakat, A.R., Lemaire, J.: Astrophys. Space Sci. **277**, 427 (2001)
Berthomier, M., Pottelette, R., Malingre, M., Khotyaintsev, Y.: Phys. Plasmas **7**, 2987 (2000)
Berthomier, M., Pottelette, R., Muschietti, L., Roth, I., Carlson, C.: Geophys. Res. Lett. **30**, 2148 (2003)
Bharuthram, R.: Astrophys. Space Sci. **202**, 337 (1993)
Bharuthram, R., Shukla, P.K.: Astrophys. Space Sci. **149**, 127 (1988)
Boubakour, N., Tribeche, M., Aoutou, K.: Phys. Scr. **79**, 065503 (2009)
Brodsky, Y., Nechuev, S.I., Slutsker, Y.Z., Fraiman, G.M.: Fiz. Plazmy **14**, 582 (1988)
Cattaert, T., Verheest, F., Hellberg, M.A.: Phys. Plasmas **12**, 042901 (2005)
Ergun, R., Carlson, B., McFadden, M.: Geophys. Res. Lett. **25**, 2025 (1998)
Fried, B.D., Gould, R.W.: Phys. Fluids **4**, 139 (1961)
Gary, S.P., Tokar, R.L.: Phys. Fluids **28**, 2439 (1985)
Gill, T.S., Kaur, H., Bansal, S., Saini, N.S., Bala, P.: Eur. Phys. J. D **41**, 151 (2007)

Hellberg, M.A., Mace, R.L., Baluku, T.K., Kourakis, I., Saini, N.S.: Phys. Plasmas **16**, 094701 (2009)

Kumar, N., Sikka, H.: Astrophys. Space Sci. **312**, 193 (2007)

Lakhina, G.S., Singh, S.V., Kakad, A.P., Verheest, F., Bharuthram, R.: Nonlinear Process. Geophys. **15**, 903 (2008a)

Lakhina, G.S., Kakad, A.P., Singh, S.V., Verheest, F.: Phys. Plasmas **15**, 062903 (2008b)

Leubner, M.P.: Astrophys. Space Sci. **282**, 573 (2002)

Mace, R.L., Hellberg, M.A.: Phys. Plasmas **8**, 2649 (2001)

Mace, R.L., Hellberg, M.A.: Phys. Plasmas **16**, 072113 (2009)

Mace, R.L., Baboolal, S., Bharuthram, R., Hellberg, M.A.: J. Plasma Phys. **45**, 323 (1991)

Mann, G., Classen, H.-T., Motschmann, U., Kunow, H., Dröge, W.: Astrophys. Space Sci. **264**, 489 (1998)

Marsch, E., Muhlhauser, K.H., Schwenn, R., Rosenbauer, H., Pillip, W., Neubauer, F.M.: J. Geophys. Res. **87**, 52 (1982)

Pakzad, H.R.: Astrophys. Space Sci. **324**, 41 (2009b)

Pakzad, H.R.: Astrophys. Space Sci. **323**, 345 (2009a)

Pakzad, H.R.: Astrophys. Space Sci. **326**, 69 (2010)

Pakzad, H.R., Tribeche, M.: Astrophys. Space Sci. (2010, in press). doi:10.1007/s10509-010-0367-1

Pottelette, R., Berthomier, M.: Nonlinear Process. Geophys. **16**, 373 (2009)

Sagdeev, R.Z.: In: Leontovich, M.A. (ed.) Reviews of Plasma Physics, vol. 4, p. 23. Consultants Bureau, New York (1966)

Saini, N.S., Kourakis, I., Hellberg, M.A.: Phys. Plasmas **16**, 062903 (2009)

Sarafopoulos, D.V., Sarris, E.T., Angelopoulos, V., Yamamoto, T., Kokubun, S.: Ann Geophys. **15**, 1246 (1997)

Scudder, J.D., Sittler, E.C., Bridge, H.S.: J. Geophys. Res. **86**, 8157 (1981)

Shizgal, B.D.: Astrophys. Space Sci. **312**, 227 (2007)

Singh, S.V., Lakhina, G.S.: Planet. Space Sci. **49**, 107 (2001)

Singh, S.V., Lakhina, G.S.: Nonlinear Process. Geophys. **11**, 275 (2004)

Tagare, S.G., Singh, S.V., Reddy, R.V., Lakhina, G.S.: Nonlinear Process. Geophys. **11**, 215 (2004)

Thorne, R.M., Summers, D.: Phys. Fluids B **3**, 2117 (1991)

Tokar, R.L., Gary, S.P.: Geophys. Res. Lett. **11**, 1180 (1984)

Tribeche, M., Boubakour, N.: Phys. Plasmas **16**, 084502 (2009)

Tribeche, M., Mayout, S., Amour, R.: Phys. Plasmas **16**, 043706 (2009)

Verheest, F., Cattaert, T., Hellberg, M.A.: Space Sci. Rev. **121**, 299 (2005)

Verheest, F., Hellberg, M.A., Lakhina, G.S.: Astrophys. Space Sci. Trans. **3**, 15 (2007)

Watanabe, K., Taniuti, T.: J. Phys. Soc. Jpn. **43**, 1819 (1977)

Younsi, S., Tribeche, M.: Phys. Plasmas **15**, 073706 (2008)